

Hydrothermal and entropy production analyses of magneto-cross nanoliquid under rectified Fourier viewpoint: A robust approach to industrial applications

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ABSTRACT

The present article has been groomed to explore the boundary-driven magnetized flow of cross nanoliquid over thin needle subject to auto catalysis chemical reactions. In addition to it, the effect of entropy optimization model is incorporated and transportation of heat under non-uniform heat source/sink, Cattaneo-Christov heat flux (rectified Fourier) viewpoint (CCHF), and non-linear thermal radiation is also taken into account. Furthermore, the Brownian and thermophoresis aspects of nanoliquid are invoked. The dimensionless governing equations are solved by apposite shooting scheme. The outcomes of the present study via demonstrated graphs and numerical benchmarks seem to indicate that controlled Sakiadis and Blasius flow pattern of cross nanofluids is attained due to incremented magnetic field strength. Temperature ratio parameter contributes the upgradation of thermal boundary layer thickness and homogenous reaction rate leads to the diminution of nanoparticles concentration. The relative values of needle velocity and the velocity of cross nanofluid are most important factor for the regulation of viscous drag force and rate of heat transportation. Augmented Weissenberg parameter and Reynolds number are the prime factor for the uplift of the flow field and the related layer thickness. Furthermore, the existence of CCHF could result in augmentation in Nusselt.

1. Introduction

Significantly, there is an increasingly acknowledgement in the research world regarding the consideration of nanoliquids as those contribute extraordinary characteristics such as upgradation of thermal conductivity and heat transportation. This is due to the stable

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Nomenclature

TC	thermal conductivity
EG	entropy generation
$TBLT$	thermal boundary layer thickness
CBL	concentration boundary layer
$CBLT$	concentration boundary layer thickness
HT	heat transfer
$NLTR$	non linear thermal radiation
DFF	Darcy Forchheimer flow
EG	entropy generation
BLF	boundary layer flow
BLR	boundary layer flow
BLT	boundary layer thickness
MHD	magnetohydrodynamics
$CCHF$	Cattaneo Christov heat flux
(u, v)	velocity along x -and r – directions
$(\rho C_p)_p$	heat capacitance of nanoparticles
$(\rho C_p)_f$	heat capacitance of base fluid
σ_f	electrical conductivity
ρ_f	density of fluid
B_0	uniform strength of magnetic field
n	flow behavior index
Γ	cross time constant
η_0	zero shear rate viscosity
μ_f	dynamic viscosity of normal fluid
k_f	TC of base fluid
σ^*	Stefan-Boltzmann constant
D_T	thermophoretic diffusion coefficient
T	fluid temperature in the boundary layer
T_w	temperature on the thin needle surface
T_∞	ambient fluid temperature
$We = \frac{\Gamma UR_0}{x}$	Weissenberg parameter
$Re = \frac{U_r}{\nu_f}$	Reynolds number
$\nu = \frac{\eta_0}{\rho_f}$	kinematic viscosity of the fluid
$M = \frac{\sigma_f B_0^2 x}{\rho_f U}$	Hartmann number
$Nb = \frac{\tau D_B C_0}{\nu_f}$	Brownian motion parameter
$Nt = \frac{\tau D_T T_\infty (\theta_f - 1)}{\nu_f T_\infty}$	thermophoresis parameter
$Pr = \frac{\nu_f}{\alpha_f}$	Prandtl number
$N_R = \frac{k^* k_f}{4\sigma^* T_\infty}$	radiation parameter
$E_c = \frac{U^2}{T_\infty (\theta_f - 1)(c_p)_f}$	Eckert number
$K_2 = \frac{k_s \nu_f x}{D_{C_1} U_r}$	heterogeneous reaction parameter
$\theta_f = \frac{T_w}{T_\infty}$	temperature ratio parameter

suspension of nanoparticles in the base fluids with minor gravitational settling over long time. Out of seven slip mechanisms proposed by Buongiorno [1], only thermophoresis and Brownian diffusion are inevitable as those enable to enhance nanoparticles' movement in the base fluid so as to regulate the HT coefficient of nanofluids. For the superior features expressed above, practical industrial applications of the nanoliquids have placed an intensified focus as coolants in the hybrid-powered engines, micro-channel heat sinks, microelectronics, pharmaceutical processes, nuclear reactor coolant, solar collector, space technology, heat exchangers, and so on. In view of such substantial applications many authors [2–15] studied in the related areas.

The Rheology of cross fluid model was established by Cross [16] and has wide range of engineering and industrial applications. Hayat et al. [17] studied the chemically reactive flow of magneto Cross nanoliquids. An axisymmetric flow and HT of Cross fluid was carried on by Khan et al. [18]. HT effect of cross nanofluid due to a circular cylinder was studied by Hina et al. [19]. In their study, they observed that increase in Weissenberg number We indicating augmented shear-deformation leads to the diminution in VBLT. Azam

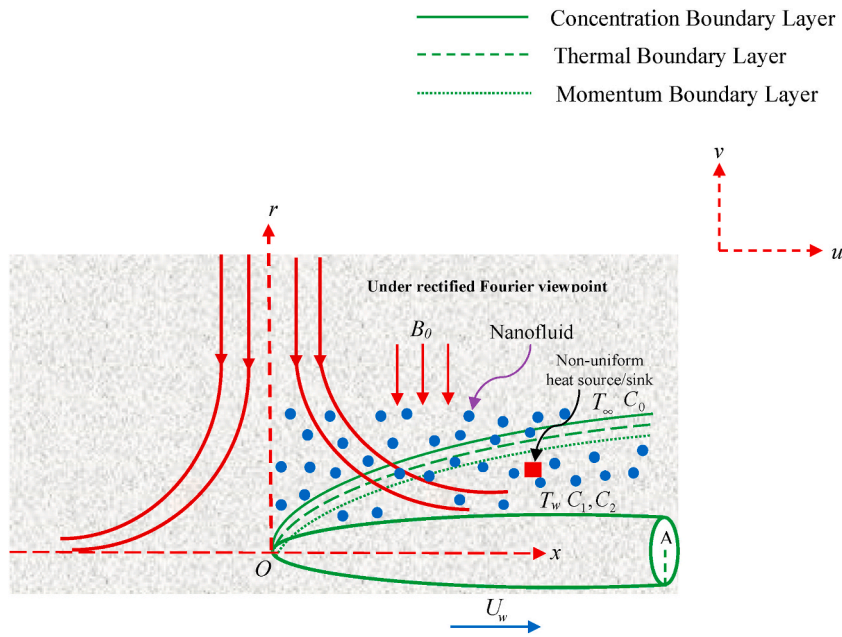


Fig. 1. Flow geometry.

et al. [20] examined the impact of Arrhenius activation energy in axisymmetric flow of radiative-Cross nanofluid. In their analysis, it is noticed that incremented activation energy parameter yielded an upgradation in nanoparticles concentration.

