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COMPREHENSIVE ANALYSIS OF SELECTED CANDIDATE BINARY SYSTEMS IN THE *TESS* FIELD

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A Dissertation Submitted to the Faculty of Sciences in Fulfillment of the Requirements for

The Award of the Master of Science in Physics of BIUST

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Acknowledgement

First and foremost, I want to express my gratitude to the Almighty God for assisting me in successfully completing my master's research within the time span allocated.

I would like to acknowledge my supervisor, **Dr. Ceren Ulusoy**, for her consistent support of my master's research and study, as well as her patience, energy, passion, and extensive expertise. During the research and writing of this thesis, her guidance was invaluable. I could not have asked for a better advisor and mentor for my master's degree. I would also like to thank her for informing me at the outset of my research.

I want to express my sincere gratitude to my co-supervisor, **Dr. Burak Ulaş**, for his insightful comments, challenging questions, and encouragement. Without his significant contribution, this thesis would not be conceivable.

I would also like to express my gratitude to my family for their unwavering support during the thesis.

Without you, my friends, this would have been a lot more difficult task. Thank you all for your continuous support and for encouraging me to relax and enjoy myself when I am stressed.

Dedication

My supportive husband, who has been at my side ever since I began my studies, has been a motivation and a rock throughout this scholastic journey. To my son, who provided me the motivation to complete this research thesis.

Abstract

In this thesis, we investigate the binary star systems AK Aqr and KZ Eri observed in the *TESS* (The *Transiting Exoplanet Survey Satellite*) field, which display light variation like binary stars. Binaries, as opposed to single stars, play a very crucial role in the provision of accurate data of light variation and radial velocity of the star. Therefore, our research is based on determining the orbital, evolutionary and absolute parameters of the stars AK Aqr and KZ Eri. The MAST (Mikulski Archive for Space Telescopes Portal) online interface was used to collect data for the targets. The PHOEBE software, established on the Wilson-Devinney binary code, is used to evaluate the light curves. A python application was created to automate the download of the enormous list of the two selected stars. The PHOEBE is used to assess the curve for AK Aqr in over contact binary. Principal component's effective temperature is set at 8000 K during the solution. The inclination i , the secondary component temperature T_2 , the mass ratio q , the surface potential values $\Omega_1 = \Omega_2$, and the primary component luminosity L_1 are all changeable factors. The limb darkening coefficients (x_2 and x_1) are selected based on temperature data from Claret (2017), which provides the *TESS* light curve values. The semi-detached configuration is assumed during the solution of KZ Eri, because the light curve exhibits obvious similarities to classical semi-detached binaries. During the solution, the surface potential values $\Omega_1 = \Omega_2$, the inclination i , the mass ratio q , the temperature of the secondary component T_2 , and the primary component luminosity L_1 are all modifiable factors. The additional parameters are specified using the same approach as for the EK Aqr analysis. The positions of the components on the mass-radius plane and Hertzsprung-Russell diagram are then represented.

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List of abbreviations

U Ma Ursae Majoris

P_{max} Maximum Period

P_{min} Minimum Period

CCD Couple Charged Device

$M_{\odot}$ Mass of the Sun

$R_{\odot}$ Radius of the Sun

$L_{\odot}$ Luminosity of the Sun

h.m.s hour minutes seconds

d.m.s degree arcminutes arcseconds (sexagesimal time from degree)

d Day

Mag Magnitude

Aqr Aquarius

Eri Eridanus

Research and Publications

Ulaş, B., Ulusoy, C., Erkan, N., Mothusi, M and **Matsete, M.**, 2022, Discovery of pulsating components in eclipsing binary systems through the TESS light curves: The cases of CPD-30 740, HD 97329, V1637 Ori and TYC 683-640-1,

<https://link.springer.com/article/10.1007/s10509-022-04053-6>

CHAPTER ONE

1. Introduction on Binary stars

1.1 Dissertation Outline

The following is the dissertation's structure:

1. Chapter 1 presents the historical background of binary stars and introduces the problem statement and framework of the thesis.
2. Chapter 2 covers the literature on binary stars, their categorization, star evolution, light curve classification, and *TESS* literature.
3. In chapter 3 we present how data were obtained through the MAST website, as well as the models utilized in light curve analysis, such as the Roche model, the Phoebe software, the Wilson- Devinney code as well as the Binary Maker software.
4. Chapter 4 includes the results of this research, as well as its interpretation and discussion.
5. Chapter 5 presents the discussion and conclusion of this thesis.

1.2 Historical Background

In the last two decades, our understanding of binary stars has greatly improved because binary stars allow scientists to compute critical data like stellar composition, mass, radius, and temperature which would be difficult to do in a single star system. Binary stars are relatively abundant in the universe. Over half of the stars in our Milky Way galaxy are binary or multiple (Kamilla et al., 2011). J. B. Riccioli, an Italian astronomer, discovered the visual binary 165 Mizar (ζ U Ma) in 1650 shortly after the first telescopes were made (Niemela, 2001). Many more were discovered shortly after the first binary star was discovered, including three components of Θ Ori by Huygens, α Cen by Father Richaud, γ Vir by Bradley & Pound α Cru by Father Fontenay,

(Niemela, 2001). However, it was not until 1767 that Reverend John Michell demonstrated that such stars were likely physically linked systems (Barnard, 2022). Michell discussed that the frequency of near stars around a given star often exceeds that expected for the observed average surface density (Zinnecker, 2001). Michell is credited with being the first person to discover these systems among the visible binary stars.

The British Astronomer, William Herschel (1738-1822) presented his famous article in July 1802 in which he invented the term "binary star" and defined it as "a real double star - the union of two stars that are found together in one system, by the laws of attraction" (Zinnecker, 2001). William Herschel was also the first to discover that gravity exists not only on the inside but also on the outside of our solar system. Herschel (1792-1871) discovered around 800 double stars, and his son John Herschel (1792-1871) carried on the investigation and cataloged over 10,000 binary and multiple star systems (Niemela, 2001). Even though Michell was unable to determine whether the pair of stars he discovered was a physical binary. Christian Mayer (1781) provided the first list of 80 binary system (Schlimmer, 2007). According to Christian Mayer, the stars in a binary system could be small suns orbiting larger suns added. However, there was no evidence to support up his claim.

1.3 Problem Statement and Framework of the thesis

Since several aspects of the evolutionary pathway of binary systems are still unknown and being explored by many researchers, the accurate identification of binary parameters enables us to draw more precise conclusions about the evolutionary history of different types of binary systems. These results will also allow us to better understand the evolution of double stars. The light variation of an eclipsing binary star system should be discovered because it discloses information about individual stars and their orbits. We expect eclipsing binaries to be classified as either contact, semi-contact, or detached binaries.

In this research, we investigate two systems in the *TESS* (*Transiting Exoplanet Survey Satellite*) field, which exhibit light variation like binary stars. We obtained orbital, evolutionary, and absolute parameters of the selected candidate binary stars by the completion of this research. The data from the targets were collected using the MAST (Mikulski Archive for Space Telescopes Portal) online interface. To do this, a Python script was built to automate the download of a large list of chosen stars. As part of the pre-analysis procedure, fluxes were transformed to magnitudes and normalized fluxes. The orbital parameters of the binaries were determined by analyzing light curves with the PHOEBE (PHysics Of Eclipsing BinariEs) software. If possible, we also looked for ways to achieve the radial velocities from the literature. The results of the analyses were used to calculate absolute parameters. We derived the evolutionary routes of the systems and debate their evolutionary features using precise parameters derived from our investigations. The *Transiting Exoplanet Survey Satellite* which is (*TESS*) is a NASA missions that examines the entire sky for exoplanets that transit nearby bright stars. Our targets are selected from among the stars with *TESS* photometric data of good quality.

CHAPTER TWO

2. Literature Review

2.1 Binary Stars

Binary stars are described as two stars orbiting a shared mass center that are held together by gravity (Batten, 1973). The brighter star is known as the primary, while the dimmer star is called a secondary or companion star. Since it is extremely difficult to obtain accurate data for a single star, binaries play a major role in providing accurate light and velocity data for binary stars on several stellar parameters (Pattnaik et al., 2011). Kallrath and Millone (2009) and Prša (2011) emphasized the importance of binaries, in that the masses of their component stars may be directly estimated by calculating their orbits. Martin et al. (1998) compiled a list of star masses of astrometric binaries which can be described as nearby stars that do not have visible companion and can be seen to wobble around a point in space). Other stellar parameters, such as radius and density, can therefore be inferred indirectly using these masses. As magnitude, in absolute terms of the constituents, can easily be determined from their luminosity, binary stars are the best distance estimators. It would be difficult to fully analyze all forms of variable, sub-stellar, stellar, and unique elements without the presence of double stars. (Prša, 2011).

2.2 Classification of Binary Stars

Binary stars are classified based on their unique observational properties (Ostlie, 2019). The next sections will go over the three different types of binary systems:

1. Visual
2. Spectroscopic
3. Eclipsing

2.2.1 Visual Binaries

Some binary stars are visible to the naked eye or with the help of telescopes. These are referred to as visual binaries. They are systems with component stars that are tangibly separated by tens to hundreds of Astronomical Units (AUs) (Pettini, 1987). Even though the stars are gravitationally connected to one other in this form of observational binary, they do not interact in the same way that stars in other tight binaries do, where a star can suck elements from the surface of another star, because they have large distances between them.

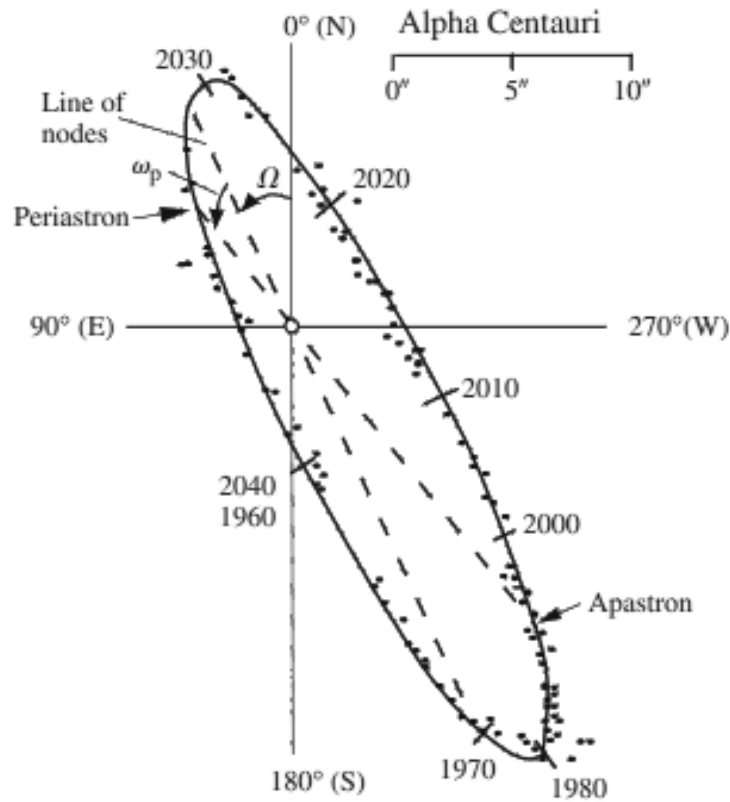


Fig. 1 α Centauri visual binary system. This is a graph of one of the two stars' relative locations projected onto the plane of the sky as a function of time (years). The radial distances are in arc seconds, and the origin is one of the components (Bradt, 2008).

Kepler's third law and conservation of momentum enable the masses of each star to be resolved, and that other factors like inclination, semi-major axis, and star size may also be calculated (Kallrath and Millone 2009). Consider two stars M_1 and M_2 that are orbiting each other. Let r_1 and r_2 be the vectors that characterize the positions of stars 1 and 2 respectively and $r = r_2 - r_1$ be the vector distance between them, with a being the elliptical orbit's semi-major axis.

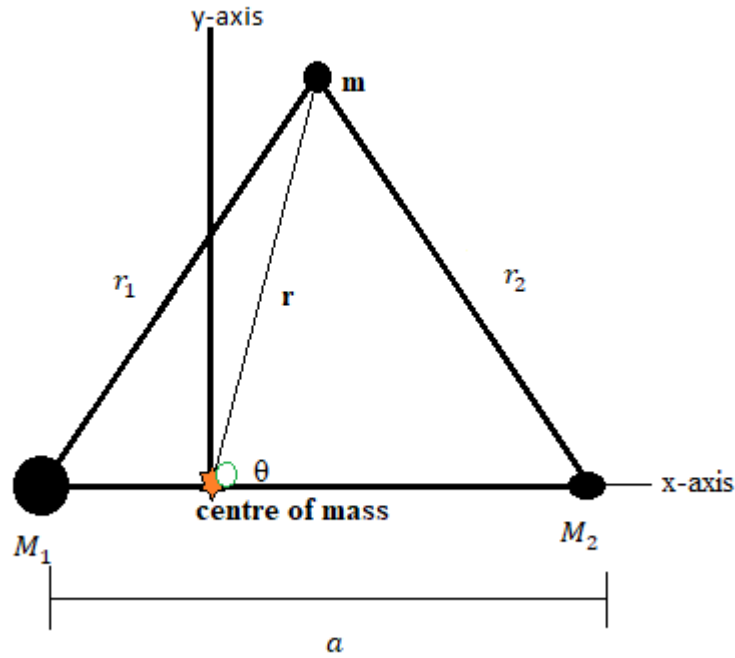


Fig. 2 M_1 and M_2 orbiting each other.

If we build up our coordinate system with the origin as the center of mass, then,

$$\frac{M_1}{M_2} = \frac{a_2}{a_1} = \frac{r_2}{r_1} \quad (1)$$

The problem is solved by calculating the distance between the two stars while they are at an angle θ in their orbits.

$$r = \frac{a(1 - e^2)}{1 + e \cos \theta} \quad (2)$$

Where e is eccentricity of the star.

The orbital period is determined by.

$$P^2 = 4\pi^2 \frac{a^3}{G(M_1 + M_2)} \quad (3)$$

(Newton from Kepler's 3rd law)

This shows how the distance between the two stars changes over time, but we are more interested in how the two stars themselves move. The distance between each of the stars and the mass in the center is calculated as follows:

$$r_1 = \frac{\mu}{M_1} r = \frac{\mu}{M_1} \left[\frac{a(1 - e^2)}{1 + e \cos \theta} \right] \quad (4)$$

$$r_2 = \frac{\mu}{M_2} r = \frac{\mu}{M_2} \left[\frac{a(1 - e^2)}{1 + e \cos \theta} \right] \quad (5)$$

Where μ is the proper motion of the star. $\theta = 0$ and $\theta = \pi$, these obviously achieve the minimum and maximum values, and the semi-major axes of the two ellipses characterizing each star's orbits are provided by half the sum of the minimum and maximum:

$$a_1 = \frac{1}{2} [r_1(0) + r_1(\pi)] = \frac{\mu}{M_2} a \quad (6)$$

Pettini (1987) mentioned that when the elliptical plane is shifted at an angle of i to the sky plane and the two planes overlap along a line parallel to the star orbit's minor axis, forming a line of nodes, one should think about the fact that we can only observe binary stars that are close to us limits this form of observation. The parameters of binary systems are often derived by astrometry, spectroscopy, or photometry and, in some circumstances, a combination of two or even these three approaches. Visual orbits have been catalogued by Worley and Douglas (1984), and speckle measurements (the ratio between the camera lens's clear aperture and focal length) by McAlister and Hartkopf (1988) and Eggleton (2006).

