



Faculty of Sciences

Department of Physics and Astronomy

GENERATION OF SPATIOTEMPORAL PATTERNS IN SPINOR EXCITON-POLARITON CONDENSATES

by

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Abstract

The focus of this thesis is to demonstrate the existence of modulational instability (MI) in nonlinear exciton-polariton condensates (EPCs). Initially, I consider a set of coupled two-dimensional (2D) driven-dissipative Gross-Pitaevskii (GP) equations that describe the spin-up and spin-down polaritons containing the photonic spin-orbit (SO) coupling terms and a pair of rate equations describing the density of incoherent excitonic reservoir density. Using the adiabatic approximation and small density fluctuations approximation, I derive a new form of the system equations for the complex polariton order parameter. The new system equations are just two coupled 2D complex Ginzburg-Landau (2DCGL) equations that both feature saturable nonlinearities. The analytical approach relies on the linear stability analysis of continuous waves (CWs) to derive an expression for the growth rate and conduct a parametric study of MI. The effect of different parameters on the growth rate spectrum is discussed with an emphasis on the photonic SO coupling and the pumping power. The predictions are verified against a direct simulation of the 2DCGL equations, and excellent agreement is found. The emergence of solitonic clusters manifests the evolution of MI. Phase diagrams on a MI spectrum for the CWs are presented against the magnetic field (B) and SO coupling strength (σ), based on which the dynamical behaviors of the emerging structures are debated, along with their response to changing σ and B . The results suggest that the photonic SO coupling and the magnetic field constitute efficient tools for nonlinear mode selection and characterization via the MI process and are equally important in any eventual experimental realization of such a process in the studied EPC system.

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List of Acronyms

AI Artificial Intelligence

BEC Bose-Einstein Condensate

DBR Distributed Bragg Reflectors

DDGPE Driven Dissipative Gross-Pitaevskii Equation

EP Exciton-polariton

GPE Gross-Pitaevskii Equation

MI Modulational Instability

ODGPE Open Dissipative Gross-Pitaevskii Equation

QW Quantum Well

SNL Saturable Nonlinearity

TE-TM Transverse Electric-Transverse Magnetic

SOC Spin-Orbit Coupling

2D Two Dimensional

Chapter 1

General introduction

In 1925, Albert Einstein established the initial prediction of a new artificial state of matter called a Bose-Einstein condensate (BEC) as a consequence of Satyendra Nath Bose's discoveries regarding the Planck rule for black-body radiation [1, 2]. Driven by these predictions, theoreticians interacted closely with experimentalists and ultimately created such a state of matter in a laboratory in 1995 [3]. The Bose-Einstein condensate was made from a gas of rubidium-87 (^{87}Rb) atoms cooled to 170 (10^{-9}K) nano-Kelvin. Later on, BECs of different atoms were demonstrated, namely Sodium-23 (^{23}Na) [4] and Lithium-7 (^7Li) [5]. However, it was not until the year 2006 that BECs of unique materials called microcavity exciton-polaritons were realized [6]. Microcavity exciton-polaritons appear as elementary excitations formed by the strong coupling of a photonic mode of a planar semiconductor and an excitonic resonance in a quantum well embedded in the antinode of a cavity mode [7]. It is worth mentioning that microcavity exciton-polaritons are not ideal bosons since they are a mixture of material excitations (excitons) with light (photons). Also, unlike the atomic BECs, it is possible to have exciton-polariton condensation at room temperature [8]. Additionally, the dynamics of the microcavity exciton-polaritons BECs are nonconservative due to the presence of strong losses of excitons and photons due to material and cavity imperfections [9].

When J. J. Hopfield was pioneering the work in exciton-polaritons in the 1950s with his work on exciton-polaritons in quantum theory [10], he could not have foreseen that it would eventually marry into Albert Einstein's work and offer any useful real-world application. Today, the people working in the exciton-polariton research field appear to be only limited by imagination when it comes to formulating functionalities for exciton-polaritons. In general, I really think some of the most exciting possibilities are in information processing devices because of their huge impact on various industries, including research, and everyday life. At the time that I am writing this thesis, deep neural networks implemented through electronic computers are excelling in performing a variety of tasks, including image and speech recognition, language processing,

and autonomous driving. However, electronic computers appear to be reaching the physical limits of *Moore's law* [12]. Moore's law describes a trend first observed by Gordon Moore whereby methods of electronic computer fabrication involved making components faster, smaller, more energy efficient, and cheaper [11]. The reason why *Moore's law* is coming to an end is that there is a fundamental size limit to how small transistors can be manufactured as they are made of atoms. Also, cooling the temperatures of transistors in very small circuits is inefficient as more energy is needed to cool the system than to actually perform the computations. Additionally, in the von Neumann computer architecture, which requires data transmissions back and forth between memory and computing units there exist limitations to data storage and retrieval speeds [13]. The end of Moore's law has prompted considerations of switching to a new superior computing paradigm characterized by high speeds, and offering intrinsic parallelism as quantum bits in quantum computing with the capacity to efficiently solve tasks of significant computational complexity [14].

The link between exciton-polaritons condensates and information processing is that theoretical and experimental descriptions of the nonlinear effects in exciton-polariton condensates suggest that exciton-polariton condensates may be an ideal platform for the implementation of advanced physical interfaces for the new superior computing paradigm that is needed. Recent realizations of artificial neural networks using exciton-polaritons have shown that exciton-polariton systems can potentially provide orders-of-magnitude improvements in accuracy, speed, and energy efficiency as compared to electronic and optical systems [15]. It was also shown that exciton-polariton can perform as a type of recurrent neural network called reservoir computing [16, 17] which does not suffer the limitations of the von Neumann computing architecture. Artificial intelligence (AI) based on optical systems is ultrafast and consumes low power compared to electronic systems [18]. However, optical systems suffer a great hindrance by low nonlinearities, while exciton-polaritons on the other hand are light-matter and have strong nonlinearities, and their photonic part still allows for high operational speeds and connectivity making them a superior platform for the realization of ultrafast low power AI [19].

The spatial localization of the steady state of the exciton-polariton BEC is called a dissipative vortex soliton [20]. The topological properties and robustness against decoherence of exciton-polariton vortices [21] plus predictions that exciton-polaritons are capable of entanglement [22] make them valuable candidates for quantum computing applications [23] at room temperature conditions. But first, we need to explore how best they can be harnessed for quantum bit manipulation, storage, and quantum communication. Since the possibility of preferential selection of vortex states through the application of an external magnetic field has been demonstrated [24] the interest in exciton-polaritons for quantum computing has grown significantly. Despite massive progress in these directions [19, 23, 25–29], the dynamical behavior of EPCs is still not completely understood. Generally, this is the only thing that

prevents the demonstration of optimal deployment of EPCs in a physically realistic system that can perform significant computational tasks.

Furthermore, the exciton-polariton BEC system considered throughout this thesis is subject to other extremely rich possible applications in the field of Astrophysics. Thinking of a scalar condensate as a model universe that can support the emergence of black-holes, then one can extend this to think of a spin-orbit coupled two component EP spinor condensate as two universes coupled by some parameters. Thus, vortex and anti-vortex pairs in the spin-up and spin-down components of the spinor EP condensates respectively allowing for the possibility to simulate astrophysical entities such as black-holes and inter-Universe and intra-Universe wormholes [30]. This system of two *wormholes* in two opposites allows for exploration and testing theories of *faster-than-light* signal propagation. As a matter of fact, the possibility of modeling and simulating galactic dark matter halos has been demonstrated using BECs [31, 32]. Another possibility is the realization of a hybrid polariton quantum gyroscope that could function as a seismometer to detect earthquakes, gravitational waves, and the rotation rate of our universe which has also been investigated in detail [33]. The key ingredient of these novel devices is the vortex superposition states.

Indeed, EP BECs are versatile objects. However, the best applications for EP BECs may still be out of reach for now because their underlying physics is not entirely understood. The best way to deploy exciton-polariton systems for applications will most likely be outrageously counter-intuitive from today's viewpoint because it is impossible to figure out the connections through the limited knowledge that exists now. Thus, I hold the perspective that, in order to effectively exploit the dynamics of a system for any specific application, one needs to utilize all the available tools in the most efficient manner. It is through this procedure of first commanding utmost knowledge in the simplest considerable systems and then proceeding to much more complicated ones, that are less obvious that holds the promise of progress in the right direction in the prospect of exploiting the rich dynamics of EP BECs for computational and pattern recognition tasks, as simulators or any other applications.

In recent years much has been done in the revelation of the underlying physics that drives the formation of localized patterns in EP BEC. In the present thesis, I build on these foundational works [9] and further the work in the research area of exciton-polariton condensates. I address a fundamental physical problem of understanding the formation dynamics, stability, and controllability of solitonic clusters in spin-orbit coupled exciton-polariton condensates. In the presence of optically defined radially symmetric annular ring-shaped trapping potentials under spatially homogeneous incoherent pumping there are clusters of bright solitons and composite clusters of dark solitons. In the context of applications of exciton-polaritons condensates, this study presents powerful tools for the on-demand generation, manipulation,

and control of solitonic states emerging in the system.

1.1 Thesis overview

The thesis consists of an introduction, three main chapters, a conclusion, an appendix section for additional resources, and finally a bibliography. It is organised as follows:

- **Chapter 1** gives a brief description of what the thesis is about, covering the thesis organization, the background of the general problem and the justification and relevance of the study. The thesis objectives are defined as well as the research methodology.
- **Chapter 2** gives a detailed introduction to EP systems, models used to describe them, parameters, and a review of some phenomena that are of main interest in this thesis.
- **Chapter 3** presents a study of MI phenomena in 2D coupled cubic CGL equations with saturable nonlinearities describing spinor EP BECs.
- **Chapter 4** presents numerical experiments via direct numerical integration of the cubic 2DCGL partial differential equations with saturable nonlinearities.
- **Chapter 5** leads to concluding remarks, with a summary of the main results of our work, and new perspectives are finally given.

1.1.1 General objectives

Study the spatiotemporal instabilities in coupled open dissipative spin-orbit coupled EP BECs.

1.1.2 Specific objectives

- Derive a system of two coupled 2D cubic dissipative Gross-Pitaevskii equations with saturable nonlinearities for EPCs under adiabatic approximation.
- Evaluate the behaviours of the growth rate spectra under the combined effects of pumping powers, the decay rate of incoherent excitons, different spin-orbit strengths, and different natures of interactions between polariton condensate with incoherent excitons.
- Compare the direct numerical simulation of the 2DCGL equations that we have derived with the linear stability analysis predictions.

1.2 Limitations of research objectives

Conceptually, studies in this thesis are comparable to some that were considered in [34]. However, one can insist on revisiting and extending such studies, now considering some additional physical parameters and taking into consideration more recent details that have been elucidated for instance the presence of TE-TM splitting parameter as well as the effects of the external magnetic field. Not only does this make previously obtained results more transparent, but it also promotes a clear physical understanding of system dynamics.

Secondly, the treatment of these problems may recover results that are well known, for instance, the Bogoliubov excitation spectrum of EP is known. Yet, it is still of interest to us to derive it again in more detail now.

Lastly, the effect of transverse-electric and transverse-magnetic (TE-TM) splitting in spinor exciton-polariton condensates was studied in [35]. However, this awakened criticism [36] and a later retaliation [37] where it was concluded that the stability of solitons and effects of TE-TM in EPCs is an extremely difficult task and that the papers involved in this debate could not provide a definitive answer to. Therefore, as far as I am concerned there is still a lot to be discovered regarding the stability and effects of TE-TM in exciton-polariton condensates.

1.3 Methodology

The adiabatic elimination [38] procedure was applied to derive the 2D coupled CGL equations with saturable nonlinearities (SNLs). Then, the linear stability analysis procedure was extensively used in Chapter Three to derive the Modulational instability criterion. The majority of code used for plotting Modulational Instability growth rate spectra on different settings was written in MATLAB and Python programming languages. The modeling of the systems of 2D coupled CGP equations with SNL involved the implementation of Split-Step Fourier transform techniques in MATLAB.

Chapter 2

Theoretical framework

2.1 Introduction

This chapter introduces the exciton-polariton (EP) system, and discusses mathematical descriptions of the exciton-polariton BEC physical system. Next, I describe the properties of EPs and their physical meaning. Then, I introduce spin-orbit coupling and explain the EP condensation mechanism. From there, I introduce the Gross-Pitaevskii equations (GPEs) and describe the effects of pumping powers and magnetic field in EPC systems. I then describe some works regarding Bogoliubov excitations in the EPC system and end the chapter with a summary.

2.2 Excitons

In a semiconductor material, it is possible, on one hand for a photon possessing energy greater than the energy gap to be absorbed. This results in the promotion of an electron from the valence band to the conduction band, which leaves a hole in the valence band. The excited electron obviously has a negative charge while the presence of a hole is interpreted as a positive charge. The entire system is referred to as *electron-hole pair*.

On the other hand, if the photon possesses energy slightly less than the energy gap, it is still possible for the incoming photon to be absorbed. However, the excited electron will not be completely promoted to the conduction band but stays just below it and remains coupled to the hole. This *hybrid* electron-hole pair is called an **exciton**. The effective mass of an exciton can be found intuitively by summing the masses of its constituents i.e., the electron and the hole ($m_\chi = m_e + m_h$). The exciton energy of the exciton is given by [39, 40]:

$$E_\chi(K, n) = E_g - E_b(n) + \frac{\hbar^2 K^2}{2m_\chi}, \quad (2.1)$$

where K wavevector of the center of mass, E_g is the energy gap, E_b represents the binding energy originating from the Coulombian interaction and

n represents the electronic density. The binding energy can be written as follows [40]:

$$E_b(n) = \frac{\hbar^2}{2m_\chi a_B n^2} = \frac{R_y^*}{n^2}, \quad (2.2)$$

with R_y^* being an effective Rydberg constant, a_B is the Bohr radius since the exciton is comparable to the hydrogen atom and is given as:

$$a_B = \frac{\hbar^2 \epsilon}{e^2 m_\chi}, \quad (2.3)$$

where ϵ is the dielectric constant of the medium, and e is the elementary charge. Semiconductors like GaAs have a high dielectric constant. Typically $a_B \approx 50 \text{ \AA}$. As a result, the binding energy in such dielectric medium reaches a few meV ($E_b \approx 4.5 \text{ meV}$ for GaAs) [48].

2.3 Microcavity system

A **Fabry-Perot cavity**, one of the most basic optical cavities was created in 1899 by Charles Fabry and Alfred Perot. Its design consists of two planar mirrors or Distributed Bragg Gray Reflectors (DBRs) separated by some distance (λ_c), parallel and facing each other. Photons with a specific wavelength are trapped within this system and go back and forth between the two DBR mirrors. A quantum well is sandwiched between the pair of DBRs as shown in Fig. 2.1.

Usually, the DBRs design consists of alternating layers of material (GaAs/AlAs) as illustrated in Fig. 2.2 and to get **reflectivity** (R) values very close to one, about 25 pairs are reportedly needed. The thicknesses and optical indexes ($n_a \neq n_b$) are engineered such that for light at a given angle of incidence and at the target wavelength (λ_0), the transmission is almost zero. Energy decay rate, which is a result of losses due to photons tunneling through the DBRs, absorption, or scattering because of the cavity imperfections can be related by the **Q-factor** given by [48]:

$$Q = \frac{\omega_{cav}}{\delta\omega_{cav}}, \quad (2.4)$$

where ω_{cav} is the resonant frequency and $\delta\omega_{cav}$ is the full width at half maximum (FWHM) of the microcavity mode. Q^{-1} indicates the fraction/percentage of the energy lost during one microcavity round-trip [42]. The photon population depletes exponentially as a result, and its lifetime is defined as:

$$\tau = \frac{Q}{\omega_{cav}}, \quad (2.5)$$

DBRs with very strong reflectivities translate to microcavities with very high Q factors up to 10^5 , making it possible to achieve photonic lifetimes of up to

several tens pico-seconds. The finesse describes roughly the number of times a photon bounces between the two DBRs before escaping the microcavity. Increasing the **finesse (F)** can increase the photonic lifetimes. The **finesse (F)** can be approximated by the relation:

$$F = \pi \frac{\sqrt{R}}{1 - R}. \quad (2.6)$$

2.3.1 Quantum well excitons

The photon trapped between the pair of DBRs can create an excitation in the QW consisting of electron-hole pairs that can exist as quasi-particles called **QW excitons**. Also, it has been shown that QW excitons are only free to move in the in-plane direction and are confined in the z direction. The dispersion curve for QW excitons is given by:

$$E_\chi(k_{||}, k_z) = E_\chi + \frac{\hbar^2 k_{||}^2}{2m_\chi}, \quad (2.7)$$

where $k_{||}$ is the in-plane momentum. This is illustrated in Fig. 2.3 as a straight horizontal line for smaller k_z . Also, a QW exciton can decay, emitting a photon. The sum of angular momentum of the electron and hole pair forming the exciton is called the exciton spin (S^X). Electrons can have spin projections $\sigma_e = \pm 1/2$ along the growth axis z , while holes can have $\sigma_h = \pm 3/2$. Therefore exciton spin can have five possible values, $S^X = \pm 2, \pm 1, 0$. The selection rules and conservation laws same as electrons and holes still apply which means that dark excitons having $S^X = 2$ do not interact with photons [9, 47].