Fair enough, chemical reactions have a key role to play in the useful industrial and technological applications. Homogeneous reaction takes place among the identical phase (liquids, gases, solids) while heterogeneous reaction is a two or more phase reaction (solid and liquid, solid and gas, etc.). Such reactions are observed through remarkable lucidity in batteries, corrosion process and electrolytic cells. Khan et al. [21] carried on a comparative study of Casson fluid with HHRs. Stagnation point flow with CCHF and HHRs was investigated by Hayat et al. [22]. Many others [23–27] worked in the related aspects. The NLTR phenomenon finds its implementation in nuclear reactors and rocket propulsion. The effect of linear/NLTR on various nanofluids subject to porous channel, (2D and 3D) stretched sheets, parallel plates, enclosures etc were analyzed in Refs. [28–35].

It is important to understand the significance of entropy optimization in the flow and HT of systems. In order to improve the system efficiency we control the entropy optimization of the system. Bejan [36] explored how TC of fluids behaves under the influence of entropy optimization. EG of nanofluid in semi-annulus porous enclosure was studied by Seyyedi et al. [37]. EG of Sisko nanomaterial was examined by Khan et al. [38]. Entropy optimized MHD 3D flow of nanomaterial was studied by Nayak et al. [39]. In their study, they revealed how NLTR and entropy minimization led to the good absorber of solar energy and intensification of heat transport respectively. Many others [40–42] investigated in the related aspects.

BLF over a thin needle is the booming area of research now as it has the substantial robust applications in engineering and biomedical purposes. It is found to be utilized in protected thermocouple or hot wire anemometer. It is applied in experimental basic research studies in safety relevant thermo hydraulic facilities of nuclear power plants. Such investigations are meant for analysis of accident sceneries as well as for the evaluation and further improvements of thermo hydraulic simulation codes as CFX for problems of multiphase flow media. The study on the BLF over a thin needle was initially studied by Lee [43]. BLF over a continuously moving thin needle was studied by Ishak et al. [44]. Assisting and opposing mixed convection BLF flow over vertical thin needle was analyzed by Ahmed et al. [45]. Khan et al. [46] examined the entropy minimized flow of nanofluid over thin needle. Recently, Huda et al. [47] investigated the influence of Cattaneo-Christov model on DFF of nanofluid past a thin needle. They also noticed their outcomes as the velocity fields squeezes for incremented porosity parameter and local Forchheimer number.

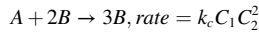
In a view that seems widely held about the investigation of the BLF and HT of nanofluids over thin needle subject to different conditions of flow. However, no researcher has been able to investigate on the entropy optimized flow of magneto-cross nanofluid over a thin needle within porous medium with considering non-uniform heat source/sink, Cattaneo-Christov heat flux viewpoint (CCHF), auto catalysis chemical reactions, and thermal radiation altogether so far. Therefore, the novelty of our present study is to investigate the impact of CCHF viewpoint, auto catalysis chemical reactions, and non-uniform heat source/sink on magneto-cross nanofluids associated with entropy optimization over a thin needle within a porous medium.

2. Mathematical formulation

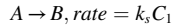
We assume the flow of viscous Cross nanofluids upon a thin needle within a porous medium with radius $r = R(x)$ and moving with a fixed velocity $U_w(x)$ (See Fig. 1). Non-uniform heat source/sink, CCHF, auto catalysis chemical reactions, and thermal radiation

altogether are considered.

The homogenous reaction in cubic autocatalysis would be



in which:



Here C_1 and C_2 would be concentrations of chemical species A as well as B, k_c and k_s would be rate of constant.

The governing equations of the viscous flow of cross nanoliquid past a thin needle considering the aforementioned hypotheses are [13,34,43–46]:

$$\frac{\partial}{\partial x}(ru) + \frac{\partial}{\partial r}(rv) = 0, \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial r} = U_\infty \frac{dU_\infty}{dx} + \nu \left(\frac{\partial}{\partial r} + \frac{1}{r} \right) \left[\left(\frac{\partial u}{\partial r} \right) \left\{ 1 + \left(\Gamma \left| \frac{\partial u}{\partial r} \right| \right)^n \right\}^{-1} \right] - \frac{\sigma B_0^2 (u - U_\infty)}{\rho_f} - \frac{\mu_f (u - U_\infty)}{\rho_f K^*} \quad (2)$$

$$\begin{aligned} & \left(u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial r} \right) + \delta_E \left(u^2 \frac{\partial^2 T}{\partial x^2} + v^2 \frac{\partial^2 T}{\partial r^2} + 2uv \frac{\partial^2 T}{\partial r \partial x} + u \frac{\partial u}{\partial x} \frac{\partial T}{\partial x} \right. \\ & \quad \left. + u \frac{\partial v}{\partial x} \frac{\partial T}{\partial r} + v \frac{\partial u}{\partial r} \frac{\partial T}{\partial x} + v \frac{\partial v}{\partial r} \frac{\partial T}{\partial r} \right) \\ & = \frac{k}{(\rho c_p)_f} \left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} \right) + \frac{\eta_0}{(\rho c_p)_f} \left\{ 1 + \Gamma \left| \frac{\partial u}{\partial r} \right|^n \right\}^{-1} \left(\frac{\partial u}{\partial r} \right)^2 \\ & \quad + \frac{16\sigma^*}{3k^* (\rho c_p)_f} \left\{ T^3 \frac{\partial^2 T}{\partial r^2} + 3T^2 \left(\frac{\partial T}{\partial r} \right)^2 \right\} \end{aligned} \quad (3)$$

$$\begin{aligned} & + \frac{\sigma B_0^2 (u - U_\infty)^2}{(\rho c_p)_f} + \frac{\mu_f}{(\rho c_p)_f} \frac{(u - U_\infty)^2}{K^*} \\ & + \frac{(\rho c_p)_p}{(\rho c_p)_f} \left[D_B \frac{\partial C_1}{\partial r} \frac{\partial T}{\partial r} + D_B \frac{\partial C_2}{\partial r} \frac{\partial T}{\partial r} + \frac{D_T}{T_\infty} \left(\frac{\partial T}{\partial r} \right)^2 \right] + \frac{Q'''}{(\rho c_p)_f} \end{aligned}$$

$$\left(u \frac{\partial C_1}{\partial x} + v \frac{\partial C_1}{\partial r} \right) = \frac{D_{C_1}}{r} \frac{\partial}{\partial r} \left(r \frac{\partial C_1}{\partial r} \right) + \frac{D_T}{T_\infty} \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) - k_c C_1 C_2^2 \quad (4)$$

$$\left(u \frac{\partial C_2}{\partial x} + v \frac{\partial C_2}{\partial r} \right) = \frac{D_{C_2}}{r} \frac{\partial}{\partial r} \left(r \frac{\partial C_2}{\partial r} \right) + \frac{D_T}{T_\infty} \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) + k_c C_1 C_2^2 \quad (5)$$