2.2.2 Spectroscopy

Spectroscopic binaries are those discovered due to orbital motion, and they cause the radial velocities to change (Southworth, 2020). Since the orbits have a low inclination angle, the stars are sufficiently apart, and some near binaries do not eclipse each other. The double nature of stars can only be determined by identifying periodic Doppler shifts of one or both stars in the spectral lines (Bradt, 2008). The system can still be deduced by looking at the spectrum. Prša (2011), Kallrath and Millone (2009) and Kamilla et al. (2011) reviewed the spectrometric method in detail by stating that using this strategy, stars that cannot be resolved by telescopes can be detected by changes in radial velocity. Petrie (1939) also showed that spectra alone may be used to calculate the brightness of a star. We can determine the mass ratio if we have a radial velocity curve.

2.2.2.1 Velocity for Spectroscopic Binaries

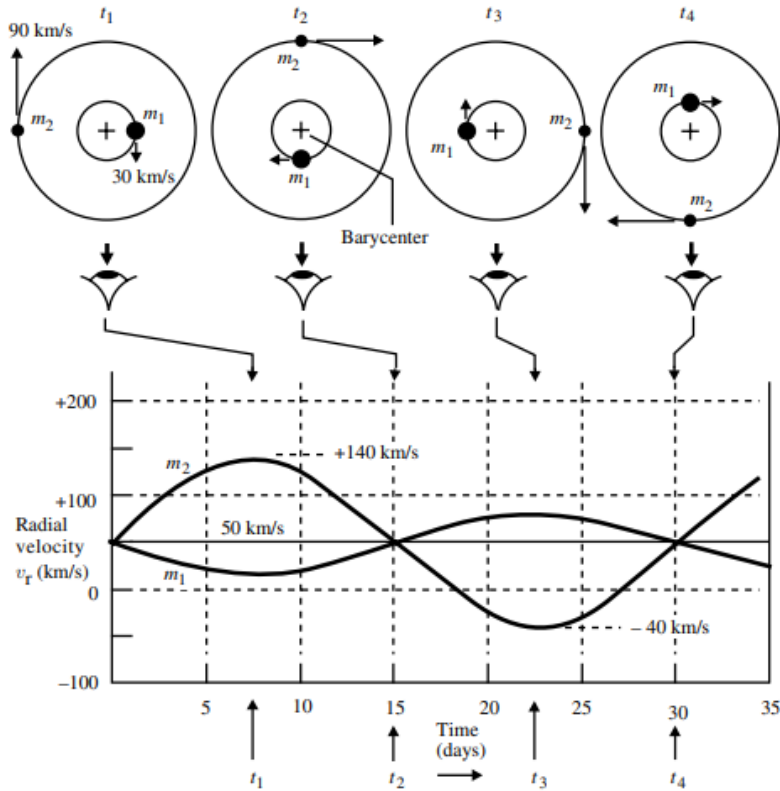


Fig. 3 Four phases of a hypothetical spectroscopic binary with circular orbits and a 30-day period. The observer is in the orbit's plane. The mass of star 1 is three times that of star 2. The two stars are always on opposing sides of the barycenter, which is traveling away from the viewer at a constant radial velocity component of + 50 km/s. In the upper left, the star speeds relative to the barycenter are displayed. Star 2 is three times farther out from the barycenter than star 1 due to its smaller mass; as a result, it must travel three times quicker to complete the orbit in the same amount of time as star 1. The astronomer's direction and observed Doppler velocities are displayed (Bradt, 2008).

The star must have sufficient radial velocity to be recognized as a spectral Doppler shift, as well as be highly luminous to provide sufficient high-resolution spectroscopic photons (Bradt, 2008). Assuming the observer is to the right, motion toward the observer results in a blue shift (negative velocity), whereas motion away from the observer results in a redshift (positive velocity) (Pattnaik et al., 2011). The eccentricity of the orbit of a double star and its time of revolution are inextricably linked to each other. When two stars have the same brightness, their spectral types are also the same. The mass of a star can only be resolved by its gravitational attraction (Pattnaik et al., 2011). Apart from the Sun and other stars that operate as gravitational lenses, binary and multiple star systems are the only places where this may happen, making binary stars an essential classification of stars. For visible binary star, the combined mass of the two stars can be calculated using the Keplerian harmonic law after the stellar parallax and orbit and of the system are known (Prša, 2011). When just one component is observed (single lined spectrometry; SB1), mass and radius cannot be determined since they require inclination, unlike double lined spectrometry; SB2, we can use the mass function to determine these values (Prša, 2011).

2.2.2.2 Double-lined spectrometry (SB2)

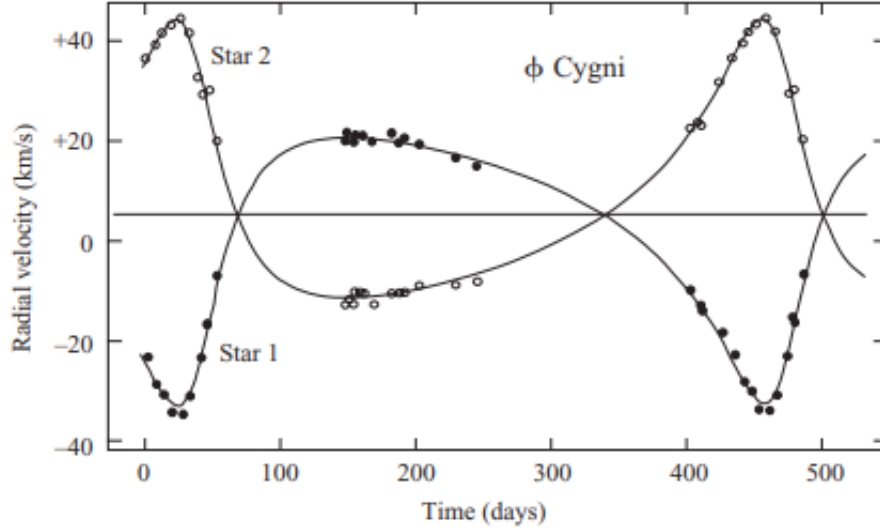


Fig.4 depicts the Doppler frequency changes of the spectral lines were used to calculate ϕ Cygni's radial velocities with respect to time. This is a double-line spectroscopic binary system made up of two almost equal-mass giants. The theoretical fits to the data points are the smooth curves. The unusual shape is caused by the orientation of the elliptical orbit relative to the optical axis and the non-constant velocities of the orbit. The observer moves away from the barycenter (Bradt, 2008).

Due to the orbital plane's undetermined inclination i to the line of sight, we only get the lower bounds to the true velocities: $v_1 \sin i$ and $v_2 \sin i$. The orbits of many spectroscopic binaries are practically circular, as tidal interactions have shorter timeframes than star lifespans, so they are more likely to circularize orbits (Pattnaik et al., 2011). When the eccentricities are modest ($e \ll 1$) the orbital velocity is practically constant: $v = 2\pi a/P$, a is the semi-major axis's radius and P is the period (Pettini, 1987).

$$\frac{M_1}{M_2} = \frac{v_2}{v_1} \quad (7)$$

In terms of the observables:

$$\frac{M_1}{M_2} = \frac{v_2 r / \sin i}{v_1 r / \sin i} = \frac{v_2 r}{v_1 r} \quad (7)$$

As with visual binaries, the mass ratio can be calculated without regard to the angle of unknown inclination i . The aggregate of the masses, on the other hand, necessitates knowledge awareness of $\sin i$. Substituting a ;

$$a = a_1 + a_2 = \frac{P}{2\pi} (v_1 + v_2) \quad (8)$$

Solving for the sum of the masses by substituting into a , we obtain

$$M_1 + M_2 = \frac{P}{2\pi G} (v_1 + v_2)^3 \quad (9)$$

Or in terms of observables:

$$M_1 + M_2 = \frac{P}{2\pi G} \frac{(v_1 r + v_2 r)}{\sin^3 i} \quad (10)$$

2.2.2.3 Single-lined spectrometry (SB1)

Pettini (1987) explained single-lined spectrometry in detail saying that in certain instances, just one of the duo's spectra is observed, but periodic shifts in its velocity profile indicate the presence of an undiscovered companion. It is indeed conceivable that this is the case unless one has done the following:

- (a) The secondary star is considered a planet
- (b) The companion is either a neutron star or black hole which are dark objects, and
- (c) The second star is significantly dimmer than the primary star.

In single-lined binaries where $v_2 r$ cannot be measured, the relation $v_2 r = v_1 r M_1/M_2$ can be substituted to obtain:

$$M_1 + M_2 = \frac{P}{2\pi G} \frac{v_1^3 r}{\sin^3 i} \left(1 + \frac{M_1}{M_2}\right)^3 \quad (11)$$

This can be reorganized so that all the measurements on the right hand of the formula are grouped together:

$$\frac{M_2^3}{(M_1 + M_2)^2} \sin^3 i = \frac{P}{2\pi G} v_1^3 r \quad (12)$$

The left-hand part of the equation is the mass function. Even if M_1 is unknown, the mass function can yield intriguing lower bounds on the mass of the unobserved companion. The question then becomes what if the distance between the stars is still so small and the inclination is so great? This is when the eclipsing binaries approach becomes useful.

2.2.3 Eclipsing Binaries

Eclipsing binaries (EBs) are often closer together and the total amount of light changes regularly (Bauer, 2018). Brancewicz & Dworak (1980) documented a catalogue of eclipsing binaries based on proto-type stars which is still in use to this day. (EBs) serve as the foundation of stellar astrophysics. Their particularly important role in determining basic stellar parameters and distances, as well as in carrying out rigorous tests for stellar evolution models has been widely appreciated. Most eclipsing objects are shot-limited which can be described as the sources that produce enough photons for poissonian scattering to predominate have a natural regime), making eclipsing binaries the most precise data collecting methods in stellar astronomy (Prša, 2011).

The relation below is used to convert fluxes to magnitudes:

$$m - m_0 = -\frac{5}{2} \log \frac{F}{F_0} \quad (13)$$

The magnitude zero-point corresponds to the flux F_0 , where m_0 is the magnitude zero-point. Binary systems can be classified in general based on whether their component stars fill their Roche lobe (which is the area of a binary system around a star where material in orbit is gravitationally linked to the star), or not (Rovithis-Livaniou, 2020). As a result, three distinct types of systems have emerged: detached, semi-detached and contact systems (Knote, 2014).

2.3 Morphological classification of close binaries

2.3.1 Detached Systems

There is no mass transfer in this system, since neither component fills their Roche lobe (Kopal, 1955). As a result, they are classified as systems in which both stars are confined to different equipotential surfaces. Since their evolution is independent of the other, detached binaries can be used as absolute physical laboratories for researching the characteristics of stars individually (Prša, 2011).

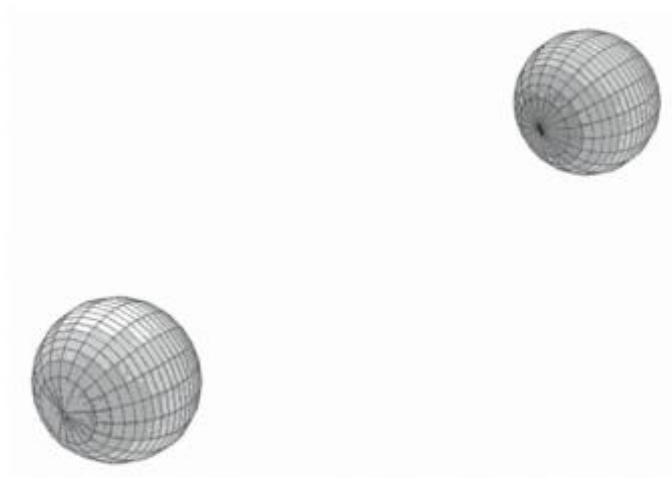


Fig. 5 An example of a detached binary system (Hilditch, 2001).

2.3.2 Semi-detached systems

When one component completely occupies its Roche lobe, mass is transferred from that component to its companion via the first Lagrangian point (Kopal, 1955). Semi-detached systems have one star occupying the Roche lobe while another is restricted to a distinct equipotential. Magnetic coupling, and mass transfer from both stars, plus interstellar material layers are common in semi-detached binaries (Prša, 2011).

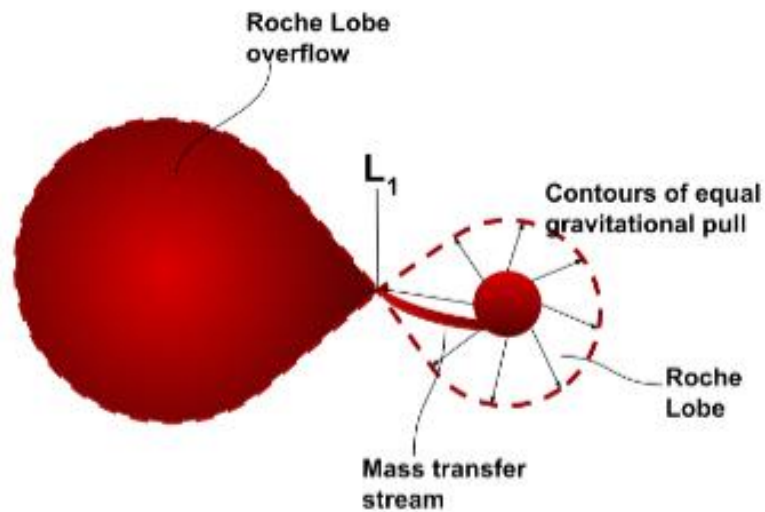


Fig. 6 A semi-detached binary system with mass transfer stream, Roche lobes and contours of equal gravitational pull (George, Misra and Ambika, 2019).

2.3.3 Contact Systems

Both components extend beyond their Roche lobes and make contact (Yakut and Eggleton, 2005). Contact binary stars (also known as W UMa-type stars) are binary stars made up of two dwarf stars with spectral types of K, F, or G (a few known examples of previous spectral types, but no known contact binary stars of the M spectral type. (Csizmadia and Klagyivik, 2004). Despite their often vastly differing masses, the constituents in a contact binary star system have roughly comparable surface temperatures and luminosities (Csizmadia and Klagyivik, 2004).

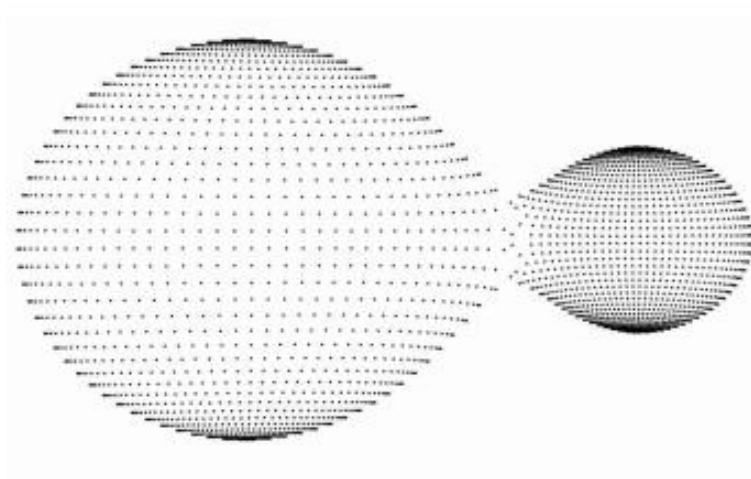


Fig. 7 A contact binary system (Wadhwa et al., 2021).