2.3.2 Cavity photons

If the microcavity is excited at an angle (θ_ϕ) with respect to the normal, then the photon is quantized in the k_z wavevector component such that:

$$k_z = 2\pi \frac{n_{cav}}{\lambda_0}. \quad (2.8)$$

The cavity photon energy can be written as

$$E_\phi = \frac{\hbar c}{n_{cav}} \sqrt{(k_x)^2 + (k_{||})^2} = \frac{\hbar c}{n_{cav}} \sqrt{\left(2\pi \frac{n_{cav}}{\lambda_0}\right)^2 + (k_{||})^2}, \quad (2.9)$$

which gives a parabolic dispersion, as illustrated in plotted in Fig. 2.3. From the dispersion curve in Eq. (2.9), the effective mass for cavity photons can be determined and is as follows:

$$m_\phi = \frac{n_{cav}^2 \hbar}{\lambda_0 c}. \quad (2.10)$$

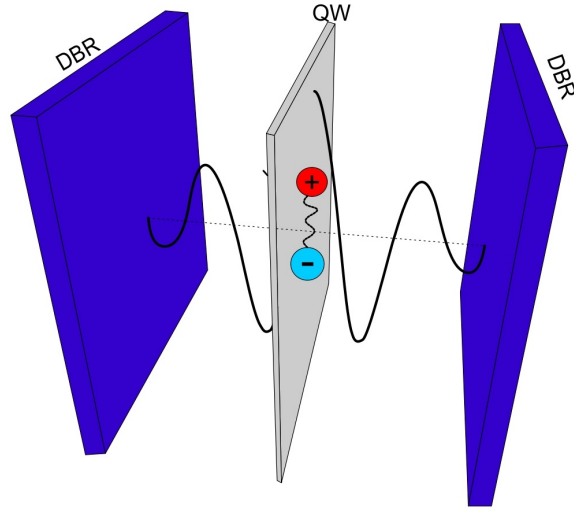


Figure 2.1: A simple construction illustration of a microcavity system showing a quantum well sandwiched by two distributed Bragg reflectors and an exciton and a photon trapped in the system.

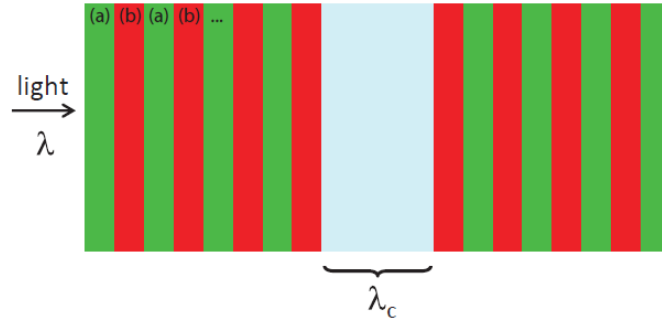


Figure 2.2: Schematic view of a microcavity. A distributed Bragg reflector mirror itself is made of two (a and b) distinct material layers stacked in alternating order. The separation between the distributed Bragg reflectors is $\lambda_c = q \frac{\lambda_0}{2n_c}$. Here q is an integer, n_c is the refractive index of the cavity medium, λ_0 is the wavelength at normal incidence, and $\lambda_c \approx 850$ nm. Adapted from [48]

It is worth mentioning that if the angle of incidence is too high, the reflectivity will dramatically drop, so the cavity photons will not be confined efficiently anymore due to the reduced finesse [41]. This is important because the idea is to effectively confine the photons and minimize losses in the system. The role of cavity photons in the exciton-polaritons systems will be clarified in the subsequent sections.

2.3.3 Microcavity exciton-polaritons

Microcavity exciton-polaritons are bosonic hybrid quasi-particles resulting from the quantum superposition of QW excitons and cavity photons. It is usually accepted to refer to these quasi-particles as polaritons.

These quasi-particles emerge when the exciton-photon coupling is strong and

the exciton and the cavity photon modes are both dressed to form new eigenstates of this hybrid system [9, 43]. The first experimental observation of such strong coupling in a QW embedded in a microcavity system was reported in 1992 [44]. The reflectivity spectra revealed the formation of new eigenstates named cavity exciton-polaritons. However, as I mentioned in the introduction the history of microcavity EPs actually dates back to the pioneering work in quantum theory of exciton-polaritons in the 1950s [10].

2.3.4 Quantum mechanical description of exciton-polaritons

With the help of a quantum mechanical approach, based on the formalism of the second quantization theory, it is possible to describe the mixing exciton-photon by treating the system as two coupled oscillators and writing the Hamiltonian system as follows [45]:

$$\mathcal{H} = \sum_k E_\phi(k) \phi_k^\dagger \phi_k + \sum_k E_\chi(k) \chi_k^\dagger \chi_k + h\Omega_R \sum_k E_\phi(k) \left(\phi_k^\dagger \chi_k + \chi_k^\dagger \phi_k \right), \quad (2.11)$$

where $\phi^\dagger$ and ϕ_k are the second quantization bosonic operators that describe the creation destruction and creation operators for the photon and likewise, $\chi^\dagger$ and χ_k are destruction and creation operators for the exciton. The parameter $|k| = \sqrt{k_x^2 + k_y^2 + k_z^2}$, while $E_\phi(k) = \hbar k^2/2m_\phi + \delta$ and $E_\chi(k) = \hbar k^2/2m_\chi$, describe the parabolic dispersions of the photon and exciton, respectively. Here, δ is the detuning parameter and Ω_R is the Rabi frequency accounting for the coupling strengths for the fields.

The Hamiltonian has a matricial form:

$$M = \begin{bmatrix} E_\phi(k) & h\Omega_R \\ h\Omega_R & E_\chi(k) \end{bmatrix}, \quad (2.12)$$

and can be easily diagonalized in a basis that will be named the polariton basis. The matrix (2.12) has two eigenvalues which correspond to the dispersions of lower and upper exciton-polariton branches. The in-plane dispersion relation for the two eigenstates (upper and lower exciton-polariton) is given by the following:

$$E_k = \frac{1}{2}(E_\phi(k) + E_\chi(k)) \pm \sqrt{[E_\phi(k) - E_\chi(k)]^2 + 4h^2\Omega_R^2}. \quad (2.13)$$

Thus, **exciton-polaritons are a quantum superposition composed of quantum well (QW) excitons and cavity photons and can be described by the quantum superposition of light and matter.** The lower polariton mode, which this thesis will be focused on has the description:

$$\Psi_{LP} = |X_\chi|\chi + |X_\phi|\phi, \quad (2.14)$$

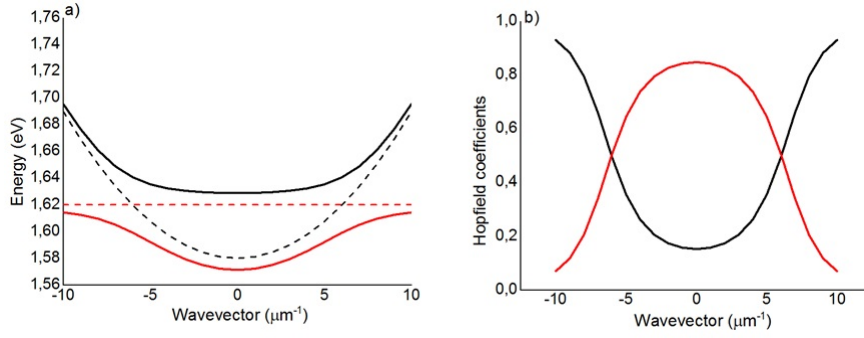


Figure 2.3: (a) Plot of energy as a function of the wavevector in a planar microcavity. The upper (UP) and lower Polariton (LP) dispersion branches correspond to those described by Eqs. (2.13) and $\Delta E = -40 \text{ meV}$. The black and red dotted lines represent cavity photon and exciton dispersions respectively. Also, (b) the corresponding Hopfield coefficients are plotted as a function of the wavevector. Adapted from [46]

where the Hopfield coefficients satisfy the normalization condition $|X_\chi|^2 + |X_\phi|^2 = 1$.

Because exciton-polaritons are composite bosonic quasi-particles i.e., half-light and half-matter, they inherit properties from photons and excitons have interesting overall properties such as extremely small effective mass (m^*) given by:

$$\frac{1}{m^*} = \frac{1}{m_\phi} + \frac{1}{m_\chi}, \quad (2.15)$$

The effective mass of exciton-polaritons is typically around $5 \times 10^{-5} m_e$. Another important thing is strong non-linear optical response from their excitonic component (see nonlinear interaction coefficient (α_1) in exciton field in Eq. (2.20)) which allow them to interact with each other through Coulomb interactions.

2.3.5 Separate exciton and photon fields representation

It is possible to describe the coupled dynamics of exciton-photon more accurately by considering the energy of the system which reads [45]:

$$E = \int \left[\frac{\hbar^2}{m_\phi} |\nabla \phi|^2 + \frac{\hbar^2}{m_\chi} |\nabla \chi|^2 + \frac{\alpha}{2} |\chi|^4 + \frac{\hbar \Omega_R}{2} (\chi^* \phi + \chi \phi^*) \right] dr, \quad (2.16)$$

here, the photonic field is given by $\phi(r, t)$, and $\chi(r, t)$ represents the excitonic field. Parameters m_ϕ and m_χ are the effective masses of the photon and exciton. The nonlinear interaction term $\alpha = 6E_b a_B^2 / \mathcal{S}$, where $\mathcal{S}$ is the normalization surface, E_b is the exciton binding energy and a_B is the corresponding Bohr radius. Also, it is worth mentioning that $m_e \gg m_\phi$, where m_e is the mass of a free electron. The dynamics of the photonic and excitonic fields can be found in the following Lagrange equations:

$$i\hbar \frac{\partial \phi}{\partial t} = \frac{\partial E}{\partial \phi^*}, \quad (2.17)$$

$$i\hbar \frac{\partial \chi}{\partial t} = \frac{\partial E}{\partial \chi^*}, \quad (2.18)$$

to obtain the following coupled set of modified Schrödinger equation and Gross-Pitaevskii equations.

$$i\hbar \frac{\partial \phi}{\partial t} = - \left[\frac{\hbar^2}{2m_\phi} \nabla^2 + i \frac{i\hbar}{2\tau_\phi} \right] \phi + \frac{\hbar}{2} \Omega_R \chi + P, \quad (2.19)$$

$$i\hbar \frac{\partial \chi}{\partial t} = - \left[\frac{\hbar^2}{2m_\chi} \nabla^2 + i \frac{i\hbar}{2\tau_\chi} \right] \chi + \frac{\hbar}{2} \Omega_R \phi + \alpha_1 |\chi|^2 \chi, \quad (2.20)$$

The parameters τ_ϕ and τ_χ account for the lifetimes of the cavity photons and excitons, respectively. EPs have a lifetime in the order of pico-seconds (typically $\tau_\phi \approx 10\text{ps}$ and $\tau_\chi \approx 400\text{ps}$) [9]. Finally, P accounts for the optical pump injecting photons into the system.

2.3.6 TE-TM splitting and photonic spin-orbit coupling

The longitudinal-transverse splitting, also called TE-TM splitting refers to an energy splitting between transverse-electric (TE) and transverse-magnetic (TM) polarized polaritons which is observed in the dispersion of exciton-polaritons for oblique angles (see Fig. 2.4). In order to explain the associated TE and TM splitting in exciton-polariton branches, a model of four oscillators (TE- and TM-polarized excitons and photons) [45], each of them having their own respective mass and dispersions can be considered:

$$E_\phi^{TE}(k) = \frac{\hbar k^2}{2m_\phi^{TE}} + \delta, \quad (2.21)$$

$$E_\chi^{TE}(k) = \frac{\hbar k^2}{2m_\chi^{TE}}, \quad (2.22)$$

$$E_\phi^{TM}(k) = \frac{\hbar k^2}{2m_\phi^{TM}} + \delta, \quad (2.23)$$

$$E_\chi^{TM}(k) = \frac{\hbar k^2}{2m_\chi^{TM}}. \quad (2.24)$$

The matrices of the system are:

$$M_{TE} = \begin{bmatrix} E_\phi^{TE}(k) & \hbar \Omega_R \\ \hbar \Omega_R & E_\chi(k)^{TE} \end{bmatrix}, \quad (2.25)$$

and

$$M_{TM} = \begin{bmatrix} E_\phi(k)^{TM} & \hbar \Omega_R \\ \hbar \Omega_R & E_\chi(k)^{TM} \end{bmatrix}, \quad (2.26)$$

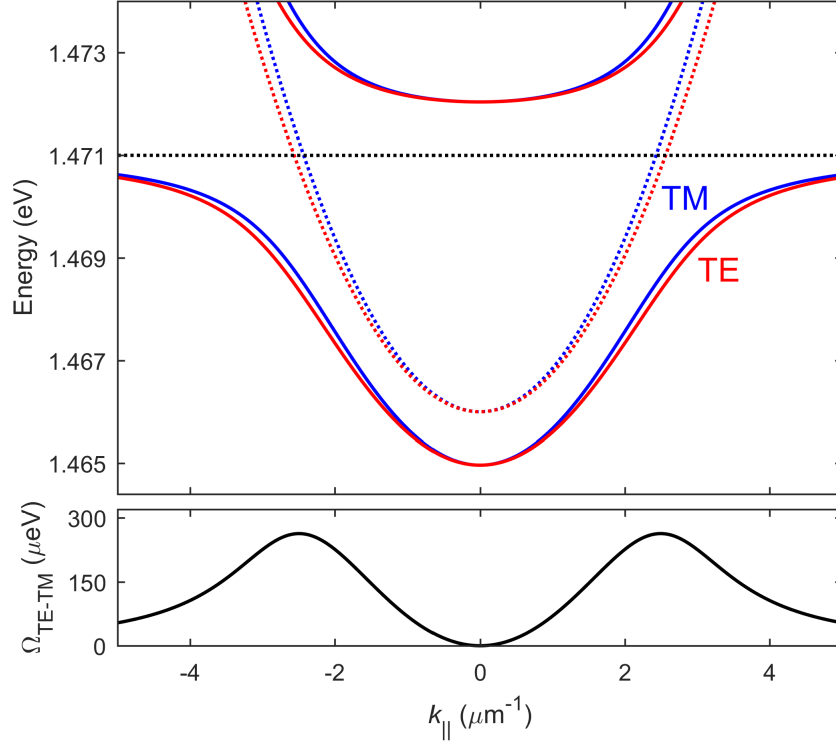


Figure 2.4: (Upper panel) typical polariton dispersion relation in the presence of large TE-TM splitting in a negatively-detuned cavity. (Lower panel) Shows the dependence of TE-TM splitting strength on in-plane wave vector

The associated TE and TM exciton-polariton branches are found from the eigenvalues of the matrices which give four exciton-polariton branches:

$$E_k = \frac{1}{2}(E_{\phi}^{TE}(k) + E_{\chi}^{TE}(k)) \pm \frac{1}{2}\sqrt{[E_{\phi}^{TE}(k) - E_{\chi}^{TE}(k)]^2 + 4\hbar^2\Omega_R^2}, \quad (2.27)$$

$$E_k = \frac{1}{2}(E_{\phi}^{TM}(k) + E_{\chi}^{TM}(k)) \pm \frac{1}{2}\sqrt{[E_{\phi}^{TM}(k) - E_{\chi}^{TM}(k)]^2 + 4\hbar^2\Omega_R^2}. \quad (2.28)$$

Then, by only focusing on the lower branches, the strength of the TE-TM splitting can be defined by:

$$\Delta_{LT} = E_L^{TE}(k) - E_L^{TM}(k) \quad (2.29)$$

The lower exciton-polariton branches have slightly different dispersion relations for polaritons polarized longitudinally (TM mode) and transversely (TE mode) with respect to the in-plane k vector as seen in Fig. 2.4 (Upper panel). The magnitude (lower panel Fig. 2.4) of TE-TM splitting increases quadratically, reaching a maximum at the angle corresponding to the point of inflection of the LPB. The difference in dispersions of the two polarization lower exciton-polariton branches means that the TE and TM exciton-polaritons have acquired different effective masses, i.e. $m_{TM} < m_{TE}$. This phenomenon actually has two known origins, the first is from an intrinsic splitting of the cavity

mode caused by an angle and polarization-dependent phase delay upon reflection from the DBRs [68], and the second is from the wave vector dependence of the long-range exciton exchange interaction [69]. It is worth mentioning that the excitonic TE-TM splitting is usually orders of magnitude lower than its photonic counterpart for this reason, it can be neglected safely. Therefore, making it possible to consider the energy of the noninteracting system that neglects the excitonic TE-TM splitting and obtain the Lagrange equations for the photonic and excitonic fields similar to Eqs. (2.19) and (2.20) that now feature an effective magnetic field term. The recovered dynamical equations allow for the derivation of the TE-TM term analytically [45]. This has also been achieved by considering the Hamiltonian of an interacting polariton system written in the basis of circularly polarized states reads as in [35]. The TE-TM splitting recovered from these studies can be written as:

$$\Delta E_{LT} = \begin{pmatrix} 0 & \beta_{LT}(k_y + ik_x)^2 \\ \beta_{LT}(k_y - ik_y)^2 & 0 \end{pmatrix}, \quad (2.30)$$

where β_{LT} is the effective TE-TM splitting constant given by:

$$\beta_{LT} = \frac{\hbar^2}{4} \left(\frac{1}{m_l} - \frac{1}{m_t} \right). \quad (2.31)$$

The strength of the TE-TM coupling is given by:

$$\sigma = \left(\frac{m_t - m_l}{m_l + m_t} \right), \quad (2.32)$$

where m_l and m_t are the longitudinal and transverse exciton-polariton masses. Finally, in real space given that $k_{x,y} \rightarrow i\partial_{x,y}$, the TE-TM splitting can be represented by $\beta(\partial_y \mp i\partial_x)^2$. The TE-TM is actually interpreted as a kind of photonic effective spin-orbit coupling (SOC). Actually, this term constitutes an important part of the next chapters.