With boundary conditions:

$$\left. \begin{aligned} u &= U_w(x), \quad v = 0, \quad T = T_w, \quad D_{C_1} \frac{\partial C_1}{\partial r} = k_s C_1, \quad D_{C_2} \frac{\partial C_2}{\partial r} = -k_s C_1, \quad \text{at } r = R(x) \\ u &\rightarrow U_\infty(x), \quad T \rightarrow T_\infty, \quad C_1 \rightarrow C_0, \quad C_2 \rightarrow 0 \quad \text{as } r \rightarrow \infty \end{aligned} \right\} \quad (6)$$

Here Q''' the non-uniform heat is absorbed or generated per unit volume and is given by

$$Q''' = \frac{ku}{xv} [A^* (T_f - T_\infty) e^{-\eta} + B^* (T - T_\infty)]$$

Where A^* and B^* are respectively the space dependent and temperature dependent parameters.

The transformations utilized for non-dimensionalization:

$$\left. \begin{aligned} u &= 2Uf'(\eta), \quad v = -\frac{v_f}{r}f + \eta \frac{v_f}{r}f'(\eta), \quad \eta = \frac{Ur^2}{v_f x} \\ \theta(\eta) &= \frac{T - T_\infty}{T_w - T_\infty}, \quad \varphi(\eta) = \frac{C_1}{C_0}, \quad \chi(\eta) = \frac{C_2}{C_0} \end{aligned} \right\} \quad (7)$$

Moreover, we have:

$$T = T_{\infty} [1 + (\theta_f - 1)\theta] \quad (8)$$

Using (7) and (8), (2)-(6) take the form

$$\text{Re}[1 + (4Wef'')^n(1-n)]f''' + \frac{1}{2}[1 + (4Wef'')^n]^2 \left\{ ff'' - \frac{1}{8}(\beta + M)(2f' - \lambda) \right\} = 0 \quad (9)$$

$$\begin{aligned} \theta' + \eta\theta'' + \frac{1}{2}\text{Pr}f\theta' - \text{Pr}Y_E \left\{ \begin{aligned} &4\eta^2(f')^2\theta'' + 4\eta(f')^2\theta'' - 4\eta f' f' \theta'' \\ &+ 2\eta\lambda f' f' \theta'' - 3\eta f^2\theta' + \frac{2}{\eta}f^2\theta' \end{aligned} \right\} \\ + \frac{4}{3N_R} \left\{ \begin{aligned} &[1 + (\theta_f - 1)\theta]^3 \left(\frac{1}{2}\theta' + \eta\theta'' \right) \\ &+ 3\eta[1 + (\theta_f - 1)\theta]^2(\theta_f - 1)(\theta')^2 \end{aligned} \right\} \\ + 4\eta\text{PrEc} \left[\frac{(f'')^2}{1 + (4Wef'')^n} \right] + \eta\text{Pr}[Nb(\theta' \varphi' + \theta' \chi') + Nt(\theta')^2] \\ + \text{PrEc}(M + \beta) \left(f' - \frac{1}{2}\lambda \right)^2 + \frac{1}{2}\text{Pr}(A^* e^{-\eta} + B^* \theta)f' = 0 \end{aligned} \quad (10)$$

$$\eta\varphi'' + 2\varphi' + Scf\varphi' + 2\left(\frac{Nt}{Nb}\right)(\theta' + \eta\theta'') - \frac{1}{2}ScK_1\varphi\chi^2 = 0 \quad (11)$$

$$\Omega\eta\chi'' + 2\Omega\chi' + Scf\chi' + 2\left(\frac{Nt}{Nb}\right)(\theta' + \eta\theta'') + \frac{1}{2}ScK_1\varphi\chi^2 = 0 \quad (12)$$

$$\left. \begin{aligned} f'(A) &= \frac{1}{2}(1 - \lambda), f(A) = \frac{A}{2}(1 - \lambda), \theta(A) = 1, \\ \varphi'(A) &= K_2\varphi, \Omega\chi'(A) = -\frac{1}{2}K_2\varphi \text{ at } \eta = A \\ f'(\infty) &\rightarrow \frac{1}{2}\lambda, \theta(\infty) \rightarrow 0, \varphi(\infty) \rightarrow 1, \chi(\infty) \rightarrow 0 \text{ as } \eta \rightarrow \infty \end{aligned} \right\} \quad (13)$$

With comparable sizes of A and B, we have $\Omega = \frac{D_{C_2}}{D_{C_1}} = 1$ [48]. This leads to the relation

$$\varphi(\eta) + \chi(\eta) = 1 \quad (14)$$

Under the above assumptions, (10)–(13) will reduce to

$$\begin{aligned} \theta' + \eta\theta'' + \frac{1}{2}\text{Pr}f\theta' - \text{Pr}Y_E \left\{ \begin{aligned} &4\eta^2(f')^2\theta'' + 4\eta(f')^2\theta'' - 4\eta f' f' \theta'' \\ &+ 2\eta\lambda f' f' \theta'' - 3\eta f^2\theta' + \frac{2}{\eta}f^2\theta' \end{aligned} \right\} \\ + \frac{4}{3N_R} \left\{ \begin{aligned} &[1 + (\theta_f - 1)\theta]^3 \left(\frac{1}{2}\theta' + \eta\theta'' \right) \\ &+ 3\eta[1 + (\theta_f - 1)\theta]^2(\theta_f - 1)(\theta')^2 \end{aligned} \right\} \\ + 4\eta\text{PrEc} \left[\frac{(f'')^2}{1 + (4Wef'')^n} \right] + \eta\text{Pr}[Nb(\theta' \varphi' + \theta' \chi') + Nt(\theta')^2] \\ + \text{PrEc}(M + \beta) \left(f' - \frac{1}{2}\lambda \right)^2 + \frac{1}{2}\text{Pr}(A^* e^{-\eta} + B^* \theta)f' = 0 \end{aligned} \quad (15)$$

$$\eta\varphi'' + 2\varphi' + Scf\varphi' + 2\left(\frac{Nt}{Nb}\right)(\theta' + \eta\theta'') - \frac{1}{2}ScK_1\varphi(1 - \varphi)^2 = 0 \quad (16)$$

$$\left. \begin{aligned} f'(A) &= \frac{1}{2}(1 - \lambda), f(A) = \frac{A}{2}(1 - \lambda), \theta(A) = 1, \varphi'(A) = -\frac{1}{2}K_2\varphi(A) \text{ at } \eta = A \\ f'(\infty) &\rightarrow \frac{\lambda}{2}, \theta(\infty) \rightarrow 0, \varphi(\infty) \rightarrow 1 \text{ as } \eta \rightarrow \infty \end{aligned} \right\} \quad (17)$$