2.4 Evolution of Stars

2.4.1 The Hertzsprung-Russell Diagram (HRD)

Around 1900, astrophysicists realized that stars differ not just in terms of atmospheric temperature but also in terms of luminosity for the same atmospheric temperature, implying that two criteria are required to characterize a star (Strömgren, 2018). E. Hertzsprung and H. N. Russell discussed the distribution of stars in color-luminosity or color-spectral class diagrams and described the most important features of the distribution, based on observational material pertaining to spectral types, colors and luminosities of stars (Langer and Kudritzki, 2014). Russell discovered that as the spectral class of a star changes from O to M, its luminosity decreases. Strömgren (2018) went on to define HRD as a diagram in 1910 in which stars are represented by points according to their spectral class and luminosity, or color index and luminosity respectively. HRD is a useful instrument for researching the theory of stellar evolution (Pols, 2011). Hurley, Pols and Tout (2000) stated that stars spend majority of their time in the Main Sequence, consuming hydrogen in their cores to make helium. The original Hertzsprung–Russell diagram (Russell, 1914) was a spectral type vs. absolute visual magnitude display (Ball, 2014). Ball (2014) went on to say that in the current version, absolute magnitude takes precedence over hue, and brightness takes precedence over effective temperature.

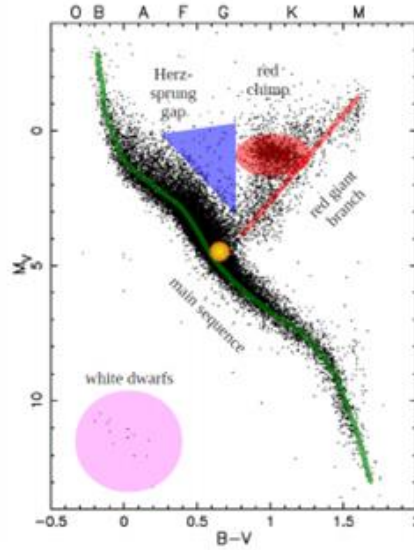


Fig. 8 Hertzsprung–Russell Diagram. This is a modern HRD, compiled using stars from the Hipparchus sample for which distances are precise to better than 5%. Several regions of interest are indicated (Ball, 2014).

Attempts to measure the temperature of a star by its color were more accurate than spectral classification categories. Spectral type OBAFGKM, frequently replace the temperature scale on the horizontal axis. This was more linear than temperature and held more information about the star than just its temperature. The conventional Stefan-Boltzmann law is an idealized scenario for point sources. Size, or surface area, is a property of real objects. Large items cool (emit energy) more quickly than small objects, hence there must be a relationship between those two. The luminosity, temperature and radius of stars are all connected by an expansion of Boltzmann's equation below.

$$L = 4\pi R^2 \sigma T^4 \quad (15)$$

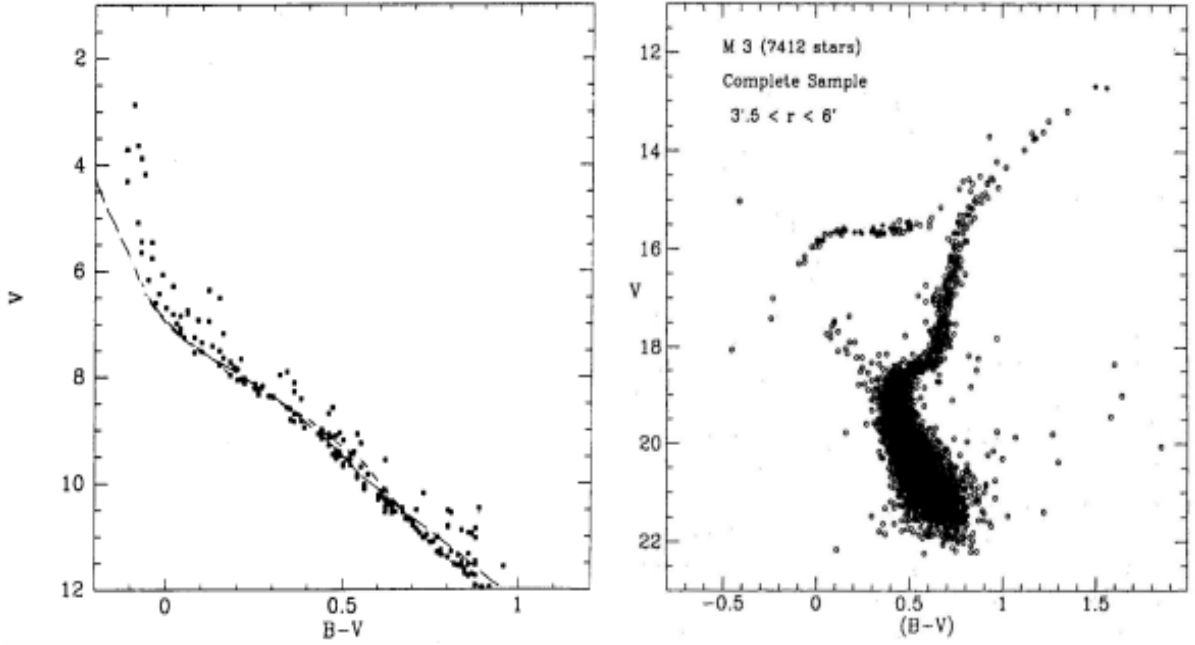


Fig. 9 Young open cluster named M45 (left panel is the Pleiades,) and an old globular cluster, M3, (right panel) (Hurley, Pols and Tout 2000).

Hurley, Pols and Tout (2000) stated that the absolute visual magnitude (M_V) is utilized as a measure of luminance in this observer's HRD, and a color index ($B - V$ or $V - I$) is employed as a measure of effective temperature. Hurley, Pols and Tout (2000) believed that star clusters give a more thorough examination of stellar evolution. Stars with well-defined properties are crucial for acquiring a better understanding of evolution (Yakut and Eggleton, 2005). The authors claimed that stars can exchange material under correct conditions, for example when one of them transforms into a giant or supergiant, or when there is a strong wind. The existence of many modes of mass transfer between the two stars characterizes the evolution of single stars to binaries (Soberman, Phinney and Van den Heuvel, 1997). The distance between the stars determines how the stars evolve; for example, if the separation distance between the stars is large, there is minimal interaction between the stars; yet, if they are nearer together, they affect each other's evolution. Both tidal and viscous forces can carry angular momentum near the Roche lobe (Savonije, 1978).

2.4.2 Mass Transfer Stability

Mass transfer is particularly fascinating when contemplating the development at least one star that is degenerating. It causes stunning effects in these circumstances, due in part to the compact object's strong magnetic and gravitational forces, nuclear burning, pulsed X-ray transmission, novae eruptions, etc. (Negu and Tessema, 2015). Negu and Tessema (2015) mentioned that the dynamical effects of mass transfer on the binary orbit, as seen in Cygnus X-3, may be evident because mass transfer rates can be significant and orbital period measurements can be accurate.

2.4.3 Evolution of a Low Mass Stars: $0.4 M_{\odot} < M < 4 M_{\odot}$

When we consider the following system of two stars: one has transformed into a white dwarf, while the other slowly transfers matter to it. As hydrogen from its companion's outer layers accumulates on the hot white dwarf's surface, it begins to build up a layer of hydrogen and heats up. As a result, the white dwarf brightens up quickly (nova) (Tutukov and Cherepashehuk, 2020), a white dwarf can be driven above the Chandrasekhar limit if it accumulates stellar matter from a companion star at a significantly quicker rate (Tutukov and Cherepashehuk, 2020). When an object's mass reaches the Chandrasekhar mass limit (exceeds $1.4M_{\odot}$), it can no longer support itself as a white dwarf and contracts, increasing its interior temperature a massive amount of fusion can occur at the same time, resulting in an explosion. During the last explosion, fusion energy is created (Tutukov and Cherepashehuk, 2020). Type **Ia** supernovae are the name given to these white dwarf explosions.

2.4.4 High Mass Stars Evolution $M > 8 M_{\odot}$

The evolution of binary stars with masses $M > 8 M_{\odot}$ has long piqued astronomers' interest due to the wide range of high-energy phenomena that accompany their evolution across all spectral ranges (Tutukov and Cherepashehuk, 2020). The authors pointed out that the system switches to the Algol stage which is when the primary component expands after forming its helium-rich core, and the initial masses of the components are near to one other ($M_2 > 0.5M_{\odot}$). After a type-II supernova explosion, a relativistic remnant emerges: a neutron star in the case of $8 M_{\odot} < M < 30 M_{\odot}$ (Negu and Tessema, 2015).

2.5 Light Curve Classification

Eclipsing Algol (EA), Eclipsing β Lyr (EB), and Eclipsing W UMa (EW) are three types of eclipsing binaries, named, respectively, after their corresponding prototype stars Algol, β Lyrae, and W Ursae Majoris (Karamanavis, 2011).

2.5.1 EA Systems - Algol Type System

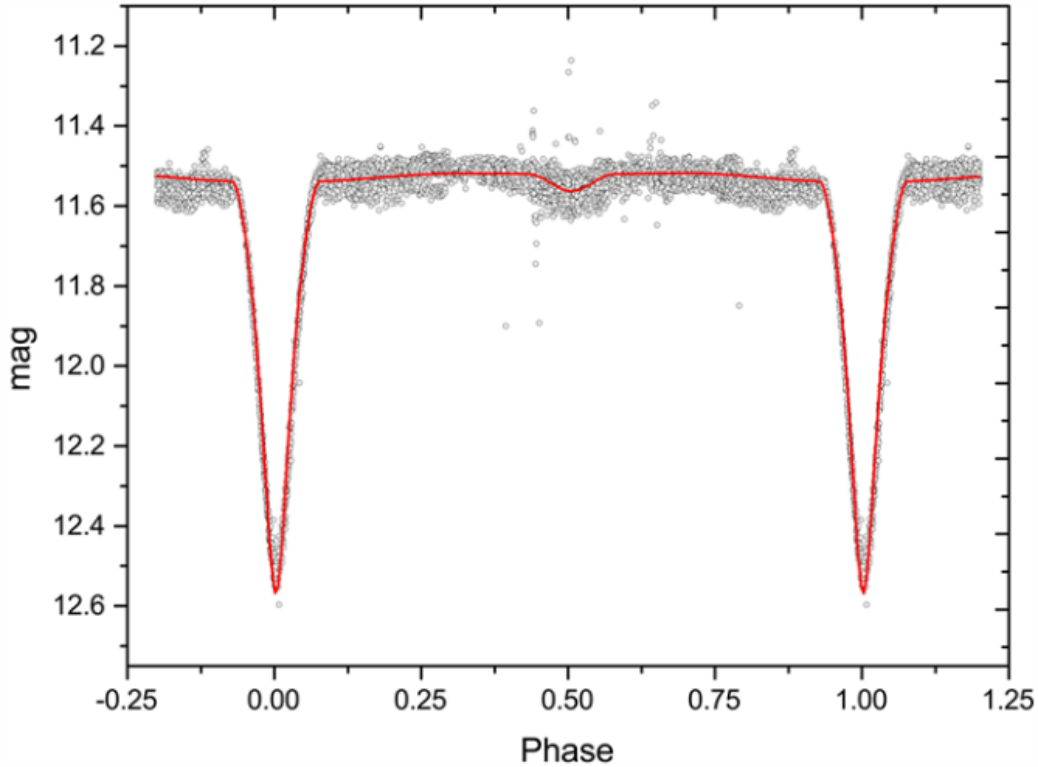


Fig. 10 The light curve of Algol type system. The continuous red line represents a synthetic light curve, and the open circles represent the measured magnitude (Manzoori et al., 2017).

An Algol type's binary system is semi-detached. Due to the filling of its Roche lobe, the less massive star transfers its mass to the more massive component, producing angular momentum and mass loss (Chehlaeh et al., 2017). Outside of eclipses (after one star passes in front of the other), there are systems with little fluctuation in the light curve. The depths of the secondary eclipses primary are varied in Algol systems, which feature flat-topped maxima but well-defined eclipses (Holcomb, 2016). An EA system's period can start from a fraction of a day to many years. The light curve may have a different structure towards the secondary minimum, showing a "reflection effect," which is a re-processing of the hotter star's radiation when it impinges on its companion's atmosphere. The redder component may be so evolved that it is filling its Roche lobe and sending

a stream of material toward its companion in severe circumstances. In fact, Algol is a good example of this. However Not all binaries with Algol-like light curves are in this state, frequently two comparable stars that are not close enough to deform each other's shapes. When stars are far away, spheres can be used to approximate their shapes. It should be noted that the spherical approximation is not suitable under all circumstances (Kallrath and Milione 2009). Between the eclipses, the flux remains generally constant; however, the depth of the eclipses varies depending on the developmental stage of the two components that compose the system (Karamanavis, 2011).

2.5.2 EB Systems - Beta Lyrae type systems

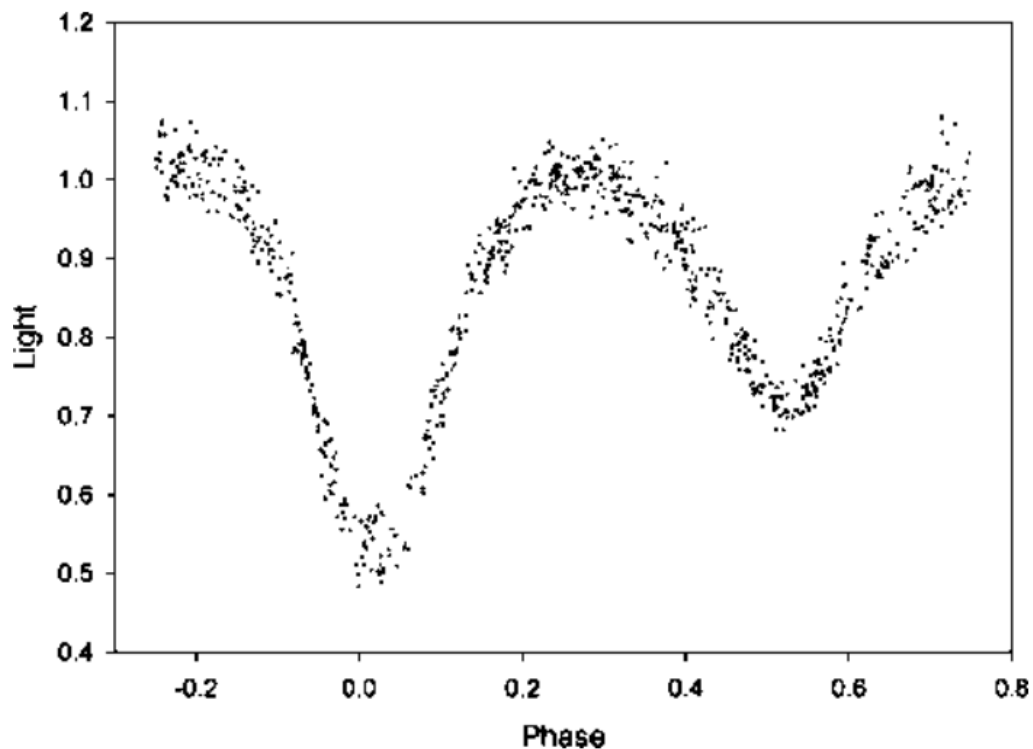


Fig.11 The light curve of β Lyrae (Terrell, 2001).

Light curves in Beta Lyrae systems are constantly changing, with both rounded peaks and minima, and primary and secondary minima of varying depths (Holcomb, 2016). Therefore, the precise periods of minima cannot be determined from the light curve since these are systems where the

stars are close enough to have gravitationally deformed each other. Rather than being spheres, the stars are ellipsoids. Hence there is a slight variation between eclipses, making it difficult to establish the exact start and end of eclipses. However, in some systems, the primary eclipse is highly noticeable and can result in a magnitude reduction of up to two. Between main and secondary eclipses, there is no period of steady light. These are generally semi-detached systems.