2.4 Bose-Einstein condensation

A Bose-Einstein condensate (BEC) is an artificial state of matter that is generated by bosons at low densities cooled to cryogenic temperatures. This fundamental principle was proposed by S. N. Bose, then A. Einstein predicted that it is possible for a substantially large fraction of noninteracting Bose gas to undergo a phase transition at ultracold temperatures and occupy the lowest energy state of a system in thermodynamic equilibrium [1, 2]. This phenomenon is known as Bose-Einstein condensation.

As described by Bose statistics, the particle number distribution (n_B) as a function of energy (ε) is given by:

$$n_B(\varepsilon) = \frac{1}{\exp\left(\frac{\varepsilon - \mu}{k_B T}\right) - 1}, \quad (2.33)$$

where k_B is the Boltzmann constant, μ is the chemical potential, and T is the temperature. As I mentioned in the introduction, BECs of different atoms have been realized at extremely low temperatures. In an ideal Bose gas, a BEC emerges when the thermal de Broglie wavelength λ_{dB} becomes comparable to the average interparticle distances. The thermal de Broglie wavelength is given by:

$$\lambda_{dB} = \sqrt{\frac{2\pi\hbar^2}{mk_BT}}, \quad (2.34)$$

where m is the mass of the boson. We can infer that λ_{dB} is inversely related to the mass. Therefore, the critical temperature (T_c) required to obtain a BEC is higher for a boson with a very small effective mass. This suggests that it could be possible for exciton-polaritons to achieve Bose-Einstein condensation at temperatures very close to room temperature. Indeed, exciton-polariton can achieve condensation at room temperature [8].

2.5 Bose-Einstein condensation of exciton-polaritons

After so much effort had been devoted to developing a solid-state system that can support the occurrence of BEC [50], compelling experimental evidence for BEC of exciton-polaritons was finally presented [6]. It was demonstrated that **exciton-polaritons can relax into a macroscopically occupied ground state to create an exciton-polariton condensate**. This is possible for the lower branch exciton-polariton (LP) which is illustrated in Fig. 2.3, and the $k \simeq 0$ region of the dispersion curve. Occupation of the ground state developing from a exciton-polaritons can be seen in fig. 2.5. The process of creating an EPC requires an incoherent optical pump, the role of the pump is to excite (create) excitons which in turn relax into high-energy excitons that eventually feed and maintain the condensate. It is worth noting that, exciton-polariton condensation is accompanied by the formation of an incoherent excitonic reservoir.

This *condensation* of exciton-polaritons is different from standard BEC, because exciton-polaritons are not ideal bosons, and the system is driven far from thermal equilibrium. Moreover, the exciton-polariton system is nonconservative since exciton-polaritons continuously leak out of the system because of microcavity imperfections.

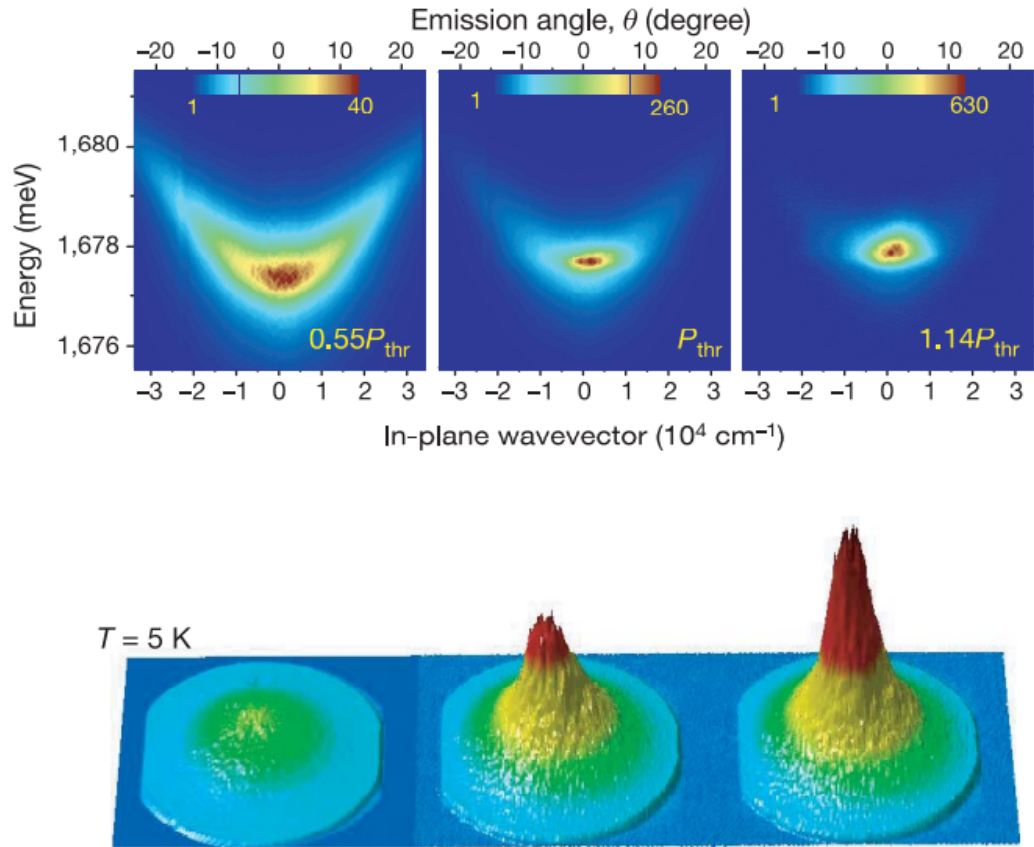


Figure 2.5: Evidence of exciton-polariton condensation. Above pumping threshold, the emission exclusively appears to be coming from lowest energy state. Adopted from [6].

2.6 Dynamical equations for exciton-polariton condensates

The macroscopic fraction of particles that populate the condensate mode at temperature (T) much smaller than the critical condensation temperature (T_c) behave collectively as one and the quantum atomic matter field ψ behaves as a classical field. The properties BECs are therefore modelled by classical partial differential equations resembling nonlinear Schrödinger equation (NLSE) describing the macroscopic wave function called the Gross-Pitaevskii equation (GPE), which incorporates the trap potential as well as the interactions among atoms (in this context, excitons). The GP equation has the form [9]:

$$i\hbar \frac{\partial \psi(r, t)}{\partial t} = \left[-\frac{\hbar^2}{2m} \nabla^2 + V(r) \right] \psi(r, t) + g|\psi(r, t)|^2 \psi(r, t), \quad (2.35)$$

where ψ represents the wave function, ∇^2 is the Laplacian, $r = (x, y, z)$ and t represent the spatial and temporal coordinates respectively, m is the effective mass of exciton-polaritons, $V(r)$ represents the external potential, and g represents polariton-polariton interactions.

2.6.1 Driven dissipative complex Gross-Pitaevskii equations for exciton-polariton Bose-Einstein condensates

Under coherent driving, the classical partial differential equation governing the dynamical properties for the lower branch EP BEC order parameter can be written as [9]:

$$i\frac{\partial \psi_{LP}}{\partial t}(r, t) = \left[\omega_{LP}^o - \frac{\hbar}{2m_{LP}} \nabla^2(r) + V_{LP}(r) \right] \psi_{LP}(r, t) + g_{LP} |\psi_{LP}(r, t)|^2 \psi_{LP}(r, t) - i\frac{\gamma_{LP}}{2}(r, t) \psi_{LP}(r, t) + i\eta_{LP} E^{inc}, \quad (2.36)$$

where m_{LP} is the lower branch EP mass, ω_{LP}^o is the frequency of the bottom of the lower exciton-polariton branch, $V_{LP}(r)$ is the external potential felt by the exciton-polaritons, g_{LP} is the polariton-polariton interaction parameter represented by:

$$g_{LP} = \frac{6E_b |X_\chi|^2 a_B^2}{\mathcal{S}}, \quad (2.37)$$

the η_{LP} parameter quantifies the coupling of the exciton-polariton to incident radiation. The exciton-polariton loss rate is represented by γ_{LP} and the incident field is $E^{inch}(r, t)$.

2.6.2 Open driven dissipative complex Gross-Pitaevskii equations for exciton-polariton Bose-Einstein condensates

As mentioned earlier, there is leakage of photons from microcavities and also transfer between the uncondensed and condensed EP portions. Therefore, it is necessary to address losses and gain within the system. The system in description is now dissipative. Dissipative systems are much more complicated than Hamiltonian systems, in the sense that they take into consideration these extra factors.

In thesis, I will be dealing with condensation under incoherent pumping. Formation of the exciton-polariton BEC in this setting is simultaneously accompanied by formation of a bath of incoherent excitons (Reservoir) n_R . This induces a complex repulsive trapping potential for exciton-polaritons [51, 54, 59].

$$V(x, y) = V'(x, y) + iV''(x, y), \quad (2.38)$$

where

$$V'(x, y) = P(x, y) + g_R n_R, V''(x, y) = R n_R - \gamma_c. \quad (2.39)$$

Where $P(x, y)$ is the pumping γ_c is the condensate relaxation rate, $R n_R$ determines the rate of the exciton-polariton condensate replenishment by the exciton relaxation from the exciton reservoir and g_R is the parameter accounting for interaction between exciton-polariton and the reservoir. The GP equation introduced earlier takes an additional $\dots + \frac{i}{2} R [n_R(r, t)] \psi_{LP}$ terms to now take the form [9]:

$$\begin{aligned} i \frac{\partial \psi_{LP}}{\partial t}(r, t) = & \left[\omega_{LP}^o - \frac{\hbar}{2m_{LP}} \nabla^2(r) + V_{LP}(r) \right] \psi_{LP}(r, t) + g_{LP} |\psi_{LP}(r, t)|^2 \psi_{LP}(r, t) \\ & - i \frac{\gamma_{LP}}{2} \psi_{LP}(r, t) + i \eta_{LP} E^{inc}(r, t) + \frac{i}{2} R [n_R(r, t)] \psi_{LP}(r, t). \end{aligned} \quad (2.40)$$

Where the parameter R couples the condensate and exciton reservoir, describes scattering rate from incoherent excitons into the EP condensate. The incoherent reservoir density equation is of the form:

$$\frac{\partial n_R(r, t)}{\partial t} = P - R [n_R(r, t)] |\psi_{LP}(r, t)|^2 - \gamma_R n_{LP}(r, t), \quad (2.41)$$

here, the term γ_R is the reservoir relaxation rate and $R |\psi(r, t)|^2 n_R(r, t)$ describes the scattering of reservoir polaritons into the condensate. Even though they are similar, there exists a more commonly used form/version of system equation in literature which is as follows [59]:

$$i \frac{\partial \psi}{\partial t} = \frac{1}{2m} \nabla^2 \psi + \frac{i}{2} (R n_R \psi - \gamma_c) \psi + \tilde{g} n_R + g |\psi|^2 \psi, \quad (2.42)$$

$$\frac{\partial n(r, t)}{\partial t} = P(r, t) - (\gamma_R + R |\psi|^2) n, \quad (2.43)$$

In words, the condensate is described by a system of two equations as the GP equation is now coupled to a rate equation of the exciton reservoir. A straightforward linear stability analysis [59] of this model provides remarkably detailed insights about the condensation of EPs. The spontaneous emergence of the EP condensate is mostly governed by the pumping powers. The incoherent pumping must compensate for the radiative losses for condensation to occur. Simply put, EP condensation is only possible when the pumping power is greater than the threshold value. For pumping values below the threshold ($P > P_{th}$), the uncondensed solution $|\psi_0|^2 = 0$, exists with $n_0 = P/\gamma_R$. Beyond the pumping threshold $P_{th} = (\gamma_c/\gamma_R)/R$, the condensate with density $|\psi_0|^2 = (P/\gamma_c) - (\gamma_R/R)$ appears and the exciton density now becomes $n_0 = \gamma_c/R$. Indeed, this is consistent with the observation in Fig. 2.5, where the EP condensate at $k \simeq 0$ is almost perfectly defined for the pumping powers greater than the threshold.

2.6.3 Adiabatic approximation in the Gross-Pitaevskii equations

Under the assumption adiabatic approximation is fulfilled, it is possible to reduce the open-dissipative Gross-Pitaevskii model Eqs. (2.40) and (2.43) to a much simpler model equation [9] by expressing n_R as:

$$n_R = \frac{P(r, t)}{[\gamma_R + R |\psi_{LP}(r, t)|^2]}, \quad (2.44)$$

2.6.4 Validity of the adiabatic approximation

The adiabatic approximation is said to be fulfilled when the reservoir density n_R is able to quickly adjust to the condensate density $|\psi(r, t)|^2$ (*adiabatically following the density*). Indeed, the reservoir density n_R is able to quickly adjust to the condensate density $|\psi(r, t)|^2$ provided the exciton-polariton condensate $|\psi(r, t)|^2$ is constant, when this happens Eq (2.43) has the solution [38]:

$$n_R = \frac{P(r, t)}{\gamma_R + R |\psi_{LP}(r, t)|^2} + C e^{-(\gamma_R + R |\psi(r, t)|^2)t}. \quad (2.45)$$

and the timescale of the reservoir is then given by:

$$\tau_R = \frac{1}{(\gamma_R + R |\psi(r, t)|^2)}. \quad (2.46)$$

The limits of the validity of the adiabatic approximation in the ODGPE have been investigated by comparing the characteristic timescales existing in the

dynamical system [38]. It was discovered that for the adiabatic approximation to be consistent all the terms in Eq. (2.42) necessarily need to be smaller than $\hat{\tau}_R = \tau_R^{-1}$; namely $\frac{\hbar k^2}{2m}$, $\frac{1}{2}[Rn_R - \gamma_c]$, and $g_R n_R$ must be smaller than $\hat{\tau}_R$.

then Eqs. (2.40) and (2.43) become:

$$i\frac{\partial\psi_{LP}}{\partial t}(r,t) = \left[\omega_{LP}^o - \frac{\hbar}{2m_{LP}}\nabla^2(r) + V_{LP}(r) \right] \psi_{LP}(r,t) + g_{LP} |\psi_{LP}(r,t)|^2 \psi_{LP}(r,t) - i\frac{\gamma_{LP}}{2}\psi_{LP}(r,t) + \frac{iP(r,t)/2}{\gamma_R + R|\psi_{LP}(r,t)|^2}\psi_{LP}(r,t), \quad (2.47)$$

This equation is the complex GP equation with saturable nonlinearities. It worth noting that some authors refer to it as the complex Ginzburg-Landau equation (CGLE) for the exciton-polariton condensates. The CGL equation is a well known equation in the theory of pattern formation in nonlinear dynamical system [9].

Infact, the adiabatic approximation is put into action in this next Chapter. Also, with the assumption that the spin relaxation time is much shorter than all other scales, adiabatic approximation can be particularly useful [52].

2.6.5 Optical pump

Now, the role of the pump in formation of the EP condensate is established. The next step is to study the dependence of EP condensate structure on optical pumping profiles, intensities and pumping positions. In these studies, a remarkable degree of control of EP condensate structure has been demonstrated [53]. Furthermore, It has been demonstrated that through varying intensity and the spatial pump profile, it is possible to access a rich class of solitons [20] in exciton-polariton condensates. In the next chapters where I am interested in explaining the physical origin of spin-orbit solitons in the EP system, I shall consider a radially symmetric annular pumping profile since it was demonstrated that it can support the formation of spin-orbit coupled topological solitons [24]. The radially symmetric annular pumping profile I am considering has the form:

$$P = P_0 \left[\exp\left(-\left(\frac{r - R_p}{W_p}\right)\right) + \exp\left(-\left(\frac{r + R_p}{W_p}\right)\right) \right], \quad (2.48)$$

Where R_p is the radius of the pump, W_p is the width of the pump, and $r = \sqrt{x^2 + y^2}$. The pump profile offers some interesting dynamics, as creates a bath of incoherent excitons which produces both a ring trapping potential and positive gain for coherent exciton-polariton condensate located in the center of the ring [54]. The ring shape is seen in Fig. 2.6, it is this geometrical distribution of intensity that allows the condensate to accumulate toward the center (also see Fig. 2.7).