Where

$$\left. \begin{aligned} We &= \frac{\Gamma U Re}{x}, Re = \frac{Ur}{v_f}, M = \frac{\sigma_f B_0^2 x}{\rho_f U}, v = \frac{\eta_0}{\rho_f}, \Omega = \frac{DC_2}{DC_1}, \lambda = \frac{U_\infty}{U}, Y_E = \frac{\delta_E U}{x} \\ \beta &= \frac{v_f x}{UK^*}, N_R = \frac{k_f k_f^*}{4\sigma^* T_\infty}, N_l = \frac{\tau D_T T_\infty (\theta_f - 1)}{v_f T_\infty}, K_1 = \frac{k_c C_0^2 x}{U}, K_2 = \frac{k_s v_f x}{D_{C_1} U r} \\ Pr &= \frac{v_f}{\alpha_f}, Sc = \frac{v_f}{D_{C_1}}, Nb = \frac{\tau D_B C_0}{v_f}, Ec = \frac{U^2}{(c_p)_f (\theta_f - 1) T_\infty}, \theta_f = \frac{T_w}{T_\infty} \end{aligned} \right\} \quad (18)$$

3. Entropy production analysis

In the present study local entropy generations EG could be displayed by

$$\begin{aligned} \dot{S}_{gen} &= \underbrace{\frac{k_f}{T^2} \left[\left(\frac{\partial T}{\partial r} \right)^2 \right] + \frac{k_f}{T^2} \left[\frac{16\sigma^* T^3}{3k^* k_f} \left(\frac{\partial T}{\partial r} \right)^2 \right]}_{\text{Heat transfer irreversibility}} + \underbrace{\frac{\eta_0}{T} \left\{ \frac{\left(\frac{\partial u}{\partial r} \right)^2}{1 + \lambda \left| \frac{\partial u}{\partial r} \right|^n} \right\}}_{\text{Viscous dissipation irreversibility}} + \underbrace{\frac{\sigma_f B_0^2 (u - U_\infty)^2}{T}}_{\text{Joule heating irreversibility}} + \underbrace{\frac{\mu_f (u - U_\infty)^2}{TK^*}}_{\text{Porous medium Irreversibility}} \\ &+ \underbrace{\frac{RD_{C_1}}{T} \left(\frac{\partial C_1}{\partial r} \frac{\partial T}{\partial r} \right) + \frac{RD_{C_1}}{C_1} \left\{ \left(\frac{\partial C_1}{\partial r} \right)^2 \right\} + \frac{RD_{C_2}}{C_2} \left\{ \left(\frac{\partial C_2}{\partial r} \right)^2 \right\} + \frac{RD_{C_2}}{T} \left(\frac{\partial C_2}{\partial r} \frac{\partial T}{\partial r} \right)}_{\text{Mass transfer irreversibility}} \end{aligned} \quad (19)$$

The EG number would be the non-dimensional type of the EG rate which expresses the ratio between the actual EG rate $\dot{S}_{gen}$ and characteristic EG rate $\dot{S}_0$.

Using eq. (7) the dimensionless EG rate N_G becomes:

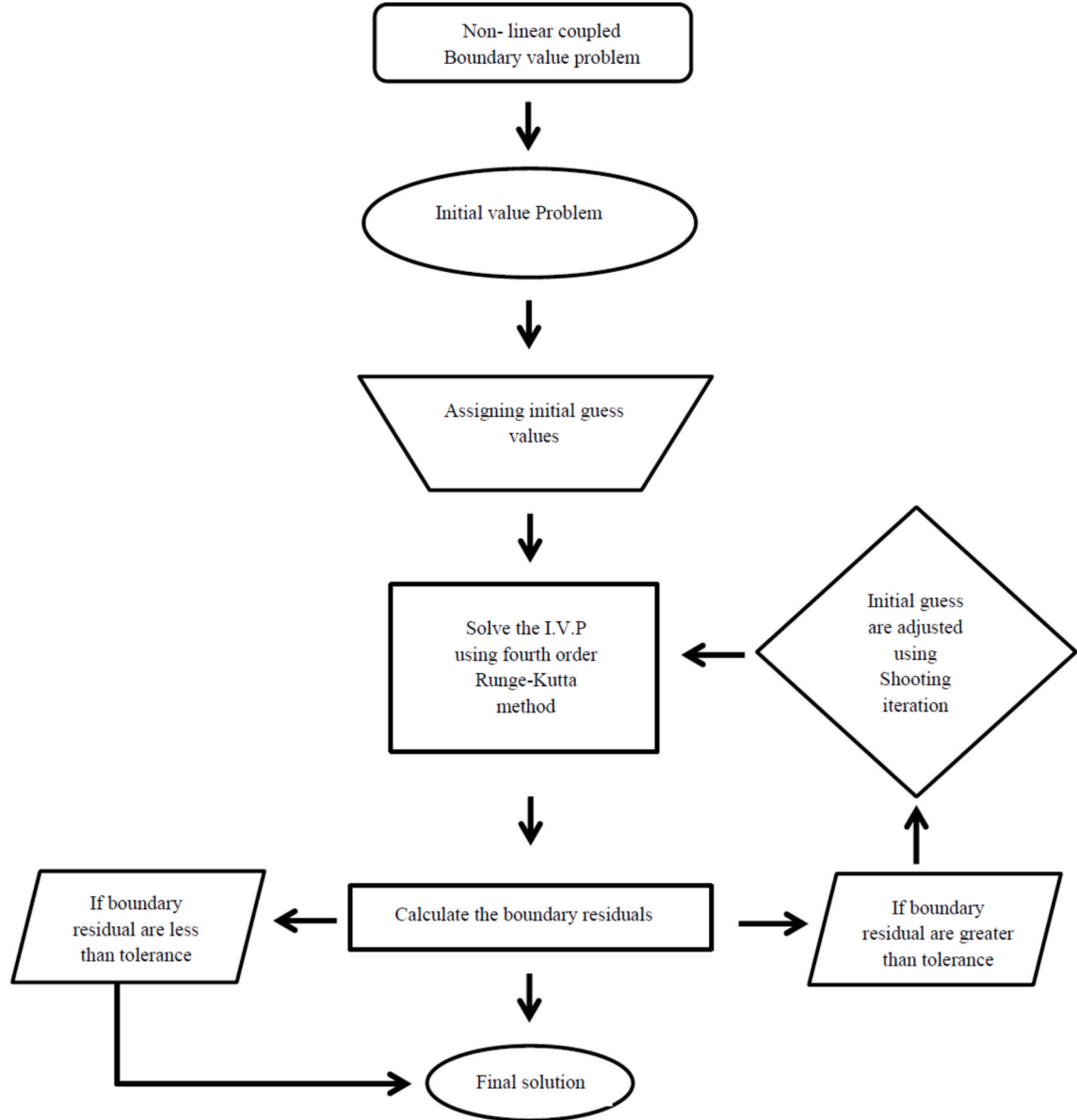
$$\begin{aligned} N_G &= \frac{\dot{S}_{gen}'''}{\dot{S}_0'''} \\ &= Re^2 \left\{ 1 + \frac{4}{3} Rd [1 + (\theta_f - 1)\theta]^3 \right\} \frac{(\theta')^2}{[1 + (\theta_f - 1)\theta]^2} \\ &+ \frac{4Br}{(\theta_f - 1)[1 + (\theta_f - 1)\theta]} \left\{ \frac{(f'')^2}{1 + (4We f'')^n} \right\} \\ &+ \frac{(\beta + M) Br Re}{(\theta_f - 1)[1 + (\theta_f - 1)\theta]} \left(f' - \frac{1}{2}\lambda \right)^2 + \frac{\Omega_1 Re^2}{(\theta_f - 1)} \varphi' \left(\theta' + \frac{\varphi'}{\varphi} \right) \\ &+ \frac{\Omega_2 Re^2}{(1 - \varphi)} (\varphi')^2 - \frac{\Omega_2 Re^2}{(\theta_f - 1)[1 + (\theta_f - 1)\theta]} \theta' \varphi' \end{aligned} \quad (22)$$