2.5.3 EW Systems - W UMa type systems

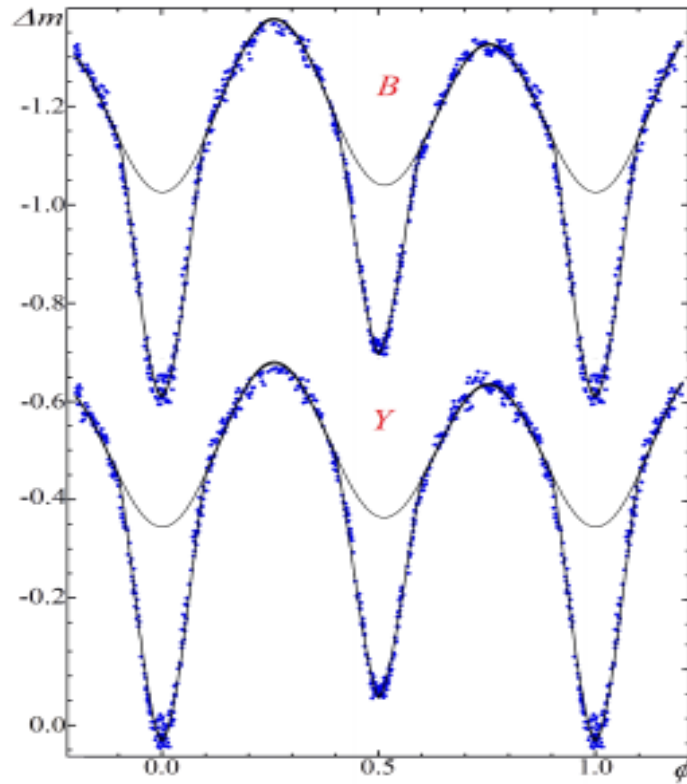


Fig. 12 Light curve phase of W UMa in blue (B) and yellow (Y) filters (Tkachenko, Andronov and Chinarova 2016).

W UMa-type binaries, which are also referred to as EW stars are eclipsing binaries and have orbital periods ranging from 0.2 days to one day (Karamanavis, 2011). W UMa type variables are divided into A-type and W-type subclasses (Oklahoma, 1999).

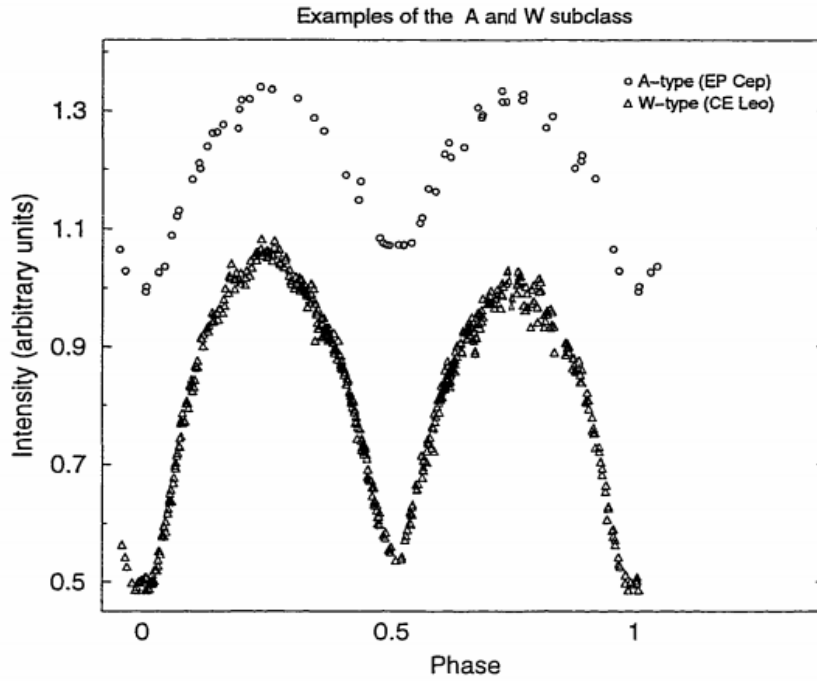


Fig. 13 The subclasses A-type and W-type of Ursae Majoris (Karamanavis, 2011).

The light curve shape at phase = 0.5 is one of the identifying features between the A-type and W-type (Oklahoma, 1999). The author continues to state that near this region, the curve is flat, indicating that the smaller star is in front and near this phase, the W-type curve is not flat. However, near phase = 0.0, flatness can be seen. For a W-type, this is where the smaller star is in front of the larger one (McCartney, 1999). The variation in light curve shape during primary and secondary eclipses demonstrates the distinction between the two types. Both stars are main sequence stars with spectral classifications ranging from A to K (Oklahoma, 1999). Karamanavis (2011) stated that during an orbital period, the spectral color or type of a W UMa star does not change. This means that the common cover is visually thick and maintains a relatively constant temperature. The mass ratio (q) ranges from 0.08 to 0.8 (McCartney, 1999). Only 542 EW systems are included in the GCVS (General Catalogue of Variable Stars). For a small percentage of those, light curves are available, and radial velocity measurements are available for even fewer (Karamanavis, 2011). Light curves also change continuously, but the depth of the minima tend to be equal (Holcomb, 2016). The stars are in contact, or almost in contact are eclipsing binaries. The stars are warped, and they may share a material envelope. Eclipses are not predictable at the beginning or at the end,

as the light curve changes constantly. They usually have amplitudes of less than 0.8 magnitudes. The orbital period of the stars might be as short as a few hours because they are so close together (Li et al., 2007).

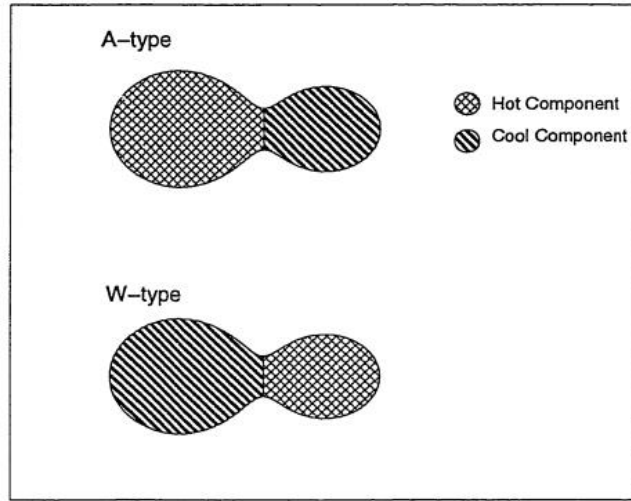


Fig.14 Two classes of the W UMa-type binaries. Physically, the larger star is hotter and more massive than for the A types. The physically larger star is cooler and more massive for the W types. The Roche lobe depiction was constructed here, as well as in all subsequent plots, using C.H. Kim's program (Holcomb, 2016).

The typical paradigm for explaining these disparities is the temperature direction (McCartney, 1999). Based on whether either primary or secondary has a greater temperature, two sub-classes (A-type and W-type systems) can be distinguished. The more substantial component in A-type systems (spectral types A to F) has the higher temperature, whereas the smaller component in W-type systems (spectral types G to K) has the higher temperature (Holcomb, 2016).

2.6 *Transiting Exoplanet Survey Satellite (TESS)*

In this study, we investigate *TESS* targets that show light variation like binary systems. The targets' data are collected using the MAST (Mikulski Archive for Space Telescopes Portal) online

interface. Prior to *TESS*, NASA's *Kepler* mission revolutionized exoplanetary science by finding that planets between the sizes of Earth and Neptune were plentiful and appeared to have a diverse range of compositions and fascinating orbital patterns (Ricker et al., 2014). According to the authors, most of *Kepler*'s stars are too feeble for detailed follow-up studies. Parker et al. (2018) stated that *TESS* follow *Kepler*'s lead, and search for transiting planets among the nearest and brightest stars. According to Ricker et al. (2014), the primary goal of *TESS* is to find hundreds of transiting exoplanets that are smaller than Neptune, with primary stars bright enough to estimate planetary masses and compositions and for follow up spectroscopy.

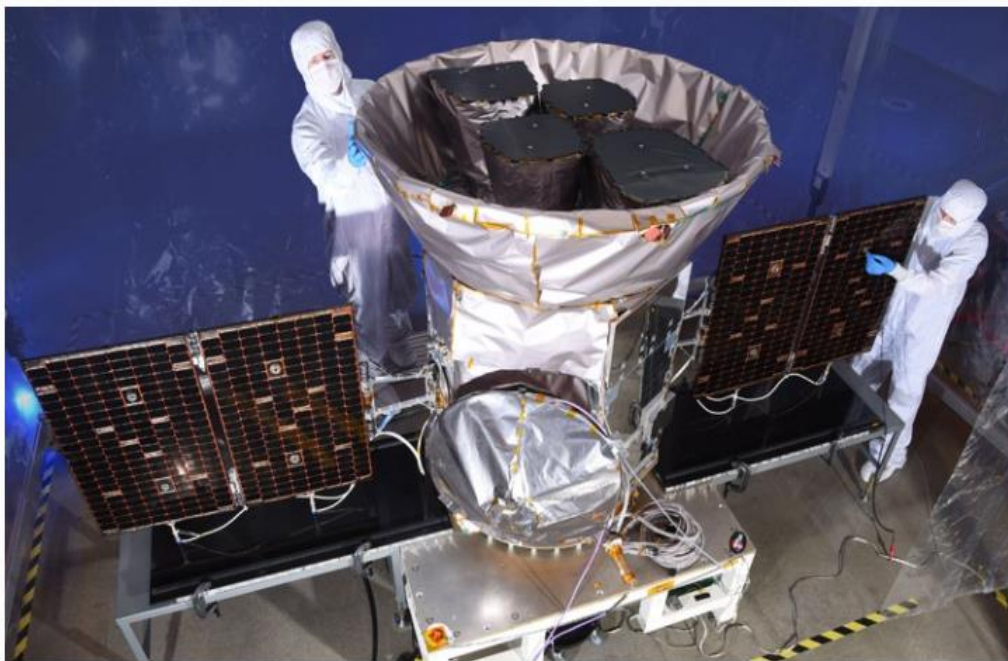


Fig. 15 Shows the fully integrated *Transiting Exoplanet Survey Satellite (TESS)* (Rinehart, 2018).

2.7 *TESS* Mission

The *Transiting Exoplanet Survey Satellite (TESS)* is a NASA Explorer project that searches for exoplanets using the transit method (Gregory et al., 2017). The authors indicated that *TESS* is the first telescope to survey the entire sky and spend two years looking for exoplanets by studying the

brightest stars around the earth. *TESS* has provided excellent targets for additional instrument characterization.

The key science requirements for the *TESS* mission are:

1. Its purpose is to explore over 200,000 stars for planets (Rinehart, 2018) with epochs of 10 days and radii of 2.5 Earth radii revolving around the brightest stars, as well as planets with radii of at least 2.5 Earth radii spread around the celestial sphere (Barclay, 2017).
2. Search the 10,000 stars near the ecliptic poles for traversing planets with cycles up to 120 days. (Ricker et al., 2014)
3. Determine at least 50 planets' masses with radii not larger than 4 Earth radii. Barclay, 2017)

2.8 Design Consideration

2.8.1 Sky Coverage

TESS observes about 85% of the sky, and that is 400 times more than *Kepler* 's did during its main two-year mission. (Barclay, 2017). In comparison to the *Kepler* Mission, *TESS* observes many nearby bright stars. As the brightest stars are dispersed over the entire sky, the *TESS* mission relies on an all-sky survey (Ricker et al., 2014).

2.8.2 Period sensitivity

TESS travels around the Earth in a high-inclination, high-eccentricity orbit with a period equal to the Moon's sidereal orbital period (Pál, Molnár and Kiss, 2018). The mission's goal is to find planets with a wide scale of orbital periods, with P_{min} close to the Roche limit of just few hours and P_{max} of a year or more (Ricker et al., 2014). The authors further stated that the identification of habitable-zone planets near Sun-like stars is ruled out if P_{max} is set to less than 10 days.

Therefore, it is possible to acquire access to planets with habitable zone near M stars if at least one percent of the sky is studied with $P_{max} > \sim 40$ days. Barclay (2017) stated that if a field is closer to the ecliptic, it will have a maximum coverage of 27 days; this means that the orbital period of the planet will have to be shorter than this.

2.8.3 Types of Stars Monitored by *TESS*

Spectral classes from F5 to M5 main-sequence dwarfs are the most abundant genuine stars in the cosmos, which includes our sun (Ricker et al., 2014). The authors continue to add that dwarfs with spectral types lower than F5 revolve quickly, which broadens their spectral lines and makes reliable radial-velocity observations difficult. The M dwarfs are the most frequent stars in the Milky Way, accounting for around 75% of all stars (Maldonado et al., 2020).

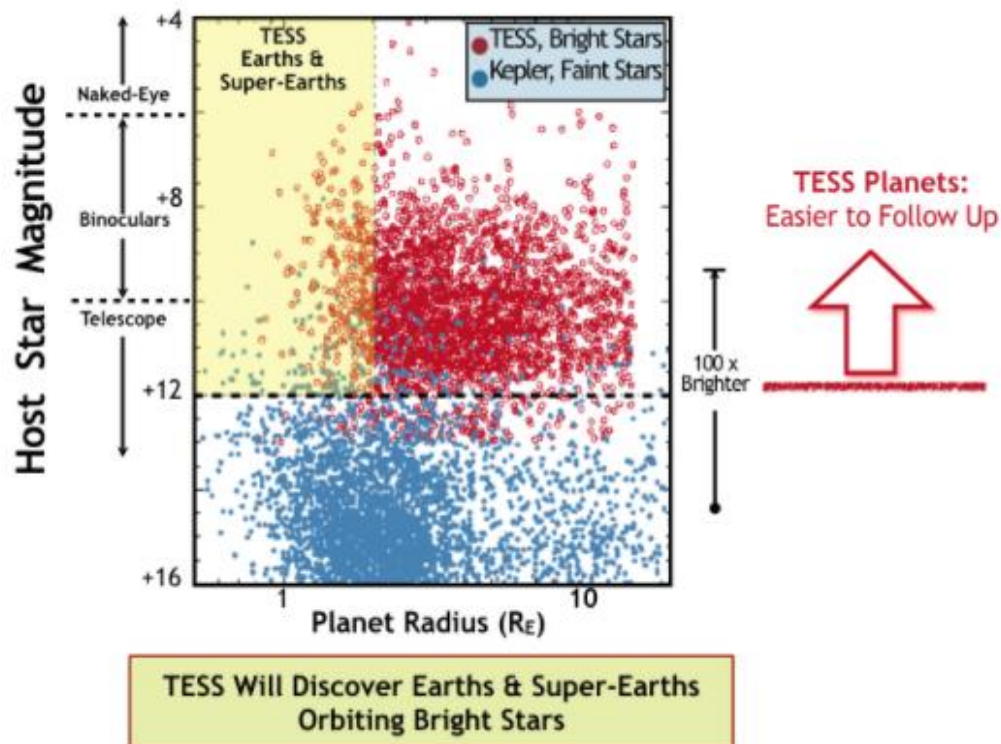


Fig. 16 The types of stars monitored by the naked eye, binoculars, *TESS*, and *Kepler* (Ricker et al., 2014).

2.8.4 Detector

Silicon charge-coupled devices (CCDs) Gregory et al. (2017) are the best astronomical detectors in the visible band (Ricker et al., 2014). They feature excellent linearity and dynamic array for a long history of space flight and highly bright objects (Barclay, Pepper and Quintana 2018).