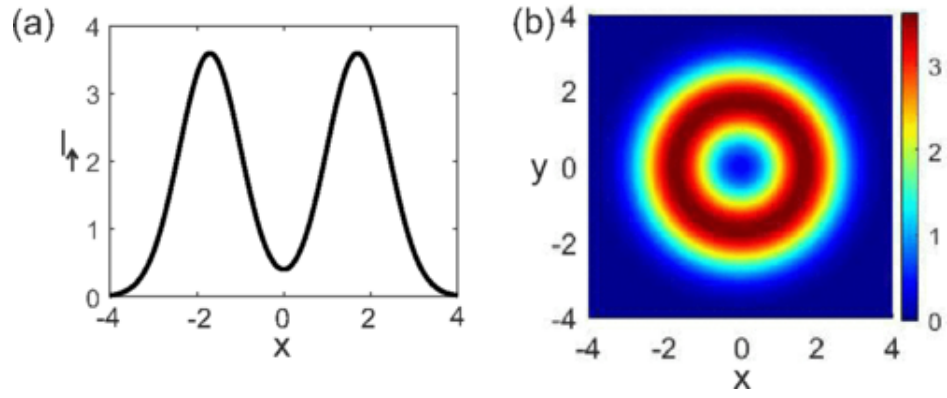


Figure 2.6: The distribution of the intensity of the external pump. Adopted from [24].

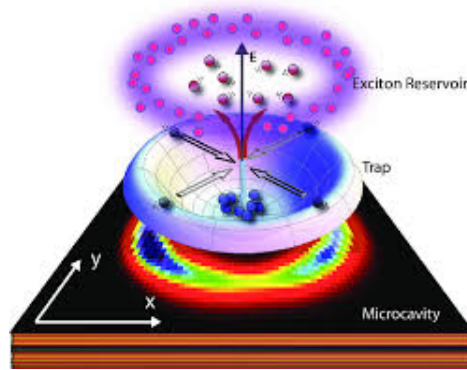


Figure 2.7: Illustration of the exciton-polariton self trapping mechanism. Adopted from [54].

2.6.6 Magnetic field

Application of an external magnetic field on a spinor condensate affects the system's excitation spectrum as:

$$E_\chi(B) = E_\chi \pm \frac{g\mu_B}{2}B, \quad (2.49)$$

where B is the applied magnetic field, g is the polariton g -factor, and μ_B is the Bohr magneton. The bare Zeeman splitting is given by:

$$2\Omega = g\mu_B B. \quad (2.50)$$

Making this substitution of Eq. (2.49) into Eq. 2.13, one gets the expression for the Zeeman splitting of the energy levels of EP condensate in the LP branch:

$$E_{LP}(B) = \frac{1}{2}(E_\phi(k) + E_\chi(B)) \pm \sqrt{[E_\phi(k) - E_\chi(B)]^2 + 4\hbar^2\Omega_R^2} \quad (2.51)$$

Also, the application of a magnetic field has a suppressing effect on the superfluidity of the EP BEC [55]. Furthermore, beyond the critical magnetic field $\Omega_c = \frac{g\mu_B B_c}{2}$, the EP condensate becomes fully circularly polarized. It is worth mentioning that the spontaneous circular polarization of a nonequilibrium condensate of exciton-polaritons is related to a phase synchronization of the spin-up and spin-down polarized EP condensates [56–58].

2.6.7 Two component open driven dissipative Gross-Pitaevskii equations

It has been known that exciton-polaritons have two allowed projections on the structure growth z -axis making it a *spinor condensate*. Thus, the order parameter of the two component condensate should be written as a complex two component vector $\psi = [\psi_\uparrow, \psi_\downarrow]^T$ of the 2D coordinates and time. I can now present the most general form of the Open-Dissipative Gross-Pitaevskii equations that will be used throughout this thesis which reads as follows [24]:

$$i\hbar \frac{\partial \psi_\uparrow}{\partial t} = \left[-\frac{\hbar^2}{2m^*} \nabla^2 + i\frac{\hbar}{2}(R\rho_\uparrow - \gamma_c) + g_R\rho_\uparrow + \tilde{g}_R\rho_\downarrow + \frac{g_e\mu_B}{2}B + g_c|\psi_\uparrow|^2 + g_x|\psi_\downarrow|^2 \right] \psi_\uparrow \\ + \beta \left(\frac{\partial}{\partial y} + i\frac{\partial}{\partial x} \right)^2 \psi_\downarrow, \quad (2.52)$$

$$i\hbar \frac{\partial \psi_\downarrow}{\partial t} = \left[-\frac{\hbar^2}{2m^*} \nabla^2 + i\frac{\hbar}{2}(R\rho_\downarrow - \gamma_c) + g_R\rho_\downarrow + \tilde{g}_R\rho_\uparrow - \frac{g_e\mu_B}{2}B + g_c|\psi_\downarrow|^2 + g_x|\psi_\uparrow|^2 \right] \psi_\downarrow \\ + \beta \left(\frac{\partial}{\partial y} + i\frac{\partial}{\partial x} \right)^2 \psi_\uparrow, \quad (2.53)$$

$$\frac{\partial \rho_{\uparrow}}{\partial t} = P_{\uparrow} - (\gamma_R - R|\psi_{\uparrow}|^2)\rho_{\uparrow}, \quad (2.54)$$

$$\frac{\partial \rho_{\downarrow}}{\partial t} = P_{\downarrow} - (\gamma_R - R|\psi_{\downarrow}|^2)\rho_{\downarrow}. \quad (2.55)$$

The TE-TM splitting or SOC (see Eq. (2.31)) induces some exchange and dependence between $\psi_{\uparrow}$ and $\psi_{\downarrow}$ wavefunctions of the spin-up and spin-down componen. The coefficients, g_c accounts for nonlinear interaction between exciton-polaritons belonging to the same polarizations and g_x account exciton-polariton interaction with exciton-polaritons of the orthogonal polarization. Similarly, g_R is the coefficient of the exciton-polariton interaction with the reservoir excitons of the same polarization and $\tilde{g}_R$ accounts for cross polarizations interactions between exciton-polaritons and reservoir excitons.

2.7 Bogoliubov excitations in exciton-polariton condensates

Bogoliubov excitations of EPCs have been found in the scalar GP equation [59] which takes the form:

$$i\frac{\partial \psi}{\partial t} = \left[-\frac{\hbar}{2m_{LP}}\nabla^2 + \frac{i}{2}(Rn_R - \gamma_c) + g|\psi|^2 + 2\tilde{g}_R n_R \right] \psi, \quad (2.56)$$

It is coupled to the equation for the density written as:

$$\frac{\partial n_R}{\partial t} = P - \gamma_R n_R - Rn_R|\psi|^2 + D\nabla^2 n_R, \quad (2.57)$$

This model bears a very close resemblance to the ones we just discussed above. The only difference here is that the spatial diffusion of reservoir polaritons takes place with a diffusion constant D , otherwise, the rest of the parameters are the same as introduced. The steady state under a continuous-wave and incoherent pumping is obtained by substituting the following ansatzes:

$$\psi(x, t) = \psi_0 e^{-i\mu_T t}, \quad (2.58)$$

$$n_R(x, t) = n_R^0, \quad (2.59)$$

where, ψ_0 is the amplitude, μ_T is the chemical potential. The perturbations around the steady state solution take the form:

$$\psi(r, t) = \psi_0 e^{-i\mu_T t} \left[1 + \sum_k u_k e^{i(kr - \omega t)} + \sum_k v_k^* e^{i(kr - \omega t)} \right], \quad (2.60)$$

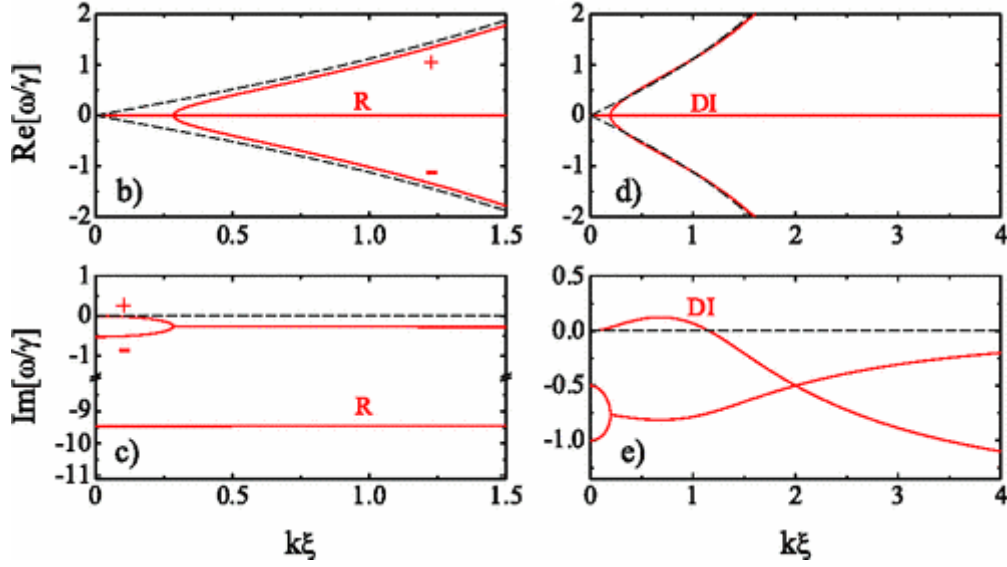


Figure 2.8: Real (Upper panel) and imaginary (bottom panel) parts for dispersion of the excitations in the scalar condensate case. Adopted from [59].

$$n_R(x, t) = n_R^0 \left[1 + \sum_k w_k e^{i(kr - \omega t)} + \sum_k w_k^* e^{i(kr - \omega t)} \right]. \quad (2.61)$$

Here, ω is a complex modulation frequency of the mode with k wave numbers, and u , v , w are small amplitudes. Substituting Eqs. (2.60) and (2.61) into Eqs. (2.56) and (2.57) and linearising around the steady state i.e., retaining only the first order terms of fluctuations, we obtain the eigenvalue problem for the fluctuations which has the simple matricial form $\omega \mathcal{U}_k = \mathcal{L}_k \mathcal{U}_k$. Where $\mathcal{U}_k$ is the perturbation vector and is given by $\mathcal{U}_k = [u, v, w]^T$.

The dispersion of elementary excitations is seen in Fig. 2.8 when the reservoir adiabatically follows the condensate's evolution. The dispersion relation is found by solving the eigenvalue polynomial, one finds real and imaginary solutions as shown in Fig. 2.8 (b) and (c).

Also, the dispersion of Bogoliubov excitations of a spinor EP BECs has been analysed in the absence TE-TM splitting [60]. Here, the linear Bogoliubov-like dispersion can only be recovered when certain compensatory effects are active i.e., the blueshift must exactly compensate the detuning. Most studies before neglected the effect of TE-TM splitting and the dynamics of the exciton reservoir. However, it is known that TE-TM splitting has a non-trivial effect on the dispersion of exciton-polaritons.

2.7.1 Vortices and Solitons

Conservative solitons are solitary waves whose shape is conserved during propagation in a dispersive medium. They are formed when dispersion is compensated by non-linearities in the system. It was predicted that spatially localized

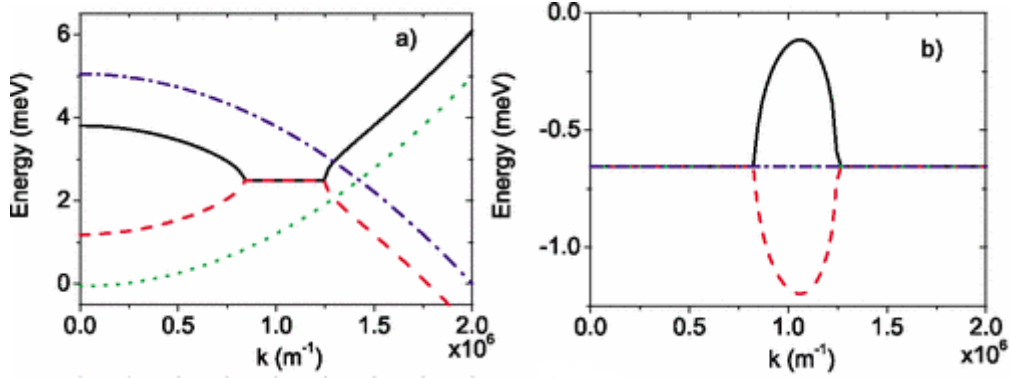


Figure 2.9: Real (left) and imaginary (right) parts for dispersion of the excitations in the case of a circularly polarized EP BECs. Adopted from [60].

steady states of the exciton-polariton BECs are dissipative solitons [20]. Dissipative solitons are different from their conservative counterpart in that they exist thanks to the compensation of dispersion by non-linearities in the system and additional compensation of losses by gain [61]. The most basic solitonic structures are the bright solitons which are characterized as a localized intense peak above a continuous wave background, followed by dark solitons which are identified by a density dip and an associated phase gradient. The transformation of the ring-shape condensates of the polaritons in cylindrical pillar microcavities into several bright lobes, indicating the presence of quantum states with nonzero angular momenta has been reported [62]. Stable vortex solutions in incoherent ring-shaped traps have been reported in EPCs [63]. These vortex states in EPCs exist in 2D systems and are given by:

$$\psi_V = f(r)e^{im\theta}, \quad (2.62)$$

where $f(r)$ is the radial function of the vortex which can be evaluated variationally and numerically. The characteristic orbital angular momentum or topological charge is represented by m , which describes the change in phases of the corresponding wavefunction after one closed path revolution around the vortex central position [45]. The topological charges m take integer multiples of 2π and there is a topological singularity at the center of the vortex. It has been shown that the structure and polarization properties of vortices in linearly polarized EPCs, possess two winding numbers, being either integer or half-integer simultaneously, and that four half vortices have the smallest energy [64]. The intrinsic lifetime of half-vortices in a spinor cavity EPCs with spin-anisotropic interactions has been given, while the texture of the vortices in the system has been analyzed [65]. The stabilization of the steady state by the polariton energy relaxation in an inhomogeneous pumping profile has been investigated in EPCs [66]. Furthermore, dissipative effects and the feedback induced by the inhomogeneity in the EPCs have been elucidated [67]. Analogy between half-integer topological defects in spinor polariton condensates and magnetic monopoles has been established [71].

On the other hand, the possibility of creating oblique dark half-solitons [70] as well as the controlled formation of topologically charged, phase-locked vortices has been addressed in EPCs [28]. Other forms of stable vortex solutions in incoherent ring-shaped traps have been revealed in EPCs [72]. A chaotic EP billiard has been created using a spatially structured optical pump [74]. Dissipative effects and the feedback induced by the inhomogeneity have been elucidated [67]. Spontaneous symmetry breaking of the spin degree of freedom has been observed in a spinor EPC [75]. Accurate measurements of the polariton population distribution have shown the efficiency of the evaporating cooling mechanism in EPC in potential trap [76]. A class of dark solitons and vortices in the intrinsic bistability regime have been illustrated [77], and a variety of objects such as sources, sinks, kinks, and pulses have been obtained [78]. The formation and synchronization of vortices and spin textures have been elucidated [79]. While rotating azimuthon steady states and spin patterns in an annular trapping geometry have been presented [80]. Investigations of the stability of vortex-antivortex superposition states have shown that they can survive at a steady state in a variety of configurations [81]. The vortex structures in circularly pumped EPCs have been investigated taking into account the finite decay rate of the exciton reservoir [82]. The formation of chaos and strange attractors in the order parameters of the EPCs have been evidenced by calculating the maximum Lyapunov exponent [83]. Markovian and non-Markovian interactions have been considered in the formation of EPCs [84]. Half-vortex cores have been realized in polariton condensates excited by a ring-shaped nonresonant pump [85]. It has been demonstrated that EPCs exhibit bistability regimes between two cross-linearly polarized components [86]. Stable spiraling patterns in the form of complexes of multiple mutually interacting vortices have been found [87]. The nonequilibrium nature of polariton condensates and the spontaneous steady-state current have been exploited to form dark solitons and the vortex chains [88]. The Landau-Zener tunneling has been characterized [89], and the diffusive reservoir modes have been identified to contribute to the drag force of EPCs [90]. Periodic oscillations of the field correlation function have been observed in optically trapped EPCs [91]. It has been shown that the low-momentum collective oscillations of EPCs in a round box trap are dominated by the dipole and breathing modes [92].

2.8 Summary

To this end, I have presented a general review on EP BECs. I gave a description of EPs then, an overview of various phenomena such as spin-orbit coupling and solitons and their theoretical descriptions through the use of the GP equations for BECs. The complex GP equation has been properly introduced for saturable and nonsaturable media. I have also presented the most general system of equations that will be used in the next chapters of this

thesis. The next two chapters will focus on MI.