The Bejan number for the present problem is

$$\begin{aligned} Be &= \frac{Re^2 \left\{ 1 + \frac{4}{3} Rd [1 + (\theta_f - 1)\theta]^3 \right\} \frac{(\theta')^2}{[1 + (\theta_f - 1)\theta]^2} + \frac{\Omega_1 Re^2}{(\theta_f - 1)} \varphi' \left(\theta' + \frac{\varphi'}{\varphi} \right) + \frac{\Omega_2 Re^2}{(1 - \varphi)} (\varphi')^2 - \frac{\Omega_2 Re^2}{(\theta_f - 1)[1 + (\theta_f - 1)\theta]} \theta' \varphi'}{Re^2 \left\{ 1 + \frac{4}{3} Rd [1 + (\theta_f - 1)\theta]^3 \right\} \frac{(\theta')^2}{[1 + (\theta_f - 1)\theta]^2} + \frac{4Br}{(\theta_f - 1)[1 + (\theta_f - 1)\theta]} \left\{ \frac{(f'')^2}{1 + (4We f'')^n} \right\} + \frac{(\beta + M) Br Re}{(\theta_f - 1)[1 + (\theta_f - 1)\theta]} \left(f' - \frac{1}{2}\lambda \right)^2 + \frac{\Omega_1 Re^2}{(\theta_f - 1)} \varphi' \left(\theta' + \frac{\varphi'}{\varphi} \right) + \frac{\Omega_2 Re^2}{(1 - \varphi)} (\varphi')^2 - \frac{\Omega_2 Re^2}{(\theta_f - 1)[1 + (\theta_f - 1)\theta]} \theta' \varphi'} \end{aligned} \quad (23)$$

Table 1

Flow chart of the numerical scheme.



4. Numerical scheme

The highly coupled and complex system of equations (9), (15) and (16) is solved via shooting approach of MATLAB through transmuting boundary value problem to an initial value problem (IVP). Firstly, we should write the lower order term of differential equations (DE):

$$F''' = -\frac{1}{\text{Re}[1 + (1-n)(4Wef'')^n]} \left[\frac{1}{2} [1 + (4Wef'')^n]^2 \left\{ ff'' - \frac{1}{8} (\beta + M)(2f' - \lambda) \right\} \right] \quad (24)$$

$$\begin{aligned}
& \theta' + \frac{1}{2} \text{Pr} f \theta' + 4\eta \text{Pr} Ec \left[\frac{(f'')^2}{1 + (4We f'')^n} \right] + \text{Pr} \gamma_E \left(\frac{2}{\eta} - 3\eta \right) y_1^2 y_5 \\
& + \frac{4}{3N_R} \left\{ \frac{1}{2} [1 + (\theta_f - 1)\theta]^3 \theta' + 3\eta [1 + (\theta_f - 1)\theta]^2 (\theta_f - 1)(\theta')^2 \right\} \\
\theta'' = & - \frac{\eta \text{Pr} Nt (\theta')^2 + \text{Pr} Ec (M + \beta) (f')^2 + \frac{1}{2} \text{Pr} (A^* e^{-\eta} + B^* \theta) f'}{\eta + \frac{4\eta}{3N_R} [1 + (\theta_f - 1)\theta]^3 - \text{Pr} \gamma_E \{ 4\eta (1 + \eta) (f')^2 + 2\eta f' (\lambda - 2) \}} \quad (25)
\end{aligned}$$

$$\varphi'' = -\frac{1}{\eta} \left[2\varphi' + Sc f \varphi' + 2 \left(\frac{Nt}{Nb} \right) (\theta' + \eta \theta'') - \frac{1}{2} Sc K_1 \varphi (1 - \varphi)^2 \right] \quad (26)$$

then the equations must be reduced into a system of first order DE via below parameters:

$$y_1 = f, \quad y_2 = f', \quad y_3 = f'', \quad y_4 = \theta, \quad y_5 = \theta', \quad y_7 = \varphi, \quad y_8 = \varphi'. \quad (27)$$

Plus, the governing equations must be written in matrix structure using $y = [f, f', f'', \theta, \theta', \varphi, \varphi']^T$ as