2.8.5 Band-Pass

Wide band-passes help to reduce photon-counting noise (Ricker et al., 2014). The authors remarked that designing a wide field and optical system that are well-focused can be difficult and highlighted that greater sensitivity to red wavelengths, is perfect since planets that are small are easily spotted rotating around small stars. Following these discussions, a band-pass of 600-1000 nm was chosen for this research project. The 400 nm width was the most practicable choice for the optical design, and the red end represents the red most limit CCD sensitivity. A long-pass filter with a cut-on wavelength of 600 nm also limits the spectral response at its blue end (Sullivan et al., 2017).

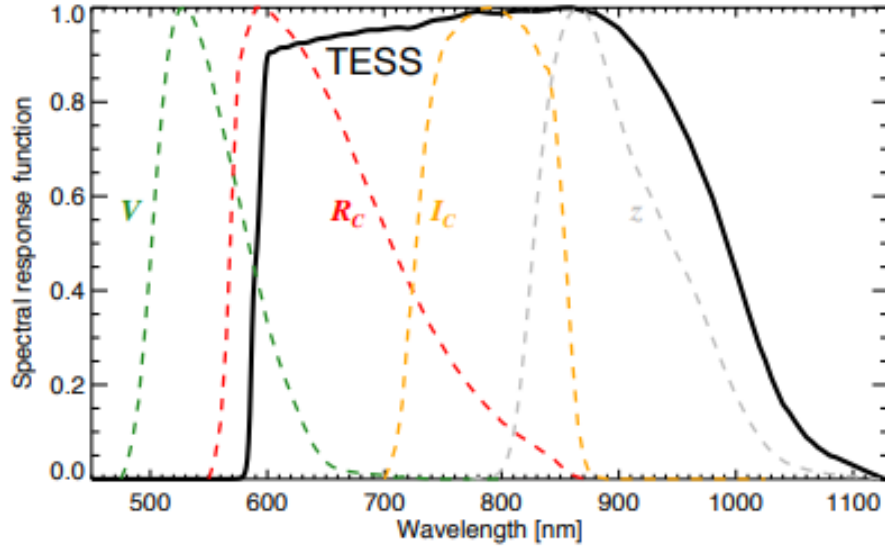


Fig. 17 The black line represents *TESS* spectral response function which is made up of the curve of the detector quantum efficiency and the curve of a long-pass filter transmission and. Johnson-Cousins V , R_c , and I_c filter curves, as well as the SDSS z filter curve, are also presented for comparison. Each function's maximum value has been scaled to unity (Ricker et al., 2014).

2.8.6 Data Handling Unit

The *TESS* Data Handling Unit (DHU) is a Space Micro Image Processing Computer (IPC-7000) (Oelkers and Stassun, 2018), and is composed of five boards: the Proton 400K, which has two Virtex-7 FPGAs that act as interfaces to the four cameras and do high-speed data processing, and the Image Processing Computer (IPC), which comprises two Virtex-7 FPGAs that perform high-speed data processing and interface with the four cameras (Swade et al., 2018)) and the Pro, a 192 GB strong storage (SSB) for huge data storage; a computer for managing, interacting with, and operating the spaceship's master avionics unit; a power supply board for the DHU; and an I/O power switchboard for controlling equipment power. (Ricker et al., 2014).

2.8.7 Summary of specifications

<i>TESS</i> summary	Specifications
Field of view of each lens	$24^\circ \times 24^\circ$
Combined field of view	$24^\circ \times 96^\circ = 2300 \text{ sq. Deg}$
Entrance pupil diameter	10.5 cm
Focal ratio (f /#)	f /1.4
Wavelength range	600–1000 nm
Ensquared energy	50% within $15 \times 15 \text{ }\mu\text{m}$ (one pixel or $0.35 \times 0.35 \text{ arcmin}$) 90% within 60×60 μm (4×4 pixels, or $1.4 \times 1.4 \text{ arcmin}$)

Table 1 Characteristics of the *TESS* lenses. Ensquared energy is the fraction of the total energy of the point-spread function that is within a square of the given dimensions centered on the peak (Ricker et al., 2014).

CHAPTER THREE

3. Data Analysis and Methodologies

3.1 Introduction

In this section, we will look at Mikulski Archive for space Telescope (MAST), which is an astronomical data archive focused on the optical, ultraviolet, and near infrared. Data obtained from the *TESS* satellite are stored on the MAST portal. MAST hosts data from more than a dozen missions, including Hubble, *Kepler/K2*, *TESS*, and *JWST*. But in our case, we will focus on data from the *TESS* mission. In addition, this chapter provides extensive background material on the analysis of light curves using models such as Phoebe and Wilson-Devinney.

Two candidate binary stars were chosen for this investigation, and their data were recorded in the MAST site.

1. EK Aqr
2. KZ Eri

3.2 How to access data from Mikulski Archive for Space Telescopes (MAST)

It was necessary to first log on to the SIMBAD website to get data from MAST. The SIMBAD astronomical database contains basic information, cross-identifications, a bibliography, and data for astronomical objects outside of our solar system.

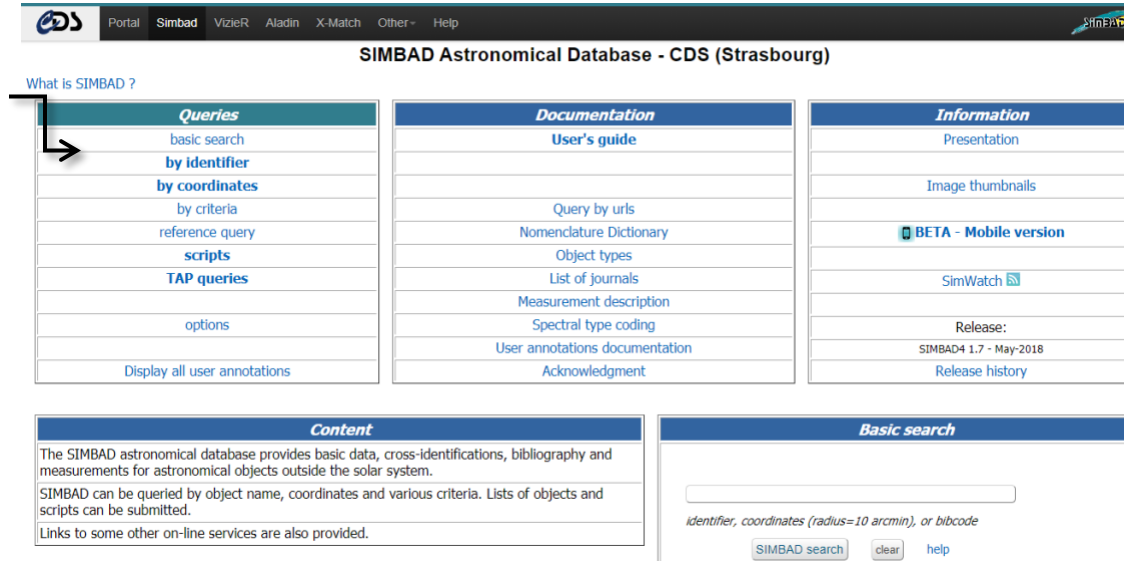


Fig. 18 Screenshot of Simbad Astronomical database website. Image credit; Simbad website

Once on the Simbad website, the basic search button in the top left corner was selected.

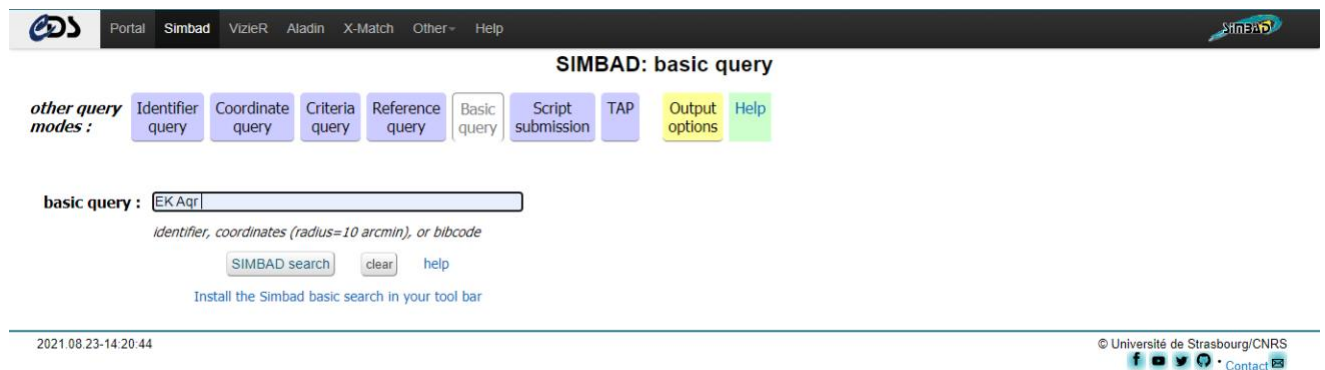


Fig. 19 Screenshot of SIMBAD: basic query. Image credit; Simbad website

Then, in the basic search, type in a binary star, such as EK Aqr, then scroll down to see TIC, also known as *TESS* Input Catalogue, under identifiers.

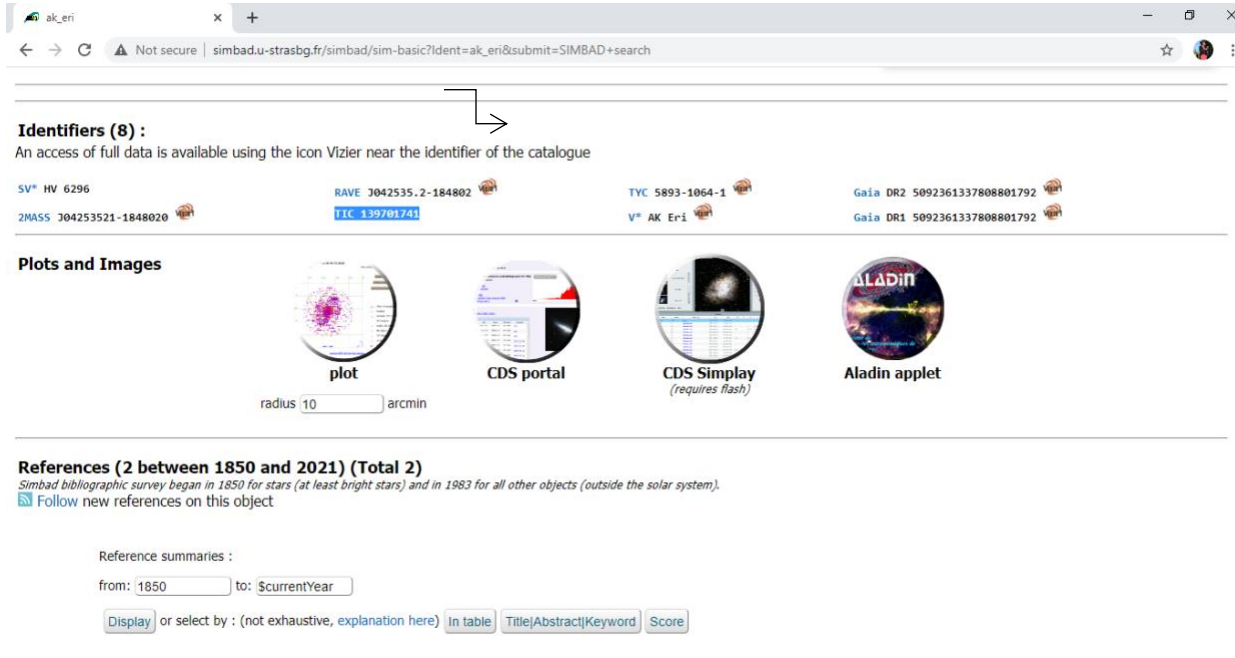


Fig. 20 Screenshot of Identifiers Image credit; Simbad website

The TIC number was then copied, and the MAST Portal was accessed in a separate tab as shown below. After logging in to the Mast Portal, the candidate binary star TIC was pasted under ‘enter the target.’

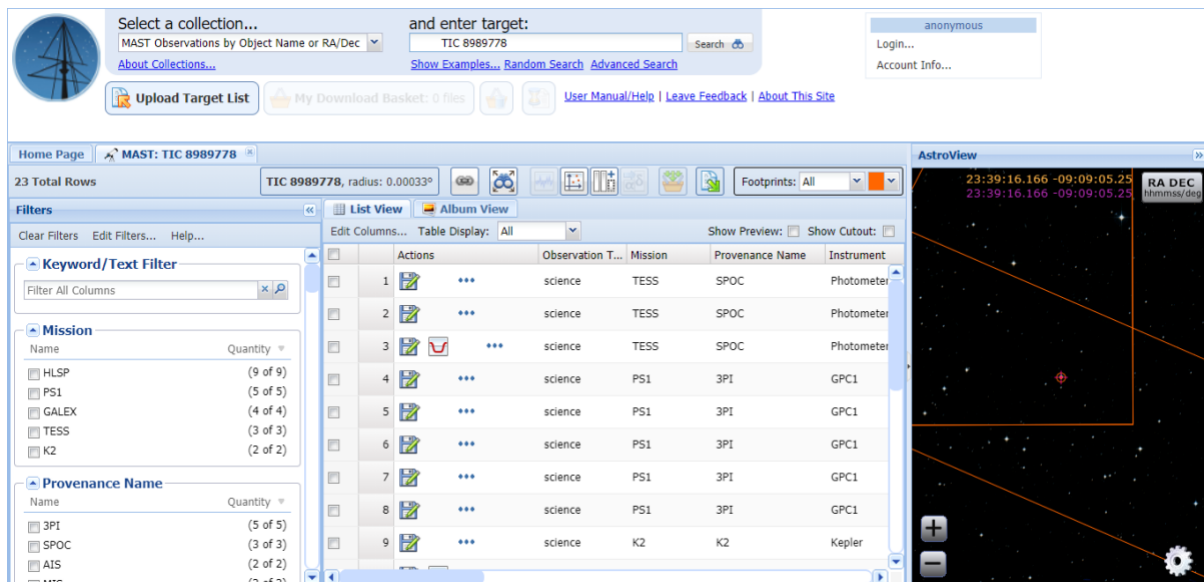


Fig. 21 Screenshot of TESS input catalogue target Image credit; MAST website

To automate the download of the massive list of the two picked stars, a Python code was built. As part of the pre-analysis procedure, fluxes were transformed to magnitudes and normalized fluxes.

3.3 Data from *TESS*

The *TESS* mission produces a variety of data packages, ranging from the *TESS* input catalog to a list of potential planet candidates (Mullally, 2020). *TESS* gathers time-series photometry at varying cadences based on sector overlap and a baseline that ranges from around 27 days to a year, the authors noted. The Full Frame Images (FFIs) are made up of the totalities of the CCDs, Target Pixel (TP) data are downloaded at a lower frequency to obtain cadence for a subgroup of targets, and the pixels around a candidate picked star are maintained as arrays in these target-pixel files, one image per time stamp (Tenenbaum and Jenkins, 2018). Each image is subjected to aperture photometry, resulting in an array of fluxes known as Light Curve (LC) files (Mullally, 2020).