Chapter 3

Modulational instability in vector exciton-polariton condensates with photonic spin-orbit coupling

3.1 Introduction

The aim of this chapter is to demonstrate the existence of Modulational Instability in the spin-orbit coupled exciton-polariton condensates. I start with a coupled generalized GP equations and rate equations for the incoherent excitonic reservoir density describing the dynamics of spinor polariton condensates trapped in ring-shaped confining potentials created by excitonic reservoirs [24]. Next, a system of two coupled 2D cubic CGL equations with saturable nonlinearities for both complex polariton order parameters are derived through an adiabatic elimination of the reservoir density, where the reservoir relaxation rate is fast compared to other time scales of condensate evolution [93]. Indeed, conditions under which the adiabatic reduction of the open-dissipative GP equation to the cubic CGL equation is justified have been derived [94]. Here, I explore the MI of spinor polariton condensates trapped in ring-shaped confining potentials, both analytically by the linear stability analysis of the MI of CW solutions, and numerically by the split-step Fourier method and direct integrations of the corresponding 2D cubic CGL equations with saturable nonlinearities. In addition, particular attention will be devoted to various physical effects on MI, such as photonic spin-orbit interactions, optical pumping powers, effects of polariton condensate interaction with incoherent exciton of the same and the orthogonal polarizations, and the effect of nonlinear polariton-polariton interactions are comprehensively discussed. Also, note that by means of Bogoliubov-de Gennes theory, the boundary of a modulationally unstable region in the parameter space was determined within the open-dissipative GP model [95]. Effects of polariton energy relaxation

and the inhomogeneous pumping profile have been taken into account in the study of MI process [96]. It has been shown that the mechanism of optical bistability in microcavities is associated with the MI process between odd and even states of the trapping potential [97]. Strong peaks of MI in semiconducting excitonic media have been found to possess gain orders of magnitude larger than that in the case of optical fibers [98]. In addition, when the pump width of the Gaussian laser beam is comparable to the unstable wavelength in which MI arises, the self-rotation of the condensate has been observed [99]. MI analysis has been used to describe a diffusive behavior of the Goldstone mode in BEC of exciton-polaritons [59]. MI of uniform states has been analyzed in the dual-core optical waveguide as an emulator for the EP system in the lossless limit [100]. MI was also predicted in two-component helicoidal spin-orbit-coupled dissipative BECs whose dynamics are described by coupled cubic CGL equations [101]. The impact of the left- and right-handed helicoidal spin-orbit couplings on MI of Bose-Bose mixtures has been discussed [102].

In the following sections we derive a system of dimensionless coupled cubic 2DCGL equations with saturable nonlinearities from eqs. (2.52-2.55). Then linear stability analysis of the MI of CW solutions will be studied on this system of equations.

3.2 The model

Evolution of the polariton condensate and the exciton reservoir is described by the coupled driven-dissipative equation of the Gross-Pitaevskii type and the rate equations with dimensionless form [24]

$$i\frac{\partial\psi_l}{\partial t} = \left[-\frac{1}{2}\nabla^2 + (-1)^{3-l}B + gn_l + \tilde{g}n_{3-l} + |\psi_l|^2 + h_{cp}|\psi_{3-l}|^2 + i\left(n_l - \frac{1}{2}\right) \right] \psi_l \\ + \sigma \left(\frac{\partial}{\partial x} + i(-1)^l \frac{\partial}{\partial y} \right)^2 \psi_{3-l}, \quad (3.1a)$$

$$\frac{\partial n_l}{\partial t} = I_l - (\Gamma + G|\psi_l|^2)n_l, \quad l = 1, 2. \quad (3.1b)$$

The operator ∇ is the nabla operator. Quantities ψ_l ($l = 1, 2$) are dimensionless order parameter functions describing the spin-up ($l = 1$) and spin-down ($l = 2$) polaritons, respectively, $n_l = \frac{R}{2\gamma_c}\rho_l$ ($l = 1, 2$) are dimensionless exciton densities describing the density of incoherent excitons that create both effective gain and repelling potential for coherent polaritons where R is the coupling parameter between reservoir and condensate, γ_c is the condensate dumping rate and ρ_l describes the densities of incoherent excitons. $B = \frac{g\mu_B}{2\hbar\gamma_c}B$ is the dimensionless magnetic field. Infact, the magnetic field B shifts the eigenfrequencies of the spin-up and spin-down polaritons in the opposite directions, leading to the eigenfrequencies difference $g_e\mu_B B$, where μ_B is the

Bohr magneton and g_e is the effective excitonic g factor. Dimensionless coefficients $g = \frac{2g_R}{\hbar R}$ and $\tilde{g} = \frac{2\tilde{g}_R}{\hbar R}$ are the parameters accounting for the polariton condensate interactions with incoherent excitons of the same and the orthogonal polarization respectively, where g_R and $\tilde{g}_R$ are the coefficients of nonlinear interaction between polaritons and incoherent reservoir excitons and $\hbar$ is the usual Planck's constant. $h_{cp} = \frac{g_x}{g_c}$ is the dimensionless coefficient accounting for nonlinear cross-polarization interactions where g_c and g_x are the nonlinear coefficients of interaction between polaritons belonging same and to the orthogonal polarizations, $\sigma = \frac{m_t - m_l}{m_t + m_l}$ is the strength of the TE-TM splitting (photonic effective spin interaction), m_t and m_l are the transverse and longitudinal masses of the polaritons. $\Gamma = \frac{\gamma_R}{\gamma_c}$ is the dimensionless decay rate of incoherent excitons where γ_R denotes the reservoir dumping rate, $G = \frac{\hbar R}{g_c}$ accounts for depletion of the exciton reservoir due to condensation into the coherent part and finally $I_l = \frac{R}{2\gamma_c} P_l$ are the dimensionless pumping powers in the right ($l = 1$) and left ($l = 2$) circular polarization, where P_l denotes the pumping powers.

In the typical case where the characteristic relaxation rate γ_R of the reservoir is much faster than all scales, one can adiabatically eliminate n_1 and n_2 respectively. The adiabatic elimination procedure consists of finding the following stationary solutions.

$$n_l = \frac{I_l}{\Gamma + G|\psi_l|^2}, \quad l = 1, 2 \quad (3.2)$$

Substituting Eqs.(3.2) into (3.1a) and (3.1b), we derive a system of two coupled 2D cubic dissipative Gross-Pitaevskii equations with saturable nonlinearities which reads as

$$\begin{aligned} i\frac{\partial\psi_l}{\partial t} = & -\frac{1}{2}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\psi_l + \left(B - \frac{i}{2}\right)\psi_l + \left(\frac{(g+i)I_l}{\Gamma + G|\psi_l|^2} + \frac{\tilde{g}I_{3-l}}{\Gamma + G|\psi_{3-l}|^2}\right)\psi_l \\ & + (|\psi_l|^2 + h_{cp}|\psi_{3-l}|^2)\psi_l + \sigma\left(\frac{\partial}{\partial x} + i(-1)^l\frac{\partial}{\partial y}\right)^2\psi_{3-l}, \quad l = 1, 2. \end{aligned} \quad (3.3)$$

On the basis of Eqs. (3.3) we would now investigate the MI phenomenon.

3.2.1 Linear stability analysis of continuous waves

Next, we use the standard linear stability analysis to study MI phenomenon in a driven-dissipative multistable system. The steady state solutions of Eqs. (3.3), corresponding to the CW signal, can be written as

$$\psi_l = \sqrt{n_{l0}}e^{-i\mu t}, \quad l = 1, 2 \quad (3.4)$$

where n_{l0} are uniform densities, respectively and μ is the common chemical potential of both polariton components. They are related by the following

relation,

$$\begin{aligned} \mu = & B(-1)^{3-l} + \frac{gI_l}{\Gamma + Gn_l} + \frac{\tilde{g}I_{3-l}}{\Gamma + Gn_{3-l}} + n_{l0} \\ & + h_{cp}n_{3-l0} + i\frac{I_l}{\Gamma + Gn_{3-l}} - \frac{i}{2}, \quad l = 1, 2. \end{aligned} \quad (3.5)$$

The linear stability of the steady state can be examined by introducing a perturbed field of the following form

$$\psi_l = (\sqrt{n_{l0}} + \delta\psi_l) e^{-i\mu t}, \quad l = 1, 2 \quad (3.6)$$

where $\delta\psi_l$ are small perturbations, with $|\delta\psi_l| \ll \sqrt{n_{l0}}$ being complex fields. Substituting Eqs. (3.6) into Eqs. (3.3) and linearizing around the unperturbed solutions lead to the following equations for the perturbed fields

$$\begin{aligned} i\frac{\partial(\delta\psi_l)}{\partial t} = & -\frac{1}{2}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\delta\psi_l + \sigma\left(\frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2}\right)\delta\psi_{3-l} \\ & + 2i(-)^l\sigma\frac{\partial^2(\delta\psi_{3-l})}{\partial x\partial y} + n_{l0}(\delta\psi_l^* + \delta\psi_l) + h_{cp}\sqrt{n_{l0}n_{3-l0}}(\delta\psi_{3-l}^* + \delta\psi_{3-l}) \\ & - \frac{gI_l n_{l0} G(\delta\psi_l^* + \delta\psi_l)}{(\Gamma + Gn_{l0})^2} - \frac{\tilde{g}I_{3-l}\sqrt{n_{l0}n_{3-l0}}G(\delta\psi_{3-l}^* + \delta\psi_{3-l})}{(\Gamma + Gn_{3-l0})^2} \\ & - i\frac{I_l n_{l0} G(\delta\psi_l^* + \delta\psi_l)}{(\Gamma + Gn_{l0})^2}, \quad l = 1, 2 \end{aligned} \quad (3.7)$$

where $\delta\psi_j^*$ are the complex conjugates of the perturbed fields $\delta\psi_j$. Now, we assume the following ansatzes for the perturbed fields

$$\delta\psi_l = a_{l1}e^{i(K_1x+K_2y-\Omega t)} + a_{l2}^*e^{-i(K_1x+K_2y-\Omega^*t)}, \quad l = 1, 2, \quad (3.8)$$

where K_1 and K_2 are the wave numbers, Ω is a complex modulation frequency of perturbations modulating the carrier signal, and a_{l1} and a_{l2} are constant complex amplitudes, respectively. The substitution of solutions (3.8) into Eqs. (3.7) gives a linear homogeneous system of equations in terms of a_{l1} and a_{l2} , i.e.,

$$M \times [a_{11}, a_{12}, a_{21}, a_{22}]^T = 0, \quad (3.9)$$

where M is a 4×4 matrix given by

$$M = \begin{bmatrix} S_1 + e + f + \Omega & S_1 + e & S_2 + g - d & S_2 \\ S_1 - e & S_1 - e + f - \Omega & S_2 & S_2 + g + d \\ S_4 + g + d & S_4 & h + S_3 + f + \Omega & S_3 + h \\ S_4 & S_4 + g - d & S_3 - h & S_3 - h + f - \Omega \end{bmatrix} \quad (3.10)$$

whose elements are as follows

$$\begin{aligned}
S_1 &= \left(\frac{gI_1G}{(\Gamma + Gn_{20})^2} - 1 \right) n_{10}, \quad S_2 = \left(\frac{\tilde{g}I_2G}{(\Gamma + Gn_{20})^2} - h_{cp} \right) \sqrt{n_{10}n_{20}}, \\
S_3 &= \left(\frac{gI_2G}{(\Gamma + Gn_{20})^2} - 1 \right) n_{20}, \quad S_4 = \left(\frac{\tilde{g}I_1G}{(\Gamma + Gn_{10})^2} - h_{cp} \right) \sqrt{n_{10}n_{20}}, \\
d &= 2i\sigma K_1 K_2, \quad e = i \frac{I_1 n_{10} G}{(\Gamma + Gn_{10})^2}, \quad f = -\frac{1}{2} (K_1^2 + K_2^2), \quad g = \sigma (K_1^2 - K_2^2), \\
h &= i \frac{I_2 n_{20} G}{(\Gamma + Gn_{20})^2}.
\end{aligned} \tag{3.11}$$

Since we seek a nontrivial solution of the system in Eq. (3.9), we require that $\text{Det}(M) = 0$. After some straightforward calculations, we obtain the following nonlinear dispersion relation

$$\Omega^4 + P_3\Omega^3 + P_2\Omega^2 + P_1\Omega + P_0 = 0 \tag{3.12}$$

with $P_\nu (\nu = 0, 1, 2, 3)$ being given in the Appendix.

The procedure adopted in this work to study the MI growth rate comprises two steps: the analytical procedure and the full numerical one. **(i)** For the analytical procedure, the following four solutions are obtained for Eq.(3.12):

$$\begin{aligned}
\Omega_{1,2} &= -\frac{P_3}{4} - \frac{1}{2} \sqrt{\frac{1}{4}P_3^2 - \frac{2}{3}P_2 + \Lambda} \\
&\pm \frac{1}{2} \sqrt{-\frac{4}{3}P_2 + \frac{1}{2}P_3^2 - \Lambda + \frac{P_3^3 - 4P_3P_2 + 8P_0}{4\sqrt{\frac{1}{4}P_3^2 - \frac{2}{3}P_2 + \Lambda}}}, \\
\Omega_{3,4} &= -\frac{P_3}{4} + \frac{1}{2} \sqrt{\frac{1}{4}P_3^2 - \frac{2}{3}P_2 + \Lambda} \\
&\pm \frac{1}{2} \sqrt{-\frac{4}{3}P_2 + \frac{1}{2}P_3^2 - \Lambda - \frac{P_3^3 - 4P_3P_2 + 8P_0}{4\sqrt{\frac{1}{4}P_3^2 - \frac{2}{3}P_2 + \Lambda}}},
\end{aligned} \tag{3.13}$$

where

$$\Lambda = \frac{1}{3} \left(\sqrt[3]{\frac{\Delta_2 + \sqrt{\Delta_2^2 - 4\Delta_1^3}}{2}} + \frac{\Delta_1}{\sqrt[3]{\frac{\Delta_2 + \sqrt{\Delta_2^2 - 4\Delta_1^3}}{2}}} \right),$$

with $\Delta_1 = P_2^2 - 3P_3P_1 + 12P_0$, and $\Delta_2 = 2P_2^3 - 9P_3P_2P_1 + 27P_3^2P_0 + 27P_1^2 - 72P_2P_0$.

The set of solutions (3.13) are such that the frequencies Ω_l ($l = 1, 2, 3, 4$) are dependent on the values of the parameters that constitute the coefficients of the dispersion relation. This implies that Ω_l may be positive, negative, or even complex, depending on values of such parameters. Additionally, the

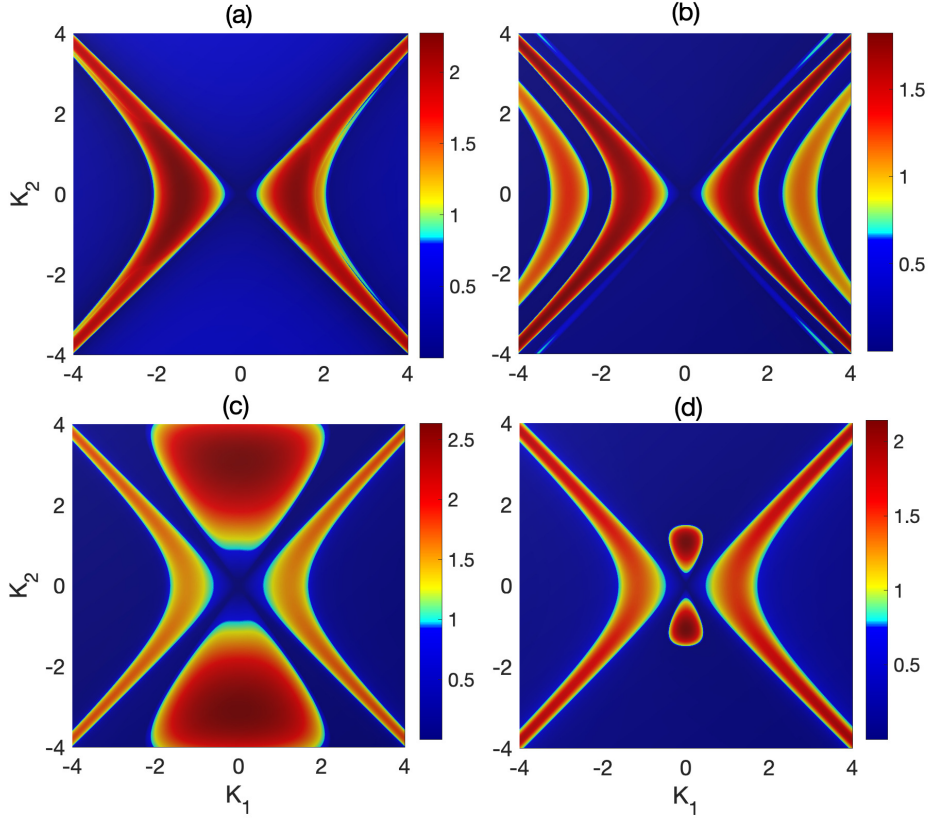


Figure 3.1: The panels show the variation of the MI gain $\zeta(\Omega)$ in the (K_1, K_2) -plane for (a): $\sigma = 0.03$, (b): $\sigma = 0.5$, (c): $\sigma = 0.7$, and (d): $\sigma = 0.9$, with the other parameters being $I = 3.0$, $G = 1.1$, $g = 3.64$, $\tilde{g} = -0.364$, $h = -0.1$, and $n_{10} = n_{20} = n_0 = 1$.

growth rate spectrum $\zeta(K_1, K_2)$ can be positive or negative, and its sign can be specified for a particular set of values of the wavenumbers K_1 and K_2 . Negative growth rate values refer to stability, a case where the system experiences vanishing of $\text{Im}(\Omega_l)$ as $t \rightarrow \infty$. Oppositely, positive values are related to the instability of the CWs, since the perturbation diverges without limit as time increases. Therefore, the sign of $\text{Im}(\Omega_l)$ informs about the stability of the CWs, and the regions of instability are commonly known as the growth rate spectrum, where the growth rate is given by $\zeta_1 = \text{Im}(\Omega_1) > 0$, or $\zeta_2 = \text{Im}(\Omega_2) > 0$, or $\zeta_3 = \text{Im}(\Omega_3) > 0$, or $\zeta_4 = \text{Im}(\Omega_4) > 0$. **(ii)** For the full numerical procedure, instead of repeating four times the calculation of finding the imaginary part of each of the solutions, one extracts numerically the largest value among such imaginary parts and compute the maximum growth rate such that

$$\zeta(\Omega) = \max\{\zeta_1, \zeta_2, \zeta_3, \zeta_4\}. \quad (3.14)$$

More explicitly, the latter expression indicates that the calculations for the MI growth rate, performed in the next section, are based on the highest positive imaginary part of the four solutions given in Eq.(3.13).