$$\frac{d}{d\eta} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \\ y_7 \end{pmatrix} = \begin{pmatrix} y_2 \\ y_3 \\ \frac{1}{\text{Re}[1 + (1-n)(4We y_3)^n]} \left[\frac{1}{2} [1 + (4We y_3)^n]^2 \left\{ y_1 y_3 - \frac{1}{8} (\beta + M) (2y_2 - \lambda) \right\} \right] \\ y_5 \\ y_5 + \frac{1}{2} \text{Pr} y_1 y_5 + 4\eta \text{Pr} Ec \left[\frac{y_3^2}{1 + (4We y_3)^n} \right] + \text{Pr} \gamma_E \left(\frac{2}{\eta} - 3\eta \right) y_1^2 y_5 \\ + \frac{4}{3N_R} \left\{ \frac{1}{2} [1 + (\theta_f - 1)y_4]^3 y_5 + 3\eta [1 + (\theta_f - 1)y_4]^2 (\theta_f - 1)y_5^2 \right\} \\ - \frac{\eta \text{Pr} Nt y_5^2 + \text{Pr} Ec (M + \beta) y_2^2 + \frac{1}{2} \text{Pr} (A^* e^{-\eta} + B^* \theta) y_2}{\eta + \frac{4\eta}{3N_R} [1 + (\theta_f - 1)y_4]^3 - \text{Pr} \gamma_E \{ 4\eta (1 + \eta) y_2^3 + 2\eta y_1 y_2 (\lambda - 2) \}} \\ y_7 \\ \left[y_5 + \frac{1}{2} \text{Pr} y_1 y_5 + 4\eta \text{Pr} Ec \left\{ \frac{y_3^2}{1 + (4We y_3)^n} \right\} + \text{Pr} \gamma_E \left(\frac{2}{\eta} - 3\eta \right) y_1^2 y_5 \right] \\ + \frac{4}{3N_R} \left\{ \frac{1}{2} [1 + (\theta_f - 1)y_4]^3 y_5 + 3\eta [1 + (\theta_f - 1)y_4]^2 (\theta_f - 1)y_5^2 \right\} \\ - \frac{1}{\eta} \left[2y_7 + Sc y_1 y_7 + 2 \left(\frac{Nt}{Nb} \right) y_5 - \frac{1}{2} Sc K_1 y_6 (1 - y_6)^2 \right] - 2 \left(\frac{Nt}{Nb} \right) \frac{\left[\eta \text{Pr} Nt y_5^2 + \text{Pr} Ec (M + \beta) y_2^2 + \frac{1}{2} \text{Pr} (A^* e^{-\eta} + B^* \theta) y_2 \right]}{1 + \frac{4}{3N_R} [1 + (\theta_f - 1)y_4]^3 - \text{Pr} \gamma_E \{ 4\eta (1 + \eta) y_2^3 + 2\eta y_1 y_2 (\lambda - 2) \}} \end{pmatrix}, \quad (28)$$

and boundary conditions could be described as

$$\begin{aligned}
y_1(0) &= \frac{A}{2}(1-\lambda), \quad y_2(0) = \frac{1}{2}(1-\lambda), \quad y_4(0) = 1, \quad y_7(0) = -\frac{1}{2}K_2y_6(0), \\
y_2(\infty) &\rightarrow \frac{\lambda}{2}, \quad y_4(\infty) \rightarrow 0, \quad y_6(\infty) \rightarrow 1.
\end{aligned}
\tag{29}$$

Then, the resulted BVP should be transformed to IVP and an ODE solver must be applied to integrate such system considering below initial condition:

$$y(0) = \left[\frac{A}{2}(1-\lambda), \frac{1}{2}(1-\lambda), S_1, 1, S_2, -\frac{1}{2}K_2S_3, S_3 \right], \tag{30}$$

where S_1 , S_2 and S_3 are unknowns and chosen based on the boundary conditions.

Thirdly, shooting approach could be utilized to detect primary guess. We should go on the procedure until attaining the convergence criteria as error $|E_i| < \text{tolerance} = 10^{-10}$ where errors may be characterized as:

$$\begin{aligned}
E_1 &= y_3(\infty, S_1) - F''(\infty), \\
E_2 &= y_5(\infty, S_2) - \theta'(\infty), \\
E_3 &= y_7(\infty, S_3) - \varphi'(\infty).
\end{aligned}
\tag{31}$$

Here, $\eta_{\max}(\eta_{\infty})$ is taken as 10. However, the system gets converged before getting $\eta_{\max}$. The flow chart of the numerical scheme have discussed in Table 1.

After calculating y_i ($i = 1, 2, 3, 4, 5, 6, 7$), we have calculated the entropy generation (from equation (22)) and Bejan number (from equation (23)) and written as

$$\begin{aligned}
N_G &= \text{Re}^2 \left\{ 1 + \frac{4}{3}Rd[1 + (\theta_f - 1)y_4]^3 \right\} \frac{y_5^2}{[1 + (\theta_f - 1)y_4]^2} \\
&+ \frac{4Br}{(\theta_f - 1)[1 + (\theta_f - 1)y_4]} \left\{ \frac{y_3^2}{1 + (4We y_3)^n} \right\} \\
&+ \frac{(\beta + M) Br \text{Re}}{(\theta_f - 1)[1 + (\theta_f - 1)y_4]} \left(y_2 - \frac{1}{2}\lambda \right)^2 + \frac{\Omega_1 \text{Re}^2}{(\theta_f - 1)y_7} \left(y_5 + \frac{y_7}{y_6} \right) \\
&+ \frac{\Omega_2 \text{Re}^2}{(1 - y_6)y_7^2} - \frac{\Omega_2 \text{Re}^2}{(\theta_f - 1)[1 + (\theta_f - 1)y_4]} y_5 y_7
\end{aligned}
\tag{32}$$

$$\begin{aligned}
Be &= \frac{\text{Re}^2 \left\{ 1 + \frac{4}{3}Rd[1 + (\theta_f - 1)y_4]^3 \right\} \frac{y_5^2}{[1 + (\theta_f - 1)y_4]^2} + \frac{\Omega_1 \text{Re}^2}{(\theta_f - 1)y_7} \left(y_5 + \frac{y_7}{y_6} \right)}{\text{Re}^2 \left\{ 1 + \frac{4}{3}Rd[1 + (\theta_f - 1)y_4]^3 \right\} \frac{y_5^2}{[1 + (\theta_f - 1)y_4]^2} \\
&+ \frac{4Br}{(\theta_f - 1)[1 + (\theta_f - 1)y_4]} \left\{ \frac{y_3^2}{1 + (4We y_3)^n} \right\} \\
&+ \frac{(\beta + M) Br \text{Re}}{(\theta_f - 1)[1 + (\theta_f - 1)y_4]} \left(y_2 - \frac{1}{2}\lambda \right)^2 + \frac{\Omega_1 \text{Re}^2}{(\theta_f - 1)y_7} \left(y_5 + \frac{y_7}{y_6} \right) \\
&+ \frac{\Omega_2 \text{Re}^2}{(1 - y_6)y_7^2} - \frac{\Omega_2 \text{Re}^2}{(\theta_f - 1)[1 + (\theta_f - 1)y_4]} y_5 y_7}
\end{aligned}
\tag{33}$$

5. Results and discussion

This section comprises the interpretation of the graphical structures of velocity, temperature and nanoparticles concentration fields under the impact of different involved parameters. The all computations have been executed for a wide range of parameters involved so called the influential parameters such as velocity ratio parameter(λ), Hartmann number(M), flow behavior index(n), Reynolds number(Re), Weissenberg parameter(We), radiation parameter (N_R), thermophoresis parameter(Nt), Brownian motion parameter(Nb), Prandtl number(Pr), temperature ratio parameter(θ_f), Brinkman number(Br), A^* and B^* are respectively the space dependent and temperature dependent parameters, heterogeneous reaction parameter(K_2), homogeneous reaction parameter(K_1), Schmidt number(Sc), CCHF parameter (γ_E), diffusion rate for heterogeneous reaction(Ω_2), diffusion rate for homogeneous reaction (Ω_1). The above parameters catered the considered values as $Re = 0.5$, $n = 0.5$, $We = 1$, $M = 1$, $\lambda = 0.8$, $Nr = 0.2$, $\theta_f = 1.2$, $Pr = 10$, $Ec = 1$, $Nb = Nt = 0.1$, $Sc = 10$, $K_1 = 0.2$, $K_2 = 0.1$, $A = 0.2$, $\Omega_1 = 0.5$, $\Omega_2 = 0.5$, $Br = 0.5$. Table 2 provides the relevant information