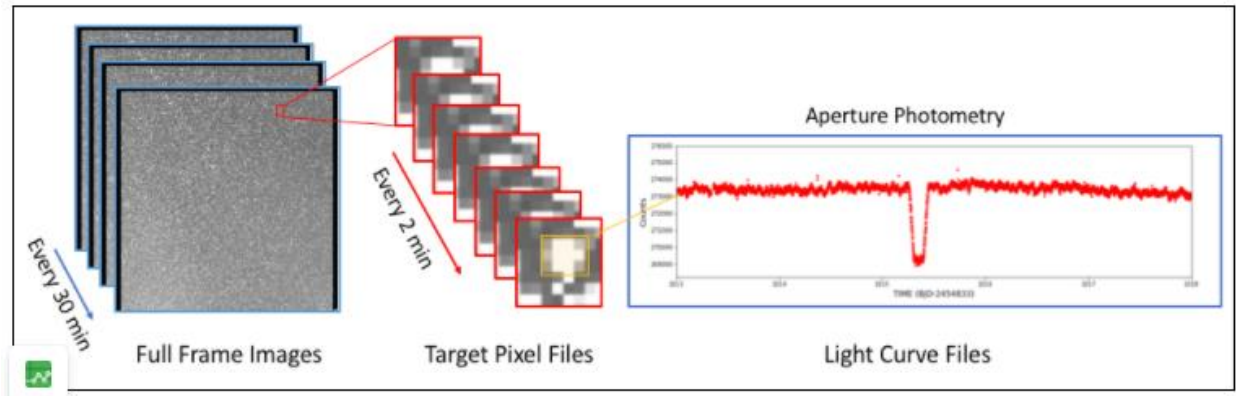


Fig. 22. A Photometric data product (Mullally, 2020).

1. Images in Full Frame (FFI): These files contain the whole field of view's 30-minute (or 10-minute) cadence time series (Oelkers and Stassun, 2018).
2. Target Pixel (TP) Files: These files contain a time series of raw and calibrated pixels downloaded at two-minute and twenty-second intervals (Twicken et al., 2020).
3. Light Curve (LC) Files: These files contain the flux time series generated from the calibrated 2-minute and 20-second target pixels (Oelkers and Stassun, 2018). These files provide simple aperture photometry, a detrended light curve (eliminating trend effects from a data set to only

display data that vary from the trend), location vectors, and quality flags the authors added. The result of the photometric extraction and subsequent systematics removal (co-trending) produced by the Science Processing Operations Center (SPOC) algorithms is stored in *TESS* light curves, which are FITS format files. The data for one target for one observation sector is contained in a single light curve file (Schlieder, 2017).

4. Co-trending Basis Vectors (CBV): These vectors define the collection of systematic trends that can be detected in each CCD's ensemble light curve data and can be used to filter out common instrumental systematics (Twicken et al., 2020).

3.4 Roche model

The Roche model has been used to investigate the exchanges and observable features of the components of binary star systems for a long time (Sepinsky, Willems and Kalogera, 2007). A mathematical expression for a star's potential in a binary system is generated using this method by approximating its interior composition with the Roche model (Pathania, Lal and Mohan, 2013). To study binary stars Kopal (1972) created the notion of the Roche equipotential.

The following patterns are used to create Roche model applications:

1. In a binary star system, the two stars interact as point sources gravitationally, which is surrounded by a massless envelope. As a result, a basic exploratory portrayal of the potential is possible (Prša, 2011).
2. As a firm body the star rotates around its axis, without any differential rotation (Sepinsky, Willems and Kalogera, 2007).
3. The immediate force field influences the structure of the binary star since the periods of completely free non-radial cycles are low when compared with the orbital period. According to this theory, the immediate force field in a binary star system governs the star's shape (Prša, 2011).

The concept of equipotential surfaces asserts that the given effective potential on a surface determines the shape and physical attributes of stars in a double star system (Prša, 2011). If we consider M_1 and M_2 as point-source masses of individual stars, and a be the distance between the

two stars, m is the test mass. Let S_1 and S_2 be the distances to M_1 and M_2 respectively. The coordinate system's origin is then set at star 1. The system revolves around an axis that runs parallel to the z -axis and passes through the center of mass (COM). The following section explains how to get the forms of binary star systems with circular paths and synchronous rotation. The findings will be extended to include orbital periods and asynchronous rotation.

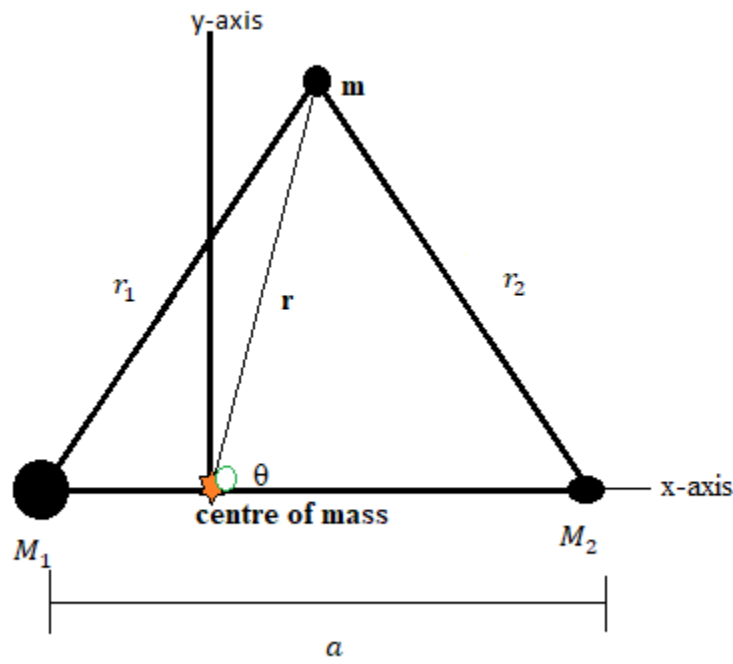


Fig. 23 Two stars orbiting each other in a binary system.

r_1 and r_2 are the distances from stars 1 and 2 to the test mass m . Therefore r_1^2 and r_2^2 are given by.

$$r_1^2 = x^2 + y^2 + z^2 \quad (14)$$

$$r_2^2 = (x - a)^2 + y^2 + z^2 \quad (15)$$

And the position of the center of mass(x_c) is given by.

$$\frac{x_c}{a} = \frac{M_2}{M_1 + M_2} = \frac{q}{1 + q} \quad (16)$$

Where $q \equiv M_2/M_1 \leq 1$ and the distance between the two stars is given by a .

Now the potential is given by equation 19 below.

$$\Phi = -G \left(\frac{M_1}{r_1} + \frac{M_2}{r_2} \right) - \frac{\omega^2}{2} [(x - x_c))^2 + y^2] \quad (17)$$

The dimensionless Roche potential is given by.

$$\Phi_N(x, y, z) = \frac{2}{(1 + q)} \frac{1}{r_1} + \frac{2q}{(1 + q)} \frac{1}{r} + \left(x - \frac{q}{1 + q} \right)^2 + y^2 \quad (18)$$

Due to the system geometry it easier to use spherical coordinates.

$$x = r \sin \theta \cos \varphi = \lambda r,$$

$$y = r \sin \theta \sin \varphi = \mu r,$$

$$z = r \cos \theta = vr \quad (19)$$

Using spherical coordinates, we can now write the effective potential.

$$\begin{aligned} r_1 &= r, \\ r_2 &= \sqrt{r^2 - 2xa^2}, \\ \hat{r} &= \sqrt{(x - x_c)^2 + y^2} \end{aligned} \quad (20)$$

$$\Phi(r, \lambda, v) = \frac{GM_1}{r} - \frac{GM_2}{\sqrt{r^2 - 2ar\lambda + a^2}} - \frac{1}{2}\omega^2(r^2(1 - v^2) - 2r\lambda xc + xc^2) \quad (21)$$

3.5 Analysis of Light Curves of Eclipsing Binaries

The measurements of brightness in relation to time are known as light curves. The variation of their light curves, which depict magnitude vs. time, generates more precise results, while the ongoing development of observational methods of eclipsing binaries produces more accurate data (Rovithis-Livaniou, 2020). Despite the vast amount of observational data from space missions and many Eclipsing Binaries, there are ways to collect the best data for their physical properties, the author added. The Wilson-Devinney model, the most extensively utilized of both the synthetic and analytic light curve modeling software, is discussed in the next section.

3.6 The Wilson-Devinney Model

The Wilson-Devinney model was initially introduced in 1971 by Robert Wilson and Edward Devinney (Knote, 2014). The author stated that for near binaries in synchronous rotations, the model is the same as the classic Roche model. The exteriors of the components are well-defined and obtained from the Roche equipotential:

$$\Omega(r; q, F, d) = \frac{1}{r} + q \left[\frac{1}{\sqrt{d^2 - 2d\lambda r + r^2}} - \frac{\lambda r}{d^2} \right] \frac{q + 1}{2} F^2 r^2 (1 - v^2) \quad (22)$$

The Wilson-Devinney binary star modeling code (WD) consists of two primary modules and is a full package for modeling binary stars and their eclipses.

1. LC module

Light and radial velocity curves, spectral line profiles, images, conjunction times, and timing residuals are all generated by this program. The "inverse problem," which can be expressed as a nonlinear least-squares problem, is the process of extracting physical parameters from eclipsing-binary light curves or radial velocity curves. Minimizing the differences between the theoretical or predicted curve and the observations is the goal (Kallrath & Milone, 1998). The system solution is the parameter vector that yields the smallest deviation; the related estimated light curve is referred to as being the best match to the data (Wilson, Devinney and Van Hamme, 2020). The sum of the squared residuals, which captures the real impact on the light-curve solution, can be weighted or unweighted, and it serves as a measure of deviation (to indicate the quality of the data data (Wilson, Devinney and Van Hamme, 2020). The standard deviation of the fit, or the standard error of a single observation of unit weight obtained from the fit, should be used to have a measure that is independent of the number of free parameters or the number of data points(Kallrath & Milone, 1998).

2. DC module

DC module adjusts light curves, velocity curves, and eclipse timings using differential corrections and the Least Squares criterion.

WD can handle eccentric orbits and asynchronous rotation (with proximity and eclipse effects) as well as compute velocity curves (Rafert and Markworth, 1986). It includes features such as detailed reflection and nonlinear (logarithmic law) limb darkening, spot parameter adjustment, and an optional provision for spots to wander around the surface, as well as the ability to trace the evolution of the light curve over several orbits, the authors added. With absolute flux solutions, Direct Distance Estimation (DDE) is achievable, and enhanced solutions for ellipsoidal variables and eclipsing binaries (EBs) with very shallow eclipses are available. Absolute flux solutions can also estimate the temperatures of both EB components under the correct conditions (Wilson, Devinney and Van Hamme, 2020). It varies from the previous model in that it includes least square analysis. Its goal is to find a solution to the challenging inverse problem. Since its introduction, the Wilson Devinney model has been modified to include a variety of parameters (Rafert and Markworth, 1986). The authors stated that the program is being used for both light curve and differential corrective focus programs. Differential corrections are a type of method for determining the precision of GPS location by comparing measurements from two or more receivers (Wilson, Devinney and Van Hamme, 2020).

3.7 PHOEBE

PHOEBE (PHysics Of Eclipsing BinariEs) is an eclipsing binary star modeling program based on the widely used WD program (Wilson and Devinney, 1971). PHOEBE's add-ons are a strong approach to enhance WD by including even more physics and solution reliability. To generate an accurate model of the binary parameters, PHOEBE combines an in-depth examination of surface and horizon processes which includes reflection, gravity brightening, and limb darkening, with a thorough exploration of the Roche potential (Kurtz et al., 2014). PHOEBE is based on the use of equipotential to approximate stellar forms. As a result, the program does not employ popular

parameters like radial-velocity curve semi-amplitudes or relative photometric radii. Instead, the values of equipotential, mass ratio and semi-major axis determine star morphologies. It's worth noting that the usage of equipotential for proximity-distorted stars means that the mass ratio is influenced not only by radial velocities (RVs) but also by photometric data.

Two tabs are split into two when the PHOEBE software is opened.

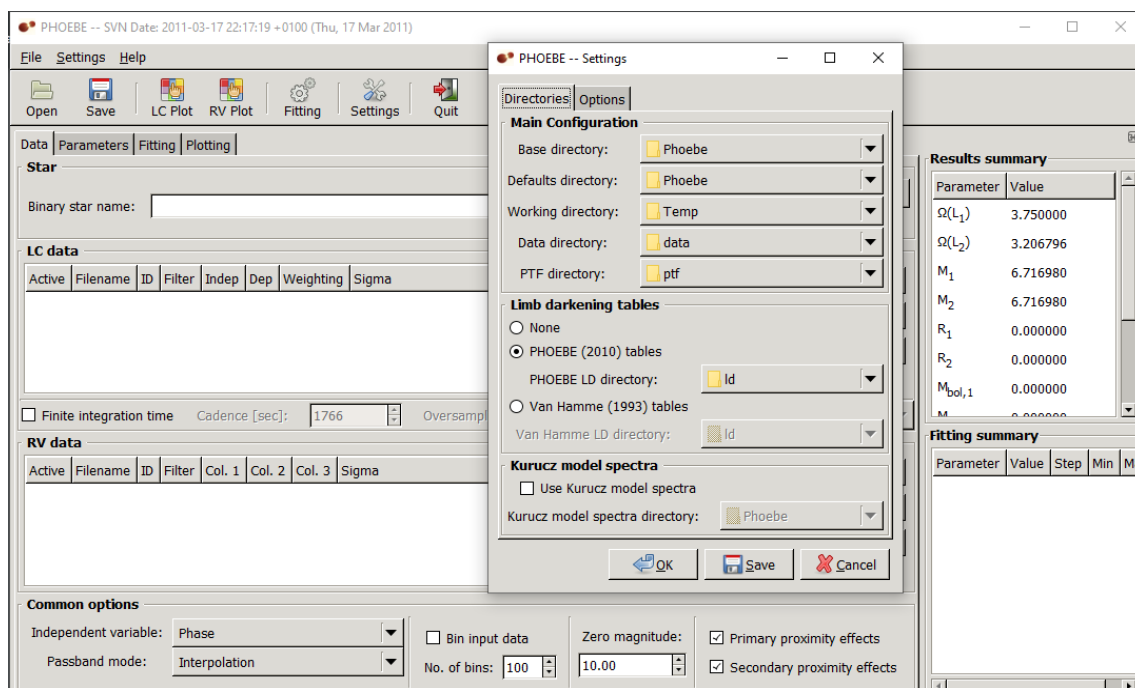


Fig. 24 Screenshot of PHOEBE software

There are two tabs for the settings: Directories and Options.

3.7.1 Directories

1. Base Directory- PHOEBE expects to find the source code in the Base directory.
2. Default Directory- This file's purpose is to make permanent changes to the default values of parameters used by PHOEBE when it starts up.
3. Working Directory- PHOEBE produces temporary files in this directory, which is a scratch space (useful for control and debugging).
4. Data Directory- This is the location where PHOEBE looks for parameter files; it's commonly set up as a parent directory with per-star subdirectories.
5. PTF Directory- It includes PHOEBE - supported Pass-band Transmission Functions.

3.7.2 Limb darkening (LD) tables

From the center to the edge, or limb, of the Sun's or another star's disk, there is a steady decline in brightness. This is called limb darkening. In Phoebe, software one can use pre-computed limb darkening (LD) tables; currently, two tables are supported: internal LD tables computed from Castelli and Kurucz's NEWODF (New opacity distribution function) models, or van Hamme (1993) tables. Alternatively, the user can enter limb darkening coefficients directly, as we did in this study. We calculated the limb darkening coefficients according to the components' effective temperature by using the tables given by Claret (2017).

3.7.3 Van Hamme (1993) tables

Band	X_1	X_2	Y_1	Y_2
Bol	0.644	0.641	0.226	0.235
V	0.739	0.721	0.259	0.271
R _C	0.667	0.648	0.272	0.280
I _C	0.583	0.565	0.264	0.271

Table 2 above shows limb darkening values from (Claret 2017).