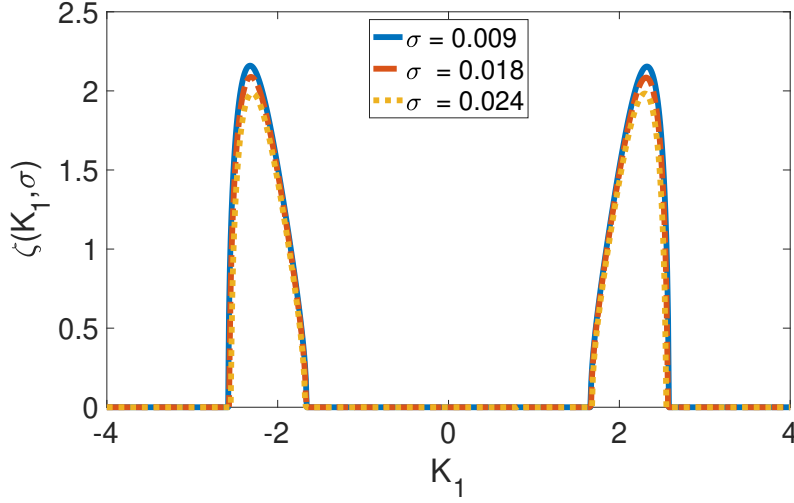


Figure 3.2: The panels show the variation of the MI gain $\zeta(\Omega)$ versus the wavenumber K_1 under the effect of weak values of the photonic SO coupling for $K_2 = 2$ and the parameter values $I = 3.0$, $G = 1.1$, $g = 3.64$, $\tilde{g} = -0.364$, $h = -0.1$, and $n_{10} = n_{20} = n_0 = 1$.

3.3 Parametric analysis of the growth rate of instability

We know that the MI is extremely sensitive to the system parameters. In the current context, it is evident from the dispersion relation (3.12) that the growth rate is highly influenced by the photonic SO interaction strength (σ) and the pumping power ($I_1 = I_2 = I$). The absence of the magnetic field parameter B in the dispersion relation for instability means that it cannot contribute to the properties of the MI spectrum. In this work, we would like to discuss the interplay between different system parameters in the instability spectrum. For a comprehensive study, we consider different combinations of the physical parameters in question and present the other cases. For the calculations, we consider two polariton condensate components with equal densities $n_{10} = n_{20} = n_0$. For convenience, we assume these densities to equal one, that is, $n_0 = 1$. Also, we only consider the case of equal pumping power in both polarizations $I = I_1 = I_2$. Thus, following the mathematical calculation pertaining to the nonlinear dispersion relation corresponding to the system's instability, we dedicate the subsequent sections to the study of various effects in the instability spectra of our system.

3.3.1 Effect of the photonic spin-orbit interactions (σ) on MI

In order to highlight the effect of the photonic SO coupling, we have first represented, in Fig. 3.1, the MI growth rate in the (K_1, K_2) -plane for different values of σ . From the initial spectrum obtained in Fig. 3.1(a), with $\sigma = 0.03$,

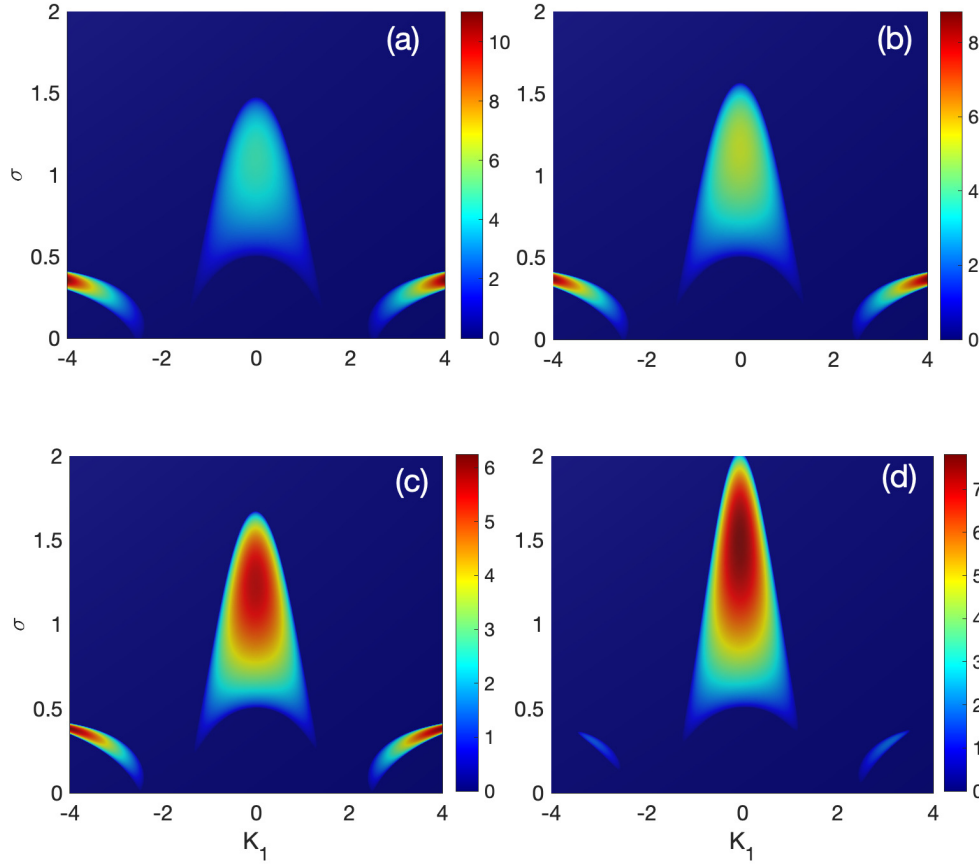


Figure 3.3: The panels show the variation of the MI gain $\zeta(\Omega)$ versus the perturbation wavenumber K_1 and the photonic SO coupling strength σ for (a): $I_1 = I_2 = 1$, (b): $I = 3$, (c): $I_1 = I_2 = 5$, and (d): $I_1 = I_2 = 6$, with $K_2 = 1.5$, $G = 1.1$, $g = 3.64$, $\bar{g} = -0.364$, $h = -0.1$, and $n_{10} = n_{20} = n_0 = 1$.

one notices the appearance of additional instability regions with σ increasing. Particularly, the asymmetric lobes along K_2 , when $\sigma = 0.7$ get shrunk for $\sigma = 0.9$ as shown in Fig. 3.1(d). Interestingly, one obtains from Fig. 3.3 that the pumping power affects the MI growth rate, enhancing the instability. It is, in fact, shown in the (K_1, σ) -plane that, with increasing I , the minor tongue of instability appearing in Fig. 3.3(a) tends to become the major instability zone, therefore, eliminating the initial lateral zones of high growth rate. The cross-section of Fig. 3.3 is depicted in Fig. 3.4, where it is revealed that instability takes place even in the absence of the photonic SO coupling. Increasing σ , as already seen in Fig. 3.3, enhances the growth of instability via a single instability hump, with maximum situated at $K_1 = 0$. More interestingly, cross-sections of the MI growth rate of Fig. 3.1 are represented in Fig. 3.2 for weak values of the photonic SO coupling and $K_2 = 2$, which shows that the point $(K_1, K_2) = (2, 2)$ is a suitable point capable of giving rise to the

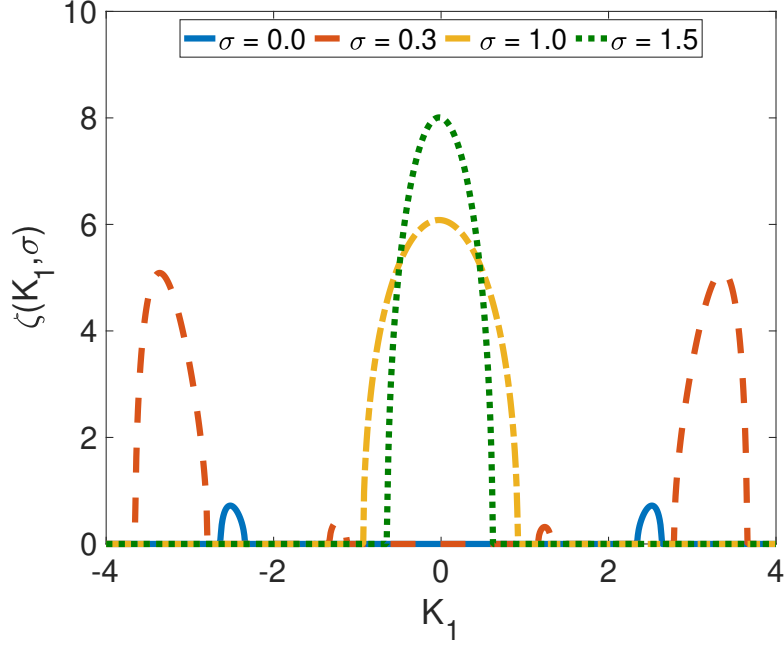


Figure 3.4: The panel shows the cross-section of the MI growth rate $\zeta(\Omega)$ versus the perturbation wavenumber K_1 for $K_2 = 1.5$. This corresponds to the results of Fig. 3.3(c), with the lines corresponding to increasing values of the photonic SO coupling strength σ . The rest of the parameter values are: $I_1 = I_2 = 5$, $G = 1.1$, $g = 3.64$, $\tilde{g} = -0.364$, $h = -0.1$, and $n_{10} = n_{20} = n_0 = 1$.

disintegration of the plane wave solutions. This particular example is treated in the numerical simulations.

3.3.2 Effect of input powers ($I = I_1 = I_2$) on MI

Since it is contained in the coefficients of the nonlinear dispersion relation (3.9), the constant input power is expected to contribute to the MI growth rate. This is effective from Fig. 3.5, where the growth rate spectrum is addressed in the (K_1, I) -plane. Following the same procedure as in the previous cases, the different panels correspond to increasing values of the photonic SO coupling σ . Initially, for $\sigma = 0.03$, the instability is two major symmetric lobes between which one perceives two minor lobes of instability that disappear when I gets stronger, leaving only the major zones [see Fig. 3.5(a)]. Interestingly, all input power values make the instability growth rate available. While the two major lobes of Fig. 3.5(a) persist, there appears between them a large zone of instability with the maximum at $K_1 = 0$, which originates from the combination of the previous two minor lobes [see Fig. 3.5(b)]. The central major instability zone shrinks for $\sigma = 0.7$ and expands with I increasing, while its maximum takes place for $I > 1.5$. In the process, its satellite sidebands lose intensity but get enhanced for $\sigma = 0.9$. The central lobe is restricted to high values of I , i.e., $I > 2$, which implies that high pumping powers promote MI in polariton systems. However, the magnitude of the

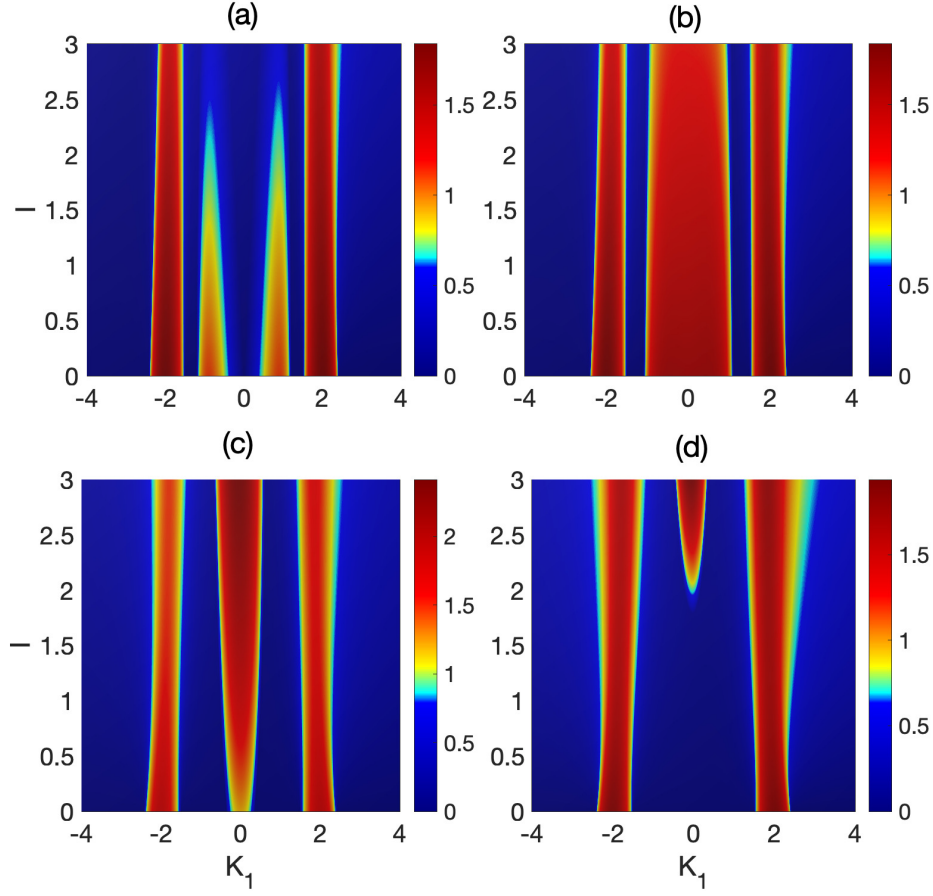


Figure 3.5: The panels show the distribution of the MI gain $\zeta(\Omega)$ versus the perturbation wavenumber K_1 and the input power $I = I_1 = I_2$ for (a): $\sigma = 0.03$, (b): $\sigma = 0.5$, (c): $\sigma = 0.7$, and (d): $\sigma = 0.9$, with the other parameters being $K_2 = 1.5$, $G = 1.1$, $g = 3.64$, $\tilde{g} = -0.364$, $\Gamma = 3$, $h = -0.1$, and $n_{10} = n_{20} = n_0 = 1$.

growth rate is highly affected by the changes in the photonic SO coupling, and the lobes, with I increasing, become asymmetric. This can be explained by the fact that in the optical fiber relying on the nonlinear Schrödinger equation, the MI gain is usually a quadratic function of the perturbation frequency, corresponding to a symmetric MI gain with respect to the origin. This has been reported even in the presence of other additional physical effects such as higher-order dispersion, higher-order nonlinearities, walk-off effects, and cross-phase modulation, to cite a few [103–105]. Nevertheless, recent studies on materials with spin-orbit coupling have shown that new effects can affect the symmetric properties of the MI gain spectrum. For example, asymmetric lobes of instability have been recently obtained by including the helicoidal spin-orbit coupling in BECs, where the strong effect of the gauge amplitude causes the MI gain spectrum to be asymmetric, while weak values of the same effect support symmetric lobes of the MI gain spectrum [106]. Very recently, the same behaviors of the MI gain were reported in Bose-Bose mixtures in the