Table 2

Local Nusselt number and skin friction for diverse parameters with $n = 0.5, Pr = 10, Ec = 1, Nb = Nt = 0.1, Sc = 10, K_1 = 0.2, K_2 = 0.1, \Omega_1 = \Omega_2 = Br = 0.5, A^* = B^* = 0.1$

λ	M	β	N_R	Re	A	We	θ_f	C_{fx}	Nu_x
0.5	1	0.1	0.2	0.5	0.2	1	1.2	0.00000000	9.47560904
0.8								0.66364829	8.35050371
1								1.03240720	7.70072169
1.2								1.34095281	7.04911109
0.8	0.5							0.84756225	8.61265064
	1.5							0.76143458	8.49663849
	2							0.54864676	8.15683183
	1	0						0.99619930	8.82404201
		1						0.84756225	8.62996930
		2						0.66364829	8.35050371
		1	0.1					0.66364829	5.80141319
			0.3					0.66364829	6.76069533
			0.4					0.66364829	11.80432843
				0.8				0.38314981	8.01839549
				1.0				0.46928795	7.86434315
				1.5				0.52467686	7.59288430
					0.1			0.96382129	10.62398013
					0.3			0.82368803	7.29297667
					0.4			0.46305193	6.64280770
						0.5		0.54503831	8.23319541
						1.5		0.60016082	8.43287894
						2		0.72788611	8.49768366
							0.8	0.66364829	8.54128249
							1	0.66364829	8.06713404
							1.5	0.66364829	7.47514144

regarding the impact of embedded parameters with their characteristic values such as with $n = 0.5, Pr = 10, Ec = 1, Nb = Nt = 0.1, Sc = 10, K_1 = 0.2, K_2 = 0.1, \Omega_1 = \Omega_2 = Br = 0.5$ on skin friction (C_{fx}) and Nusselt number (Nu_x) subject to the flow of magneto cross flows over thin needle. It is more reflective from the numerically computed Table 2 that incremented λ and Re contributed to intense viscous drag and diminution of HTR from the needle surface. Further the hike in M and A yielded reduced viscous drag as well as the rate of heat transportation while that of We caused the opposite trend for both. Interestingly, hike in N_R does not affect the skin friction while it enhances the HTR thereby augmenting the cooling which is vital for us. Further, variation of C_{fx} is independent of θ_f while it strongly influences HTR for the needle surface. Increment in θ_f rises temperature gradient and therefore belittles the HTR. It is also noticed that rise in porosity parameter β declines the SVD and the HTR. Table 3 indicates the characteristics of N_G and Bejan number (Be) under the varied estimations of each $\lambda, M, N_R, Re, \Omega_1, Br$. When it shows the impression of any these parameters, meanwhile the other parameters would be kept as fixed. As λ increases, N_G upgrades while Be declines keeping other parameters fixed as $M = 1, N_R = 0.2, Re = 0.5, \Omega_1 = 0.5, Br = 0.5$. The influence of M and Br contribute the same behavior to N_G and Be . Further, the rise in Ω_1 and

Table 3

EG and Bejan number for different parameters with $n = 0.5, We = 1, \theta_f = 1.2, Pr = 10, Ec = 1, Nb = Nt = 0.1, Sc = 10, K_1 = 0.2, K_2 = 0.1, \Omega_2 = 0.5, A^* = B^* = 0.1$.

λ	M	N_R	Re	Ω_1	Br	N_G	Be
0.5	1	0.2	0.5	0.5	0.5	1.22386514	0.53899433
0.8						2.71556608	0.20953576
1						5.24243866	0.08687502
1.2						8.64599204	0.04498241
0.8	0.5					2.04310111	0.26322413
	1.5					3.38896829	0.17613293
	2					4.06361707	0.15485269
		0.1				2.74895320	0.21765587
		0.3				2.66736316	0.20365441
		0.4				2.62244914	0.20269435
			0.8			3.81929451	0.34905766
			1.0			5.01350686	0.40150016
			1.5			9.13678640	0.45937700
				0.1		2.64560430	0.18863230
				0.8		2.76803740	0.22451991
				1		2.80301829	0.23419769
					0.8	3.72344692	0.15281759
					1	4.39536749	0.12945634
					1.5	6.07516891	0.09366130