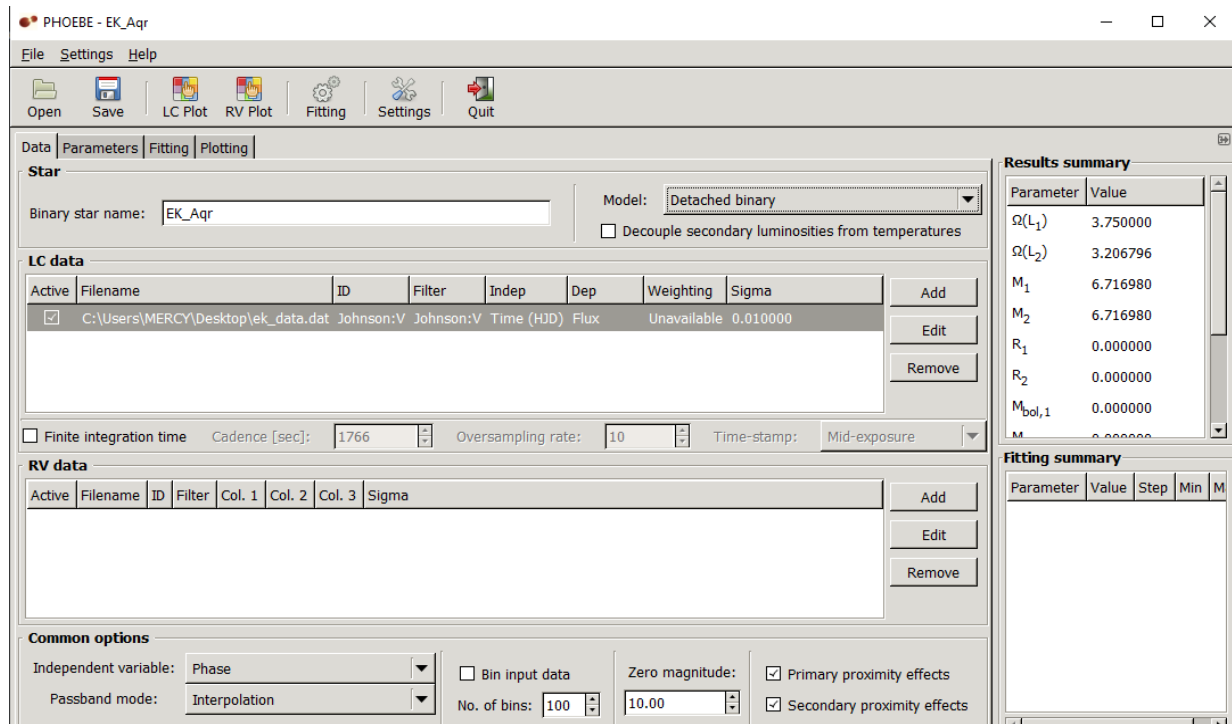


Fig. 25 Screenshot of PHOEBE software tab-EK Aqr

The star data is divided into two columns: column 1 is the time in Heliocentric Julian Date (HJD), and column 2 is the star's flux. Jonson V was chosen as the passband since it is the default value. Note that the limb darkening coefficient was not calculated based on V filter, they were derived using the table of Claret (2017).

3.7.4 Parameters in the PHOEBE software

1. Ephemeris

Heliocentric Julian Date- This is when the Julian Date (JD) has been adjusted to account for variations in the Earth's location with respect to the Sun.

Orbital Period- Orbital period is the time the star takes to make one complete orbit around the center of mass. The formula below, which is derived from *Kepler's* third law, provides the orbital period.

$$T^2 = \frac{4\pi^2}{GM} a^3 \quad (23)$$

2. System

Semi Major Axis – Both stars in a binary system orbit the center of mass in elliptical orbits (COM). The semi-major axis is one-half of the ellipse's long axis.

Mass ratio (Secondary over Primary)- dividing the mass of the secondary star by the mass of the primary star gives us the mass ratio.

Inclination- The tilt of an object's orbit around a celestial body is measured by orbital inclination. Inclination is defined as the angle between a given to our line of sight or axes of direction of the orbiting object.

3. Orbit

Argument of periastron: one of the orbital elements of an orbiting body is denoted as ω . Parametrically it is the angle between the ascending node and the body's periapsis which means the point on an orbiting object where the center of mass is closest to the body it is orbiting, and it is measured in the direction of motion.

Orbital eccentricity: an astronomical object's orbital eccentricity is a dimensionless quantity that governs how far its orbit around another body deviates from a circle.

4. Components

Primary and secondary star effective temperature: the color of the star is directly proportional to its effective temperature; the higher the temperature, the bluer the light emitted by the star. The effective temperature is one of two characteristics, along with stellar luminosity, that astronomers use to construct the standard HR diagram that allows them to classify stars.

Primary and secondary star metallicity- The quantity of elements heavier than hydrogen and helium in an object is known as metallicity. Hydrogen and helium make up most ordinary physical matter in the Universe.

5. Surface

Primary and secondary star surface albedo: the ratio of radiosity (the total radiation emitted or reflected from a surface) to irradiance (flux per unit area) received by a surface is known as surface albedo.

Primary and secondary star gravity brightening due to rapid rotation and hence an oblate shape, gravity brightening, also known as gravity darkening, is an astronomical phenomenon in which the poles of a star are brighter than the equator. When a star is oblate, the radius at its equator is bigger than the radius at its poles.

6. Limb darkening

An equation for the specific intensity of the form provides a reasonably adequate empirical approximation of the form of the limb darkening.

$$I(r) = I(0) \left[1 - u \left(1 - \sqrt{\frac{a^2 - r^2}{a^2}} \right) \right] \quad (24)$$

The solar disc's radius is a , while the vertical displacement from its center is r , and the limb darkening coefficient is u . This is frequently expressed in terms of θ (limb darkening coefficient).

3.8 Binary Maker

Binary Maker is a C-based computer program that was created as a support tool for binary star study (Bradstreet, 1990). The user-friendly program can build and display circular orbit detached, semi-detached, contact, and over contact binaries at any orbital inclination or phase, Bradstreet (1990) added. Many user options are available in Binary Maker, including the ability to create binary surfaces in spherical or cylindrical coordinates, calculate stellar volumes, and create cross sectional diagrams displaying the inner and outer crucial Roche surfaces as well as binary surfaces (Bradstreet, 1990).

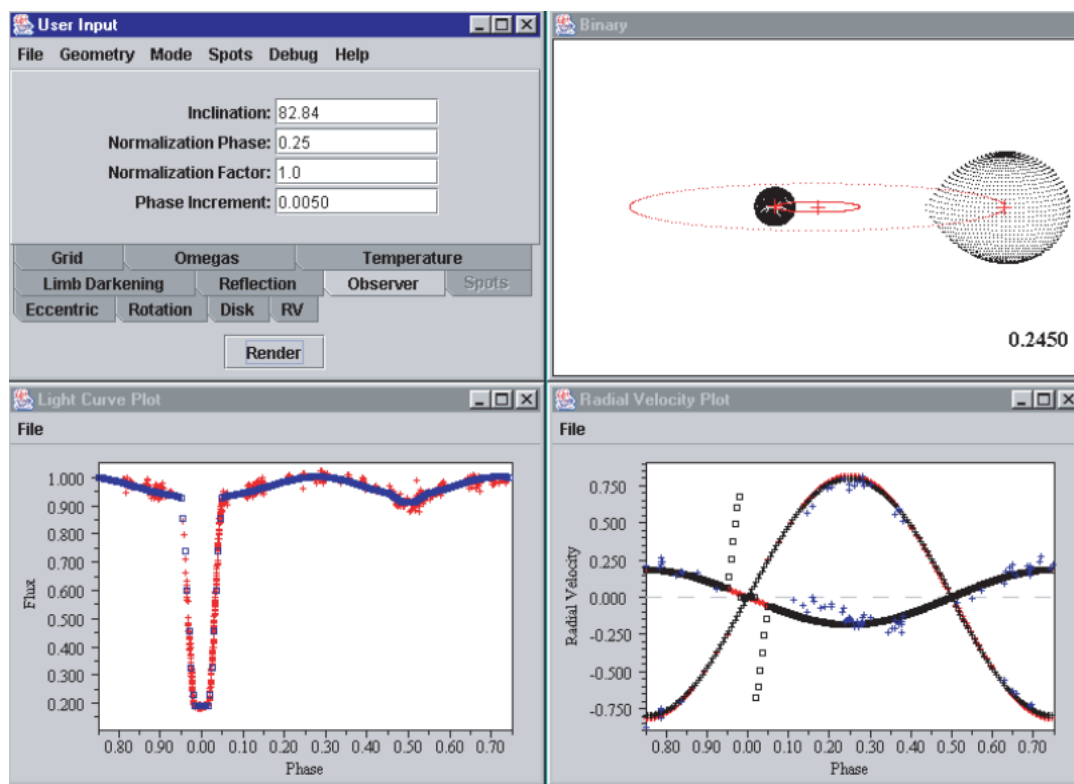


Fig. 26 Binary Maker 3 screenshot (Bradstreet, 1990).

CHAPTER FOUR

4. Results and Analysis

4.1 EK Aqr

EK Aqr is an eclipsing variable star in the constellation Aquarius, identified in the Northern Sky Variability Survey (Hoffman, Harrison and McNamara, 2009). General properties of the target are given in Table 3.

General Properties of EK Aqr	Data
Right ascension	354.81732
Declination	-09.15147
Galactic Latitude	076.0676586 deg
Galactic Longitude	-65.2273018 deg
Magnitude at Maximum brightness (600 - 1000 nm)	10.800 mag
Period	0.6127900000 d

Table 3 A summary on EK Aqr literature obtained from the VizieR Website

Parameter	Value	Error
$i(^{\circ})$	76.26	0.06
q	0.194	0.001
T_1	8000K (Kordopatis et al. 2013)	148
T_2	7616K	18
$\Omega_1=\Omega_2$	2.181	0.002
Fraction radius of primary	0.537 $R_{\odot}$	0.001
Fraction radius of secondary	0.26 $R_{\odot}$	0.01
$\frac{L_1}{L_1+L_2}$	0.837 J/s	0.001
Spot Parameters		
$\beta(^{\circ})$	45	
$\lambda(^{\circ})$	90	
r	17	
t	0.9	

Table 4 Light curve solution for EK Aqr

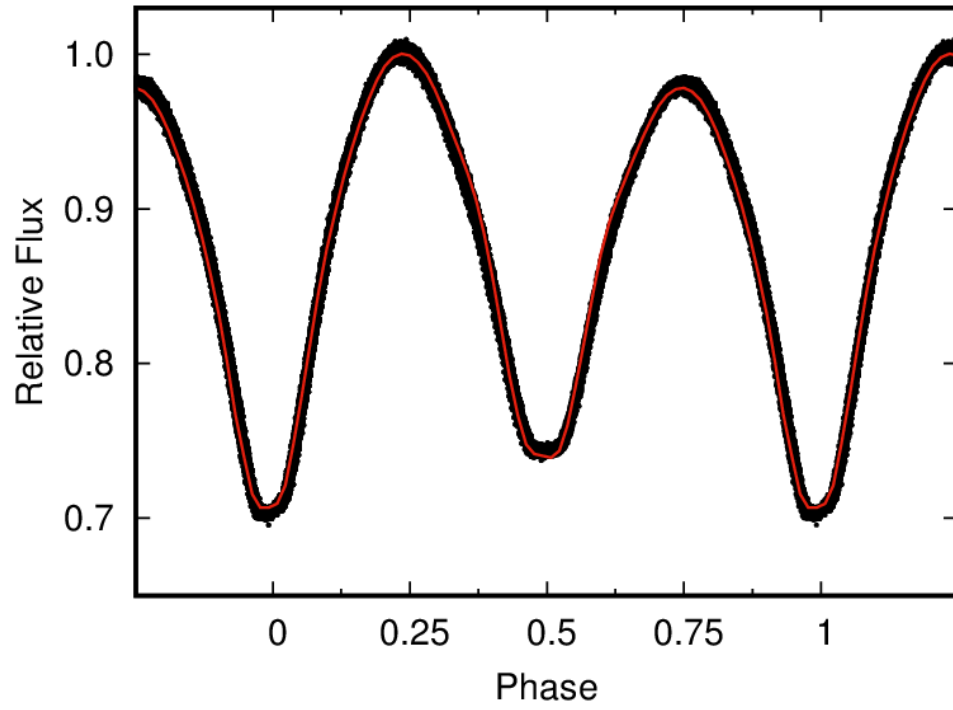


Fig. 27 The observed *TESS* light curves with the solution (red line).

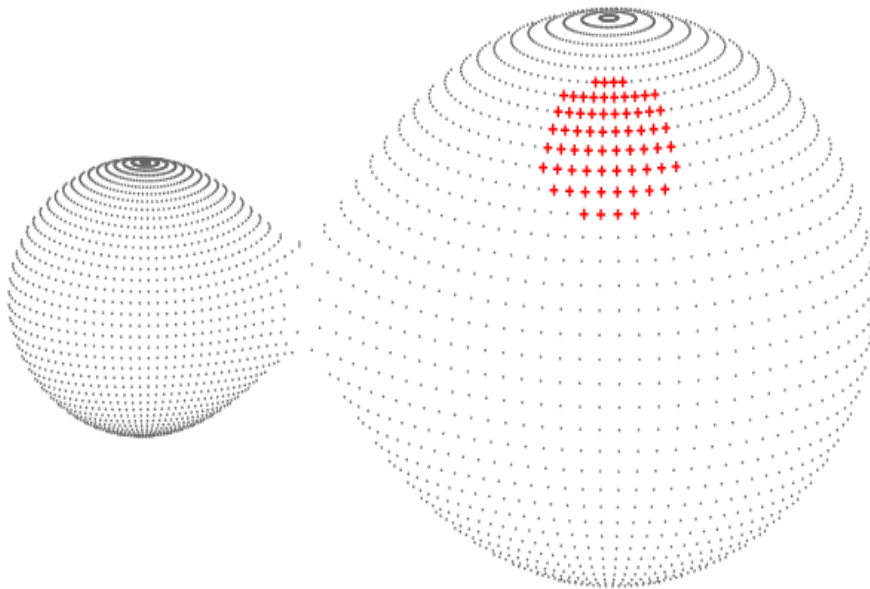


Fig. 28 Roche geometry of the system EK Aquarius

The light curve of the system shows the typical light curve of a W Uma-type binary. The curve was analyzed by using PHOEBE in “over contact binary not in thermal contact” mode. During the solution the effective temperature of the primary component of 8000 K was adopted following Kordopatis et al. (2013). The adjustable parameters are the inclination i , temperature of the secondary component T_2 , mass ratio q , surface potential values $\Omega_1 = \Omega_2$, and luminosity of the primary component L_1 during the solution. The limb darkening coefficients (x_1, x_2) are adopted according to the temperature values from Claret (2017) who gives the values for *TESS* light curves. The albedo (A_1, A_2) and gravity darkening coefficients (g_1, g_2) set to 1.0, 0.5 and 1.0, 0.32 following von Zeipel (1924) and Somosvári and Csizmadia (2006). Additionally, the magnitude difference between the two maxima of the light curve forced us to adopt a spotted area (dark patches on the outer shell of the star) on the secondary component with the parameters given in Table 4. The orbital period of the binary was adopted from Kreiner and Ashforth (2004) during the solution. The agreement between observational data and the synthetic data from the solution is given in Figure 21. The solution indicated the geometrical configuration given in Figure 22

4.2 KZ Eri

KZ Eri is an eclipsing binary star in the constellation Eridanus. General properties of the target are given in Table 5.