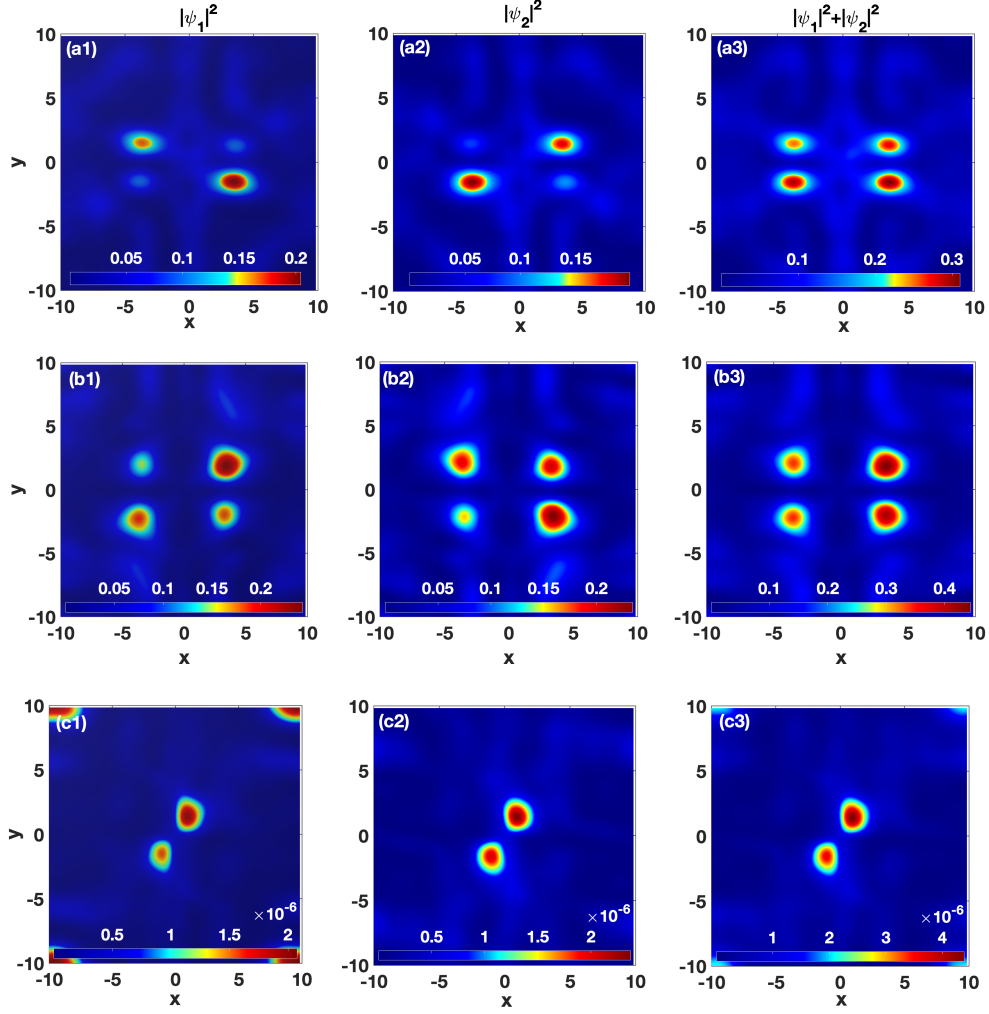


Figure 3.6: Development of MI, with the panels (a)_{j=1,2,3}, (b)_{j=1,2,3} and (c)_{j=1,2,3} corresponding respectively to $\sigma = 0.009$, $\sigma = 0.018$, $\sigma = 0.024$ and the other parameters being: $I_1 = I_2 = 2$, $G = 1.1$, $g = 3.64$, $\tilde{g} = -0.364$, $K_1 = K_2 = 2$, $\Gamma = 3$, $h = -0.1$, $n_{10} = n_{20} = n_0 = 1$, and $B = 0.02$. From left to right, the columns correspond to $|\psi_1|^2$, $|\psi_2|^2$, and $(|\psi_1|^2 + |\psi_2|^2)$.

presence of the Rashba and Dresselhaus spin-orbit couplings [107], and also under the helicoidal SO coupling [102]. The common feature of the two types of spin-orbit couplings is that they include coupling through the first-order derivatives with respect to the space coordinate of the BEC components. Contrarily, the appearance of such symmetric sidebands is also referred to as Stokes and anti-Stokes. For some input powers, Stokes and anti-Stokes sidebands can become asymmetric [108], leading to results similar to those in Fig. 3.5(d).

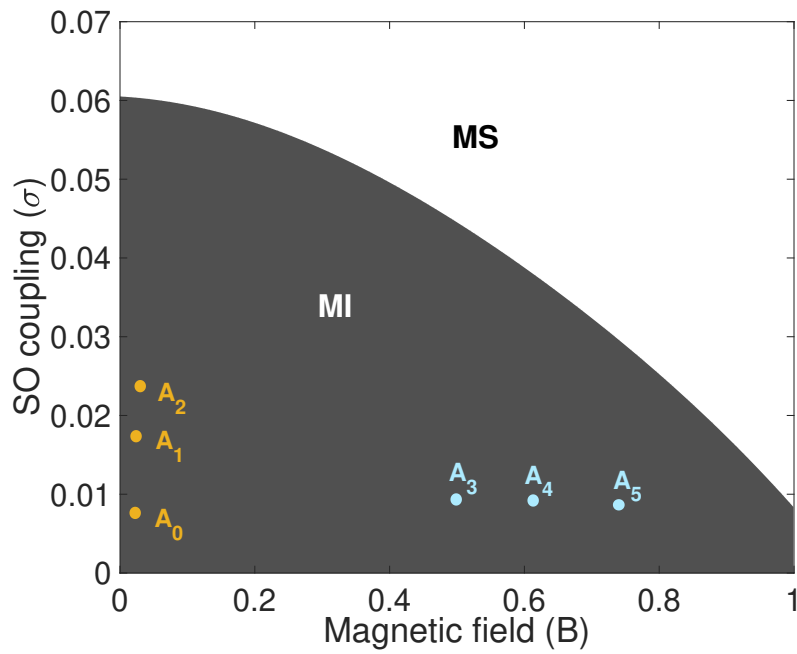


Figure 3.7: The panels show the numerically generated phase diagrams in the (B, σ) -plane for: (a) $I_1 = I_2 = 0.4$, (b) $I_1 = I_2 = 0.6$, (c) $I_1 = I_2 = 1.2$ and (d) $I_1 = I_2 = 2$. The wavenumber K_1 , K_2 , and the initial densities n_0 keep their values used to generate Fig. 3.6, while the other parameter values are: $G = 1.1$, $g = 3.64$, $\tilde{g} = -0.364$, $\Gamma = 3$, and $h = -0.1$. The points A_0 , A_1 , and A_2 which corresponds to values of B and σ were used to generate Fig. 3.6, while the points A_3 , A_4 and A_5 were used to corroborate the results of Figs. 4.1 and 4.2. The point C_0 (0.3, 0.06) was used to test the stability of the CW solutions, which gives the results of Fig. 4.3. MI indicates the region of instability, and MS (modulational stability) indicates the region of stability.

Chapter 4

Numerical experiment and discussion

The linear stability analysis relies on approximations, making it difficult to table the long-time evolution of the predicted nonlinear patterns. Therefore, the found parameter regions should be used for direct numerical simulations of the generic model to confirm their accuracy. That is performed in this section with application to Eq.(3.3) via the split-step Fourier method with periodic boundary conditions and time-step $\Delta t = 5 \times 10^{-3}$. As initial conditions, we use the following perturbed CWs:

$$\psi_l(x, y, 0) = \sqrt{n_0} + \epsilon \cos(K_1 x + K_2 y), \quad l = 1, 2, \quad (4.1)$$

where $n_0 = 1$ and $\epsilon = 10^{-3}$, with the wavenumber $K_1 = K_2 = 2$ for example, for weak values of the photonic SO coupling σ , according to Fig. 3.2. However, the wavenumber may take other values related to instability according to our analytical predictions. Information on pattern formation is recorded in terms of the densities $|\psi_l|^2$ ($l = 1, 2$), with insistence on the effect related to the TE-TM splitting due to the photonic SO coupling σ , the magnetic field B and the input powers I_1 and I_2 .

Before we proceed, let us make the precision that MI's early and non-linear stages can generate a broad class of analytical solutions of the one-dimensional nonlinear Schrödinger equation referred to as solitons on finite background [109–111]. Among those are the Akhmediev breather, the Kuznetsov-Ma soliton, and the Peregrine soliton. Besides such very rich localized solutions, the dynamical phenomena of polariton instability find their origin in the process of a reservoir-induced MI. Quantum or thermal fluctuations of the condensates or pump noise effects play a crucial role in the static and dynamic properties of homogeneous and inhomogeneous EPCs at finite temperatures. Consequently, a local increase of polariton density induces a local depletion of the reservoir density through spatial hole burning [59, 112]. Concomitantly, when such a local depletion is realized, there is an exponential increase of the initial fluctuation in time, favoured by an inherent attraction

of the condensate polaritons in the subsequent potential well. At the end of the process, the density fluctuation is then ejected from its primary position. It starts moving through the condensates due to a positive feedback loop, which is eventually broken by gain saturations and polariton dynamics. The outcome of such a process is the formation of nonlinear patterns, with features closely related to system parameters as shown in the rest of the work.

The results that show the development of solitonic clusters are depicted in Fig. 3.6, while the phase diagrams, also obtained from direct numerical simulations, are depicted in Fig. 3.7. The spectrum of behaviors that manifest the development of instability comprises clusters of two counter-symmetric solitons [see Figs. 3.6(a1) and (a2)] and clusters of four counter-symmetric solitons [see Figs. 3.6(b1) and (b2)]. For these two types of clusters, the total density leads to clusters of four bright solitons, as shown in Figs. 3.6(a3) and (b3), where the photonic SO coupling takes the values $\sigma = 0.009$ and $\sigma = 0.018$, respectively. With increasing the SO coupling strength to $\sigma = 0.024$, we obtain the instability features of Fig. 3.6(c1) and (c2), where the two condensates display similar patterns manifested by clusters of two bright solitons forming diagonal dipoles. Interestingly and expectedly, the total density adheres to the individual instability characteristics.

One factor that does not appear in the expression of the MI growth rate is the magnetic field which affects the system's dynamics. The influence of the magnetic field on confined exciton-polariton modes constitutes a recent debate. The magnetic field is an important component that can be used as a tuning parameter for photon and exciton resonance [79]. Luckily, its effect can be predicted on the emergence of the instability via numerically generated phase diagrams depicted in Fig. 3.7. For information, the points A_0 , A_1 , and A_2 in the diagram of Fig. 3.7(d) are those corresponding to the simulations of Fig. 3.6, where the magnetic field is relatively weak. For validation, the patterns of Fig. 4.1(a) and (b) correspond, respectively, to the diagram's points A_3 and A_4 . In Fig. 3.7(a), the generated instability is manifested by a cluster of four compact solitons, and the two condensates display the same dynamics. With increasing the magnetic field B , which corresponds to point A_4 , the solitonic clusters become asymmetric due to the annihilation of the two lobes, a situation that gets pronounced in Fig. 4.2, corresponding to point A_5 of the diagram, where a stronger magnetic field causes the depletion of more solitonic objects from the cluster. The reader should, however, be aware that the linear stability analysis does not permit the prediction of the system's longtime evolution but rather gives information on the onset of instability. That explains why, even though the phase diagrams of Fig. 3.7, one could not predict the type of solitonic object involved in the manifestation of instability. Moreover, MI instability allows frequency mixing, which is supposed to affect the dynamics of the obtained patterns as time increases. This gets more pronounced when we deal with a coupled system like the one under study, where there is an energy exchange between the two condensates. This

latter phenomenon has been at the origin of the emergence of different kinds of nonlinear patterns in physical systems where there is fair competition between nonlinearity and dispersion. Additional to such effects, the coupled polariton condensates involve the competition between the magnetic field and the photonic SO coupling, which may contribute to enriching their cooperative dynamics under controlled interatomic interactions. Interestingly, for the case where the conditions are not satisfied for MI to develop, the plane wave solutions will remain stable, as shown in Fig. 4.3, where the obtained patterns have been computed from point C_0 of the phase diagram delivered by Fig. 3.7. This further agrees well with our analytical predictions.

According to the linear stability analysis and the subsequent growth rate spectrum, different choices of parameters are expected to lead to more exotic behaviors in wave patterning and propagation. For example, using another set of parameters, mainly the perturbation wave numbers $K_1 = K_2 = 0.75$, the phase diagram of Fig. 4.4 is generated against the magnetic field B and the SO coupling strength σ . It is clear that nonlinear patterns will likely occur for a weak or moderate magnetic field, while the highest probability of instability allows even high values of the SO coupling strength. Interestingly, the points A_1 , A_2 , and A_3 have been used to generate the patterns of Fig. 4.5. For such points, clusters of dips, with individual objects similar to dark solitons, are obtained. In Fig. 4.5(a j) $_{j=1,2,3}$, where $\sigma = 0.075$, the dynamics of the system, through the total density, is driven by the second condensate patterns as shown in Fig. 4.5(a3). Along the same line, when $\sigma = 0.09$, the number of dips from individual condensates drops, while the global dynamics of the system is driven by the $|\psi_2|^2$ as depicted in Fig. 4.5(b3). Finally, under stronger SO coupling, the two components display the same dynamics, drastically reducing the number of defects forming the deep cluster. This confirms the behaviors already reported in Fig. 4.1.

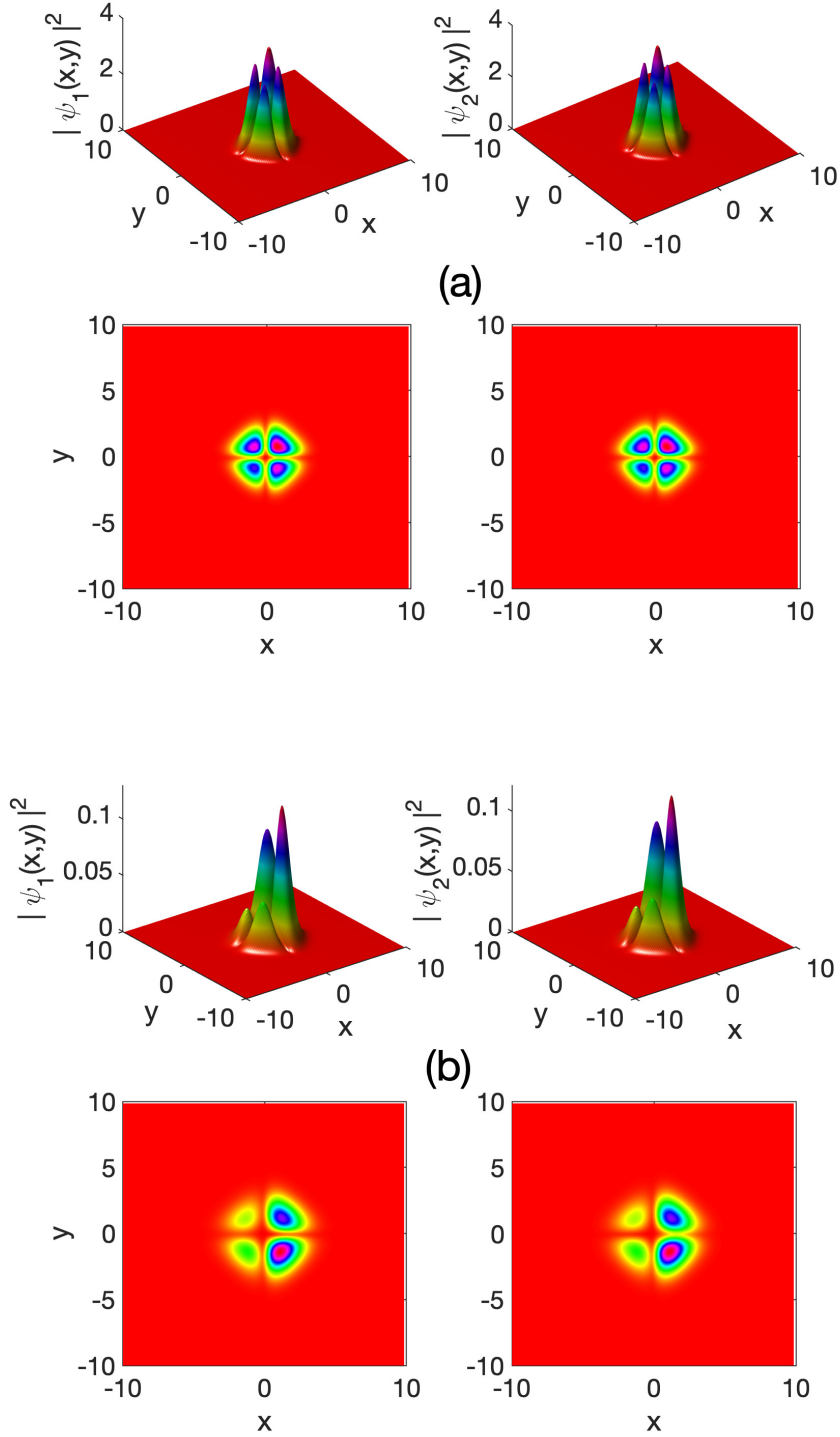


Figure 4.1: Development of MI, with the panels (a) and (b) showing $|\psi_1|^2$ and $|\psi_2|^2$, and their corresponding density plots obtained from numerical simulations using the points A_3 and A_4 of the phase diagram of Fig. 3.7. The other parameter values are: $G = 1.1$, $g = 3.64$, $\tilde{g} = -0.364$, $K_1 = K_2 = 2$, $\Gamma = 3$, $h = -0.1$, and $n_{10} = n_{20} = n_0 = 1$. The left column corresponds to $|\psi_1|^2$, and the right column to $|\psi_2|^2$.