General Property of KZ Eri	Data
Right ascension	52.97604556
Declination	-1.63929083
Galactic Latitude	-43.8975437 Deg
Galactic Longitude	-65.2273018 deg
Magnitude at Maximum brightness (600 - 1000 nm)	186.2912066 Deg
Period	0.9426900000 d

Table 5 A summary on KZ Eri literature obtained from VizieR Website

Parameter	Value	Error
$i(^{\circ})$	74.76 $^{\circ}$	0.02
q	0.441	0.001
T_1	8000K (Stassun et al. 2019)	148
T_2	4932K	0.1
Ω_1	3.986J	0.006
Ω_2	$\Omega_{cr} = 2.760J$	0.001
Fraction radius of primary	0.28 $R_{\odot}$	0.004
Fraction radius of secondary	0.285 $R_{\odot}$	0.004
$\frac{L_1}{L_1+L_2}$	0.891J/s	0.002
Spot Parameters		
$\beta(^{\circ})$	45	
$\lambda(^{\circ})$	270	
r	17	
t	0.9	

Table 6 The results of the light curve solution for KZ Eri

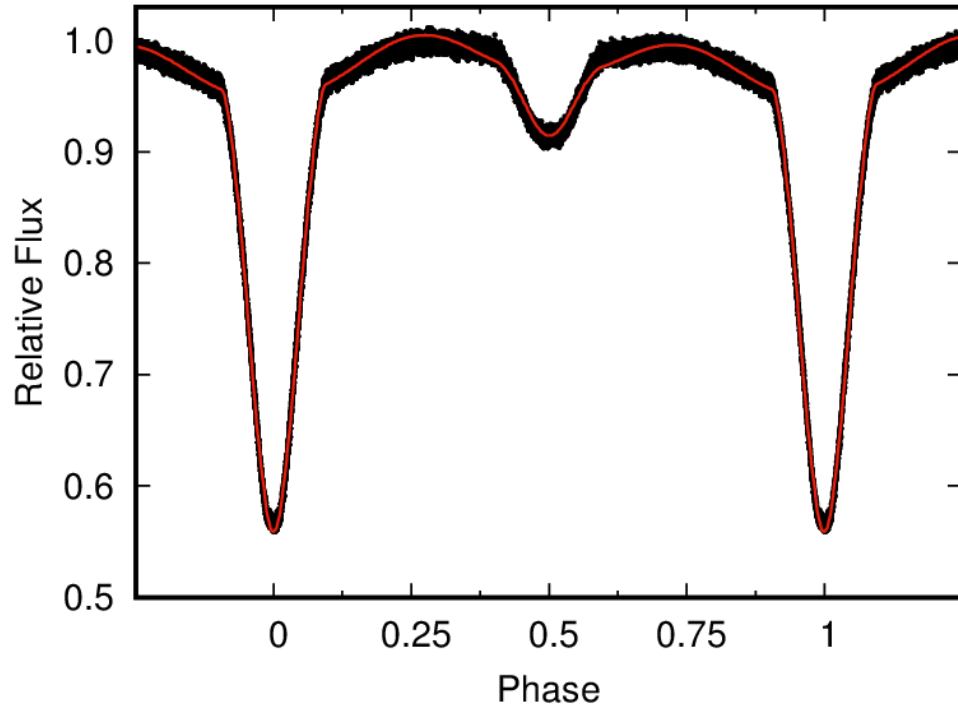


Fig. 29 The observed *TESS* light curves with the solution (red line).

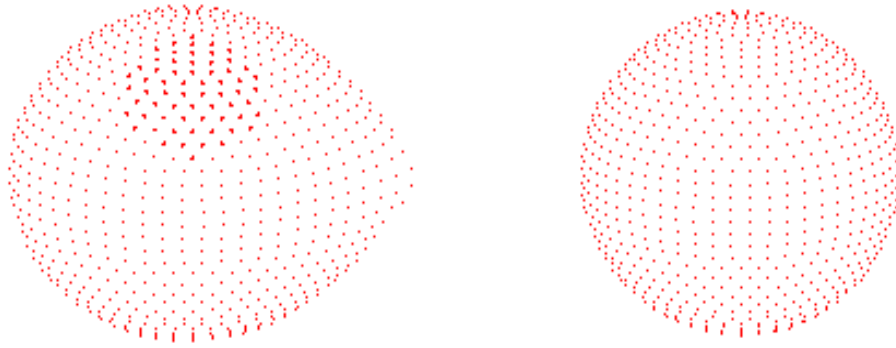


Fig. 30 The Roche geometry of the system KZ Eri

PHOEBE was used to analyze the light curve of the system. Contrary to the solution of EK Aqr, the semi-detached configuration was assumed during the solution since the light curve shows obvious similarity to a classical semi-detached binary. The adjustable parameters are the inclination i , temperature of the secondary component T_2 , mass ratio q , surface potential values Ω_1 and Ω_2 , and luminosity of the primary component L_1 during the solution. The other parameters are set in the same method used in the analysis of EK Aqr. The orbital period was taken from the ASAS database (Pojmanski, 2002). Calculated parameters are given in Table 7 and the theoretical light curve is compared to the observations in Figure 23. The geometrical configuration of the system is also shown in Figure 24.

Parameter	EK Aquarii	KZ Eridani
$M_1 (M_\odot)$	2.0	2.0
$M_2 (M_\odot)$	0.388±0.002	0.882±0.002
$R_1 (R_\odot)$	2.23±0.01	1.68±0.05
$R_2 (R_\odot)$	1.68±0.01	1.81±0.02
$L_1 (L_\odot)$	18.2±0.1	10.3±0.6
$L_2 (L_\odot)$	3.52±0.06	1.7±0.4
$a(R_\odot)$	3.52±0.06	5.893±0.004

Table 7 The absolute parameters of the systems.

4.3 Location of components on the HR diagram and mass radius plane

We locate the components of the systems on Hertzsprung-Russell diagram and mass-radius plane to compare to well-known stars of the same type and shown in Fig 25 and Figure 26. In the figures, the data for Algols are taken from İbanoğlu et al. (2006) while the data for contact binaries are given by Yildiz and Doğan (2013). Our targets show very good agreement with the known systems as can be seen from the figures.

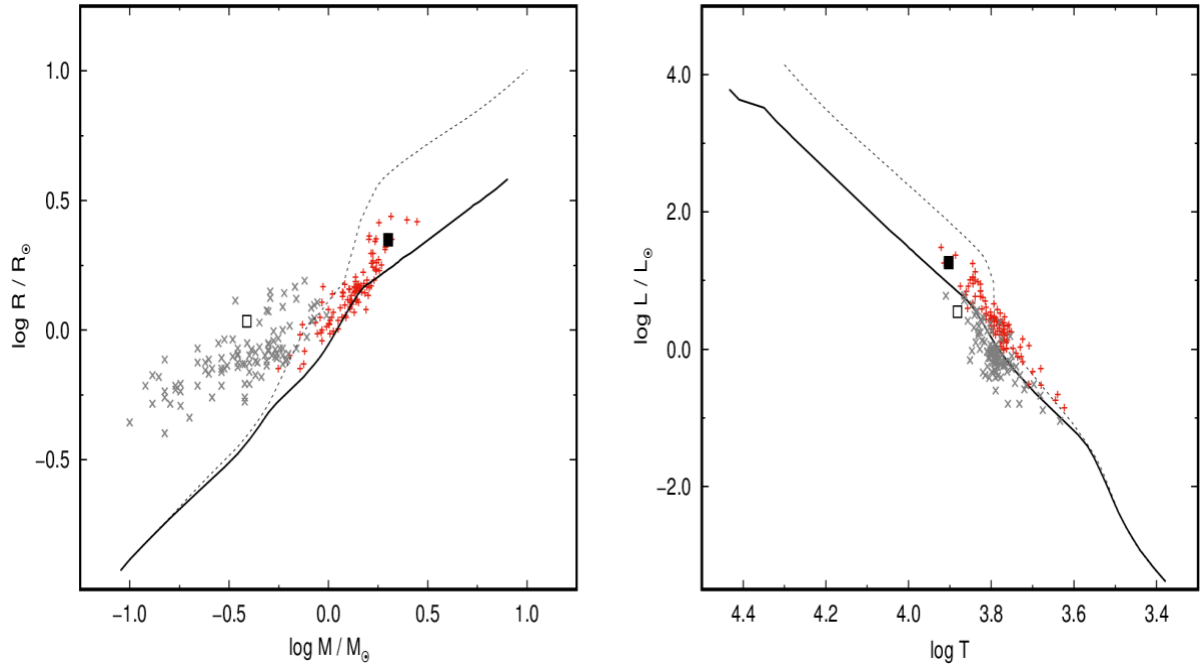


Fig. 31 The location of the components of EK Aqr on the mass-radius plane and the HRD. The plus and cross signs refer to the primary and secondary components of well-known Algols. Filled square refers to primary component while the open square signs the secondary of our targets. Solid

and dashed lines are Zero Age Main Sequence and Terminal Age Main Sequence whose parameters are from Bressan et al. (2012).

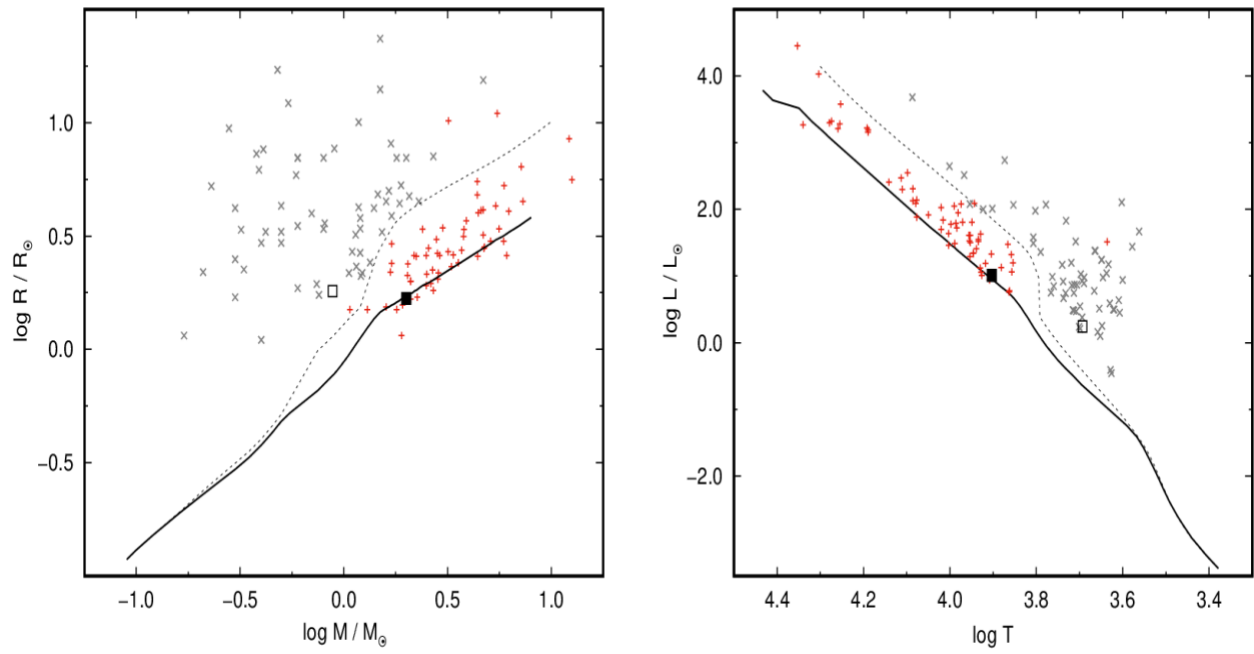


Fig. 32 is the same as Fig. 25, but for KZ Eri.

CHAPTER FIVE

5.1 Discussion and Conclusion

We present a thorough investigation of the two selected binary stars. First, binary modeling was performed utilizing high-quality *TESS* data and the results of literature radial velocity research. Light curves were extracted from *TESS* full-frame images using the light curve python program. The stellar parameters were calculated using the PHOEBE algorithm, which is based on the Wilson-Devinney code modeling, until the residuals were as low as possible. EK Aqr is found to be a contact binary star system with a small mass secondary component. KZ Eri, on the other hand, is a semi-detached binary with a secondary which component filling its Roche lobe. Based on our light curve solutions, we calculated the absolute parameters of both systems, and this are given in Table 7. The simulated light curves matched the observed ones, implying that the parameters calculated by the numerous research groups referenced in this thesis are accurate. The primary star temperature T_1 for EK Aqr was set to 8000K, while the secondary was $7616 \pm 0.28\text{K}$, implying that this is a sub type A, not a W, because the larger star was hotter and more massive. Despite their vastly varied masses, the components had slightly similar temperatures. The mass ratio for EK Aqr was 0.194 ± 0.001 , which agrees with the references in this work, which state that the mass ratio for a contact system should be between 0.08 and 0.8 (McCartney, 1999).

Then for KZ Eri we saw that outside the eclipse there was little fluctuations which meant that it had a flat-topped maximum and a well-defined eclipse (Holcomb, 2016). The mass ratio of the system was 0.441 ± 0.001 . Also T_1 and T_2 were 8000K and 4932K respectively. The positions of both components in the Hertzsprung-Russell (H-R) diagram and mass-radius plane were investigated using the obtained absolute stellar parameters. The light curve of EK Aqr shows that the more massive component is occulted during the primary minimum which corresponds that the

system is an A-type W Uma. As Stepien and Gazeas (2012) remarked this type of systems probably evolved from detached systems with the orbital period of about 2 days. The more massive star in the detached system expands and transfer mass to its companion during its evolution. The mass transfers ends when the mass ratio reversal occurs. The whole evolution may end to a single star formed by merging process.

KZ Eri is a semi-detached binary star with the main component having the filling factor of $f = \frac{r}{r_L} = 0.91$ where $r_L = \frac{0.49 q^{\frac{2}{3}}}{0.6 q^{\frac{2}{3}} + \ln(1 + q^{\frac{1}{3}})}$ (Eggleton, 1983). This gives the system a possibility to be a member of near-contact binaries whose specifications are described by Shaw (1994). If so, it is worth emphasizing that the system may evolve to an A-type W Uma binary system through its evolution.

This work has taught us that eclipsing binaries are the effective approaches to figuring out basic star parameters and allows researchers to learn a lot about how these systems evolve and, consequently, the fate of most stars. Our comprehension of the various areas in the HR diagram where variable stars are present is also based on predictions from stellar evolution models. Since some parts of evolutionary path of the binary systems are still not certain and investigate by many researchers, precise determination of binary parameters gives us to conclude the evolutionary statutes of several types of binary systems based on more accurate results. These results will also contribute very valuable knowledge to the calibration of stellar models. Because the researchers use the model's results to produce accurate calibrations, analyses on evolutionary sequences, and distance estimations, accuracy is essential.

The logical next step for these systems would be more photometric measurements. It would be possible to improve modelling and establish a stronger foundation for the orbital period by conducting more photometric observations of both systems and adding more points to the O-C plot (analyse the model's fit to the observational data after fitting it to the data). The temperature range for each system will be reduced by getting spectroscopic data for both systems, and additional static parameters will be provided for inclusion in the PHOEBE modelling process. The study of spectroscopic data would be the most essential next step in the study of these systems. This would

make it possible to use radial velocity analysis to determine the components' effective temperature, spectral type, and masses.

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