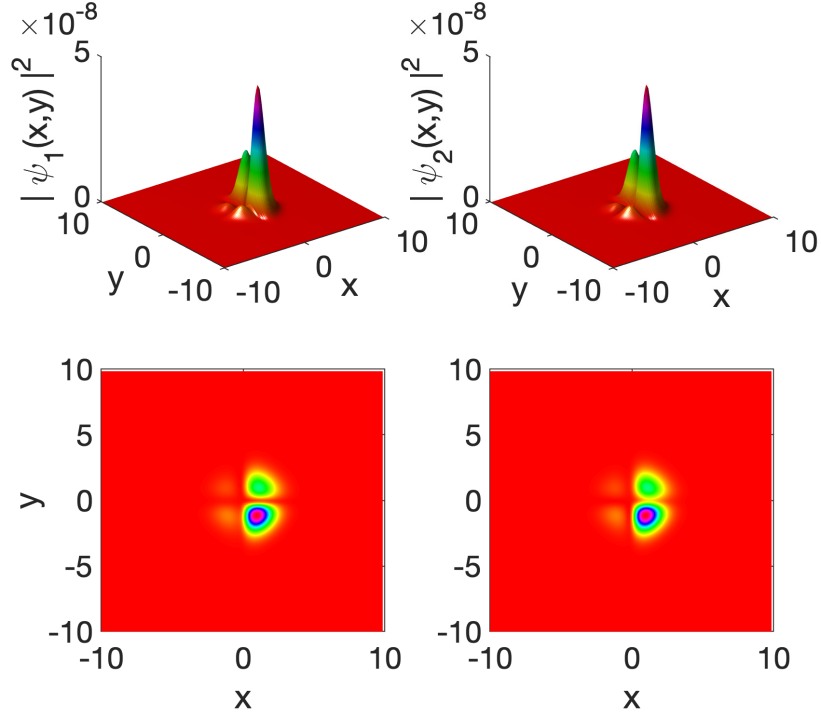


Figure 4.2: Development of MI using values of σ and B from the point A_5 of the phase diagram of Fig. 3.7. The left column corresponds to $|\psi_1|^2$, and the right column to $|\psi_2|^2$, along with their corresponding density plots. The other parameter values are: $G = 1.1$, $g = 3.64$, $\tilde{g} = -0.364$, $K_1 = K_2 = 2$, $\Gamma = 3$, $h = -0.1$, and $n_{10} = n_{20} = n_0 = 1$.

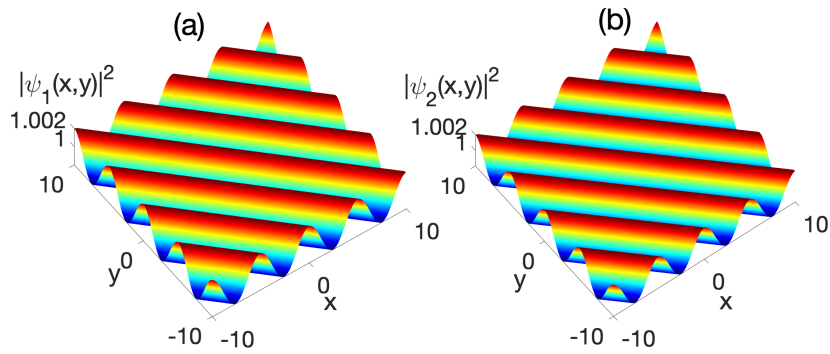


Figure 4.3: Example of modulational stability (MS) resulting from the point C_0 of the phase diagram of Fig. 3.7, panels (a) and (b) showing $|\psi_1|^2$ and $|\psi_2|^2$. The other parameter values are: $G = 1.1$, $g = 3.64$, $\tilde{g} = -0.364$, $K_1 = K_2 = 2$, $\Gamma = 3$, $h = -0.1$, and $n_{10} = n_{20} = n_0 = 1$. The left column corresponds to $|\psi_1|^2$, and the right column to $|\psi_2|^2$.

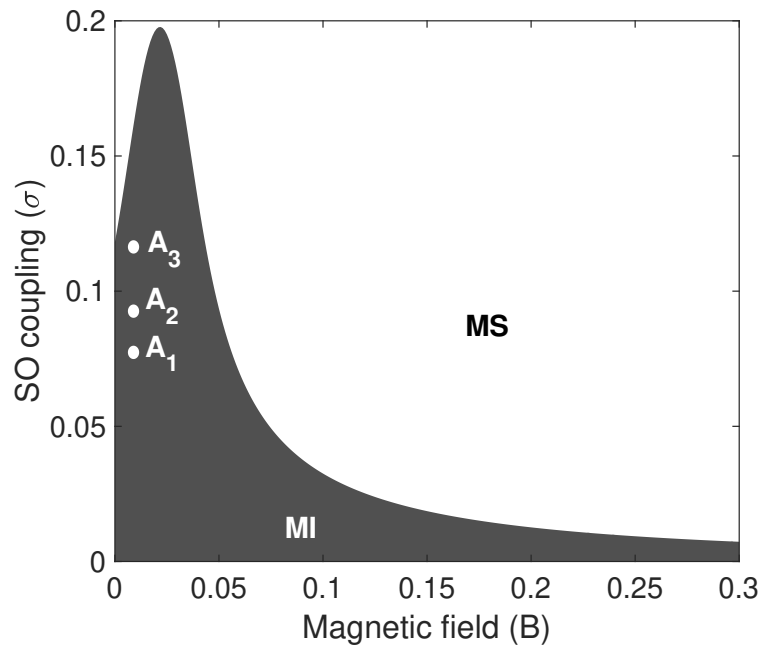


Figure 4.4: The panels show the numerically generated phase diagrams in the (B, σ) -plane for $I_1 = I_2 = 3$, the wavenumber $K_1 = K_2 = 0.75$, and the initial densities $n_0 = 1$, while the other parameter values are: $G = 1.1$, $g = 3.64$, $\tilde{g} = -0.364$, $\Gamma = 3$, and $h = -0.1$. The points A_1 , A_2 , and A_3 correspond to values of B and σ that were used to generate Fig. 4.5. MI indicates the region of instability, and MS (modulational stability) indicates the region of stability.

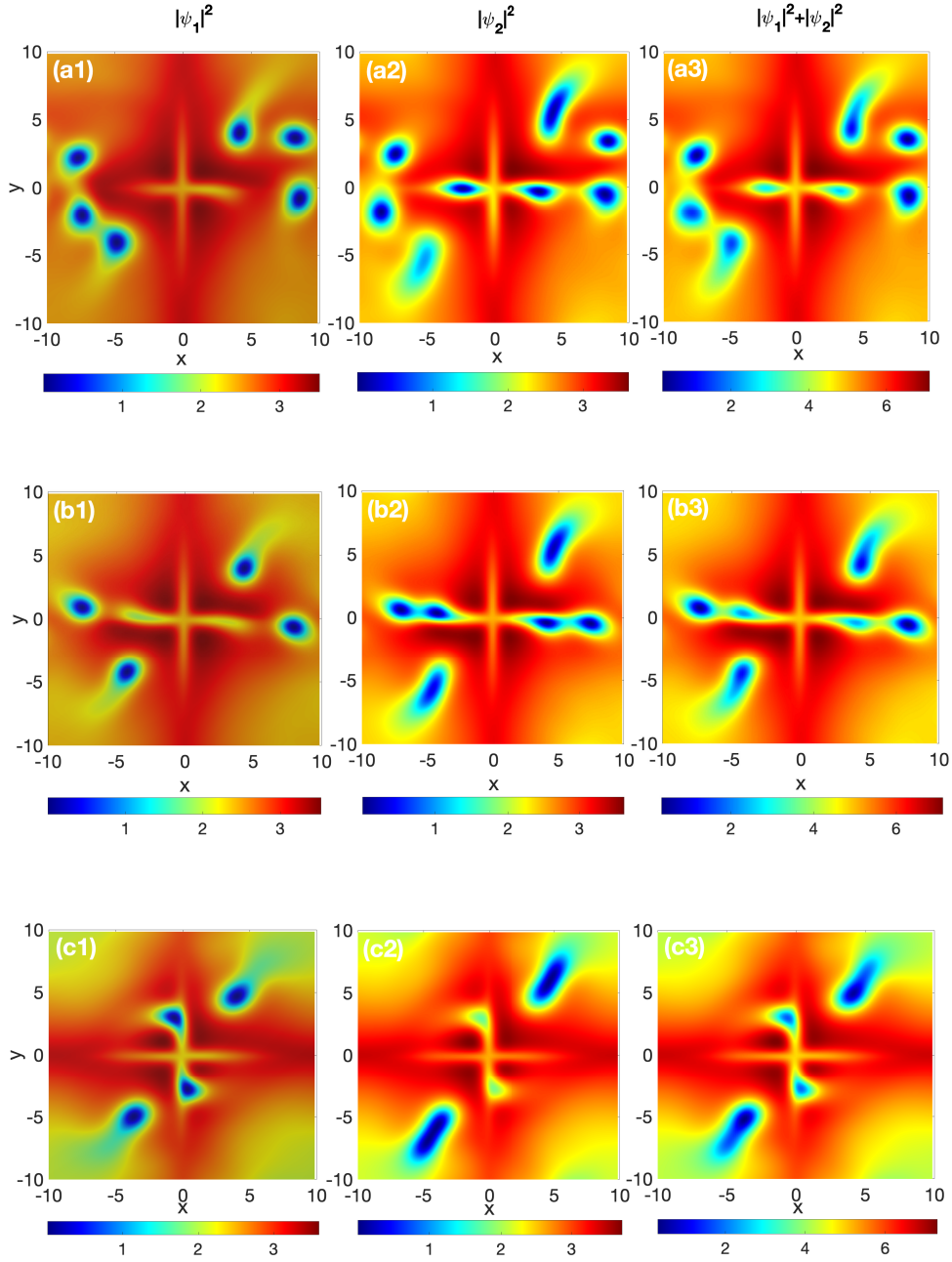


Figure 4.5: Development of MI, with the panels (a)_{j=1,2,3}, (b)_{j=1,2,3} and (c)_{j=1,2,3} corresponding respectively to $\sigma = 0.75$, $\sigma = 0.9$, $\sigma = 0.12$ and the other parameter values being: $I_1 = I_2 = 2$, $G = 1.1$, $g = 3.64$, $\tilde{g} = -0.364$, $K_1 = K_2 = 0.75$, $\Gamma = 3$, $h = -0.1$, $n_{10} = n_{20} = n_0 = 1$, and $B = 0.01$. From left to right, the columns correspond to $|\psi_1|^2$, $|\psi_2|^2$, and $(|\psi_1|^2 + |\psi_2|^2)$.

Chapter 5

Conclusions and outlooks

In summary, the theory of MI has been utilized to study the emergence of matter waves in nonlinear EPCs. Under the adiabatic approximation, the polariton condensate and the exciton reservoir dynamics have been reduced to a set of coupled 2D CGL equations. Based on the linear stability analysis of CW solutions, we have derived an expression for the MI growth rate, which has been exploited to investigate the roles of pumping powers and the photonic SO interactions on the MI gain spectrum. Except for the magnetic field that does not contribute to the analytical growth rate of MI, we have found that increasing the photonic SO coupling values brings about additional modes in the growth rate spectrum. Moreover, increasing input power promotes MI growth rate, especially when the strengths of interatomic interactions are suitably tuned. Our analytical predictions have been verified numerically, where a particular interest has been given, *primo* to the SO coupling strength, with the emergence of solitonic clusters, and, *secundo*, to the competition between the SO coupling and the magnetic field. Due to the absence of the magnetic field in the expression for the MI growth rate, its effect has been obtained via phase diagrams, computed directly using the 2DCGL equations, that have been utilized to generate more structures, among which clusters of bright solitons and composite clusters of dark solitons also known as dips. This work opens exciting possibilities in view of investigating novel driven-dissipative nonlinear phenomena with polariton condensates in a controlled platform involving the photonic SO coupling and the magnetic field, among other effects such as the dynamical reservoir regime, multimode dynamics, and instabilities [94, 96, 113].

A potential application can be the engineering of polariton lasing systems consisting of polariton laser arrays and polariton simulators, especially in the context where magnetic field and photonic SO coupling are considered. This constitutes an open question to be addressed in a forthcoming work.

We note that our study is limited to fast reservoir relaxation, where the adiabatic approximation is valid. The extension to full reservoir dynamics is necessary if the reservoir relaxation rate is less than the out-of-plane loss.

In this regime, the adiabatic elimination of the reservoir density evolution should no longer be valid. This problem, which can be solved using the non-Hermitian coupled-mode theory for EPCs [113], is an interesting topic for future investigations.

A more accurate model considering four coupled spin dependent equations for the EP condensate through the two pairs of exciton and the cavity photon fields coupled to the reservoir equations without the fast reservoir relaxation can be used to study the elementary excitations around the steady state of the system. The theoretical approach relies on the linear stability analysis of continuous waves from which a far more complicated dispersion relation can be expected and finally analyzing the real and imaginary parts of the eigenfrequencies of the Bogoliubov quasi-particles [59, 60].

Another interesting research direction is to focus on the study of topological excitations in the model, namely vortices via a variational approach. Since there is an inherent lack of a closed form solution for vortex rings, the approximate analytical solutions of the vortex radial profiles in the EP BECs are highly desirable in order to systematically describe the structure, dynamics of the vortex superposition pairs and connections between system parameters. Regarding the vortex pairs, the magnetic field is a crucial parameter in preferential selection of these states. At low magnetic fields, the most favourable configuration of vortices is a vortex in one component and an anti-vortex in the other component. Beyond the critical magnetic field and depending on the sign of the magnetic field, other combinations of vortices can be supported by the system [24]. A fascinating topic to explore with this would be the azimuthal modulational instability [114] of the different combinations of vortices on both components accessible via the magnetic field.

Vortices in BEC systems are essentially hydrodynamic in nature. Therefore, it is necessary to explore fundamental hydrodynamic instabilities, such as the Rayleigh-Taylor instability in the context of EP BECs. It was revealed that it is possible to generate quantized vortex rings under the caps of the mushroom patterns formed when the Rayleigh-Taylor instability emerges at the interface between two immiscible BECs that are pushed toward each other by a magnetic field gradient [115]. Vortex formation by the Rayleigh-Taylor instability in SO coupled EP BECs is thereby an interesting future problem that to our knowledge, has not been touched on so far. Another interesting hydrodynamic instability to consider in EP BECs is the Kelvin-Helmholtz instability, which is known to induce flutter-finger patterns of the interface, the fingers then disintegrate creating quantized vortices at each tip of the finger [116]. The Richtmyer-Meshkov instability, another fundamental hydrodynamic instability can also be extended to SO coupled EP BECs future studies. Noting that the magnetically induced Richtmyer-Meshkov instability in a two-component BEC has been investigated [117]. Other fluid instabilities to be considered in SO coupled EP BECs in the future include capillary instability also known as Plateau-Rayleigh instability [118], the counter-superflow

instability [119], and lastly the Rosensweig instability [120].

In this thesis, we investigated the case of modulational instability induced pattern formation in the SO-coupled EP BEC. However, Faraday instability is also good candidate for induced pattern formation, and it is a future work to study the Faraday instability in SO-coupled EP BECs. In BECs, Faraday patterns can be induced by periodically driving the strength nonlinearity in the system [121–123] and in SO coupled BECs, the Faraday patterns will emerge even if the interspecies and intraspecies interactions are the same [124].

The possibility of existence of rotating BEC stars at finite temperature has been considered [125]. In future, we would like to extend such studies to address SO coupled rotating EP BEC stars. Furthermore, determining the relationship between spinor EP BECs, black holes, wormholes [30] as well as dark matter, which constitutes a significant proportion of the total matter in the universe is a more challenging topic for future investigations.

Appendices

Appendix A: Coefficients of the nonlinear dispersion relation (3.12)

$$\begin{aligned} P_3 &= 2h + 2e, \quad P_2 = 2S_1f - 2S_2g - 2S_3f - 2S_4g + 2d^2 + 4eh - 2f^2 - 2g^2, \\ P_1 &= -4S_1fh + 4S_2df - 4S_2eg - 4S_3ef - 4S_4df - 4S_4gh \\ &\quad + 2d^2e + 2d^2h - 2ef^2 - 2eg^2 - 2f^2h - 2g^2h, \\ P_0 &= 4S_1S_3f^2 - 4S_1S_g^2 + 2S_1d^2f + 4S_1dh + 2S_1f^3 - 2S_1fg^2 \\ &\quad - 4S_2S_4f^2 + 4S_2S_4g^2 - 2S_2d^2g + 4S_2def - 2S_2f^2g + 2S_2g^3 \\ &\quad - 2S_3d^2f - 4S_3deg + 2S_3f^3 - 2S_3fg^2 - 2S_4d^2g - 4S_4dfh \\ &\quad - 2S_4f^2g + 2S_4g^3 + d^4 + 4d^2eh + 2d^2f^2 - 2d^2g^2 + f^4 - 2f^4g^2 + g^4 \end{aligned} \tag{5.1}$$

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List of Publications

This thesis is based on the work that led to the following publication;

Edmond B. Madimabe, Conrad B. Tabi, Camus G. Latchio Tiofack, and Timoléon C. Kofané, **Modulational instability in vector exciton-polariton condensates with photonic spin-orbit coupling**, Phys. Rev. B 107, 184502 (2023).