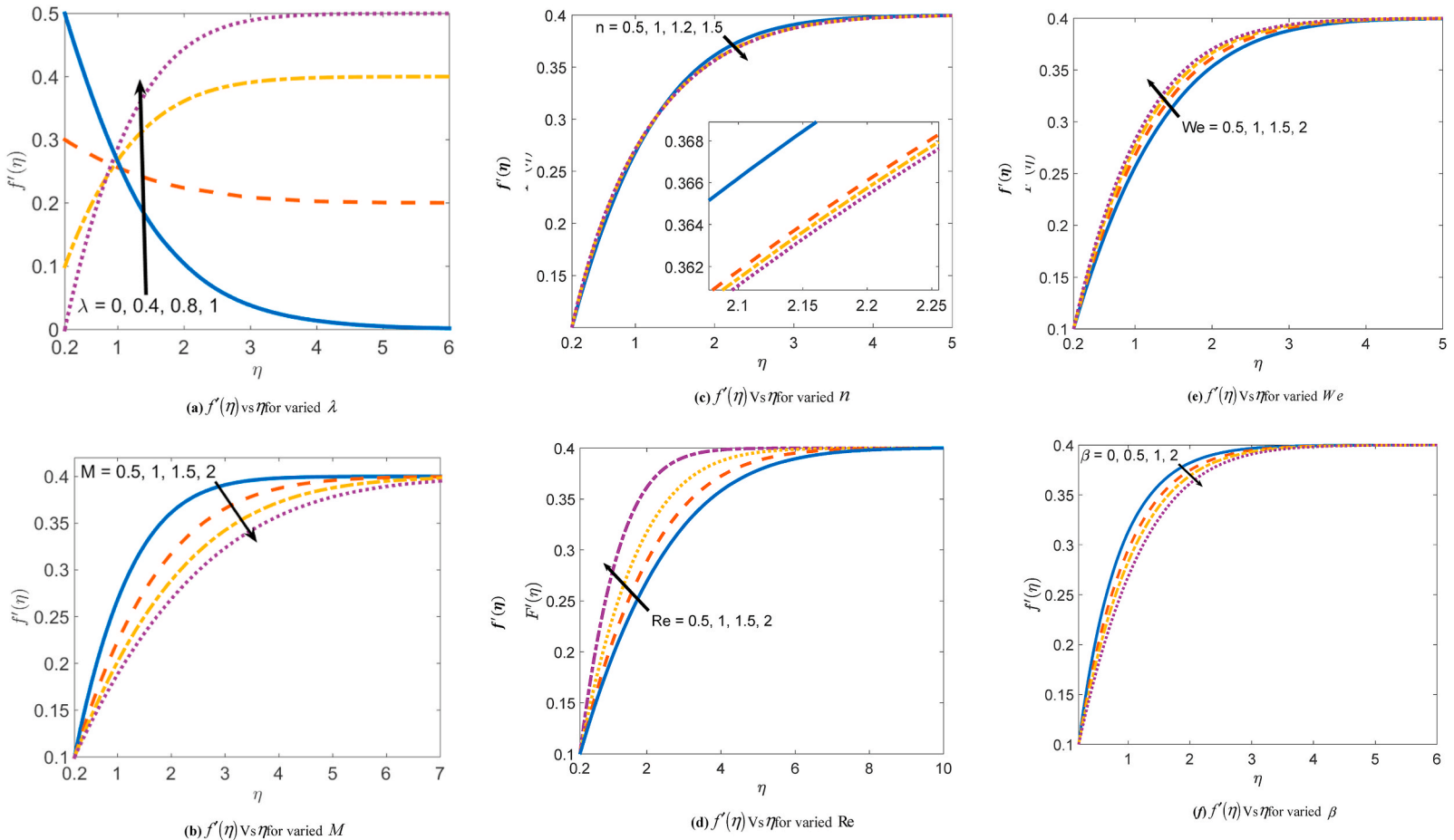


Fig. 2. (a) $f'(\eta)$ vs η for varied λ (b) $f'(\eta)$ vs η for varied M (c) $f'(\eta)$ vs η for varied n (d) $f'(\eta)$ vs η for varied Re (e) $f'(\eta)$ vs η for varied We (f) $f'(\eta)$ vs η for varied β .

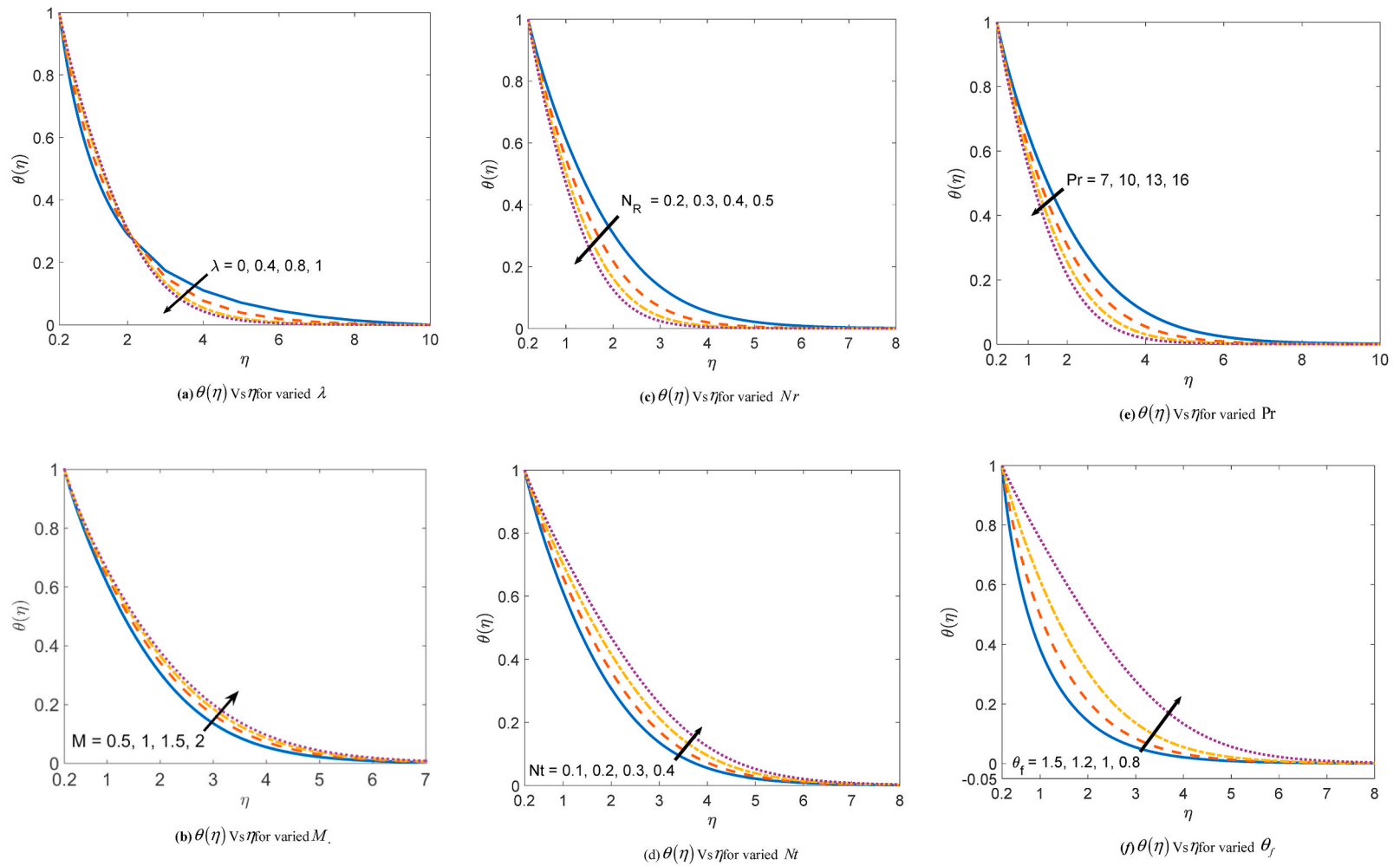


Fig. 3. (a) $\theta(\eta)$ Vs η for varied λ (b) $\theta(\eta)$ Vs η for varied M (c) $\theta(\eta)$ Vs η for varied Nr (d) $\theta(\eta)$ Vs η for varied Nt (e) $\theta(\eta)$ Vs η for varied Pr (f) $\theta(\eta)$ Vs η for varied θ_f (g) $\theta(\eta)$ Vs η for varied We (h) $\theta(\eta)$ Vs η for varied β (i) $\theta(\eta)$ Vs η for varied A^* (j) $\theta(\eta)$ Vs η for varied B^* (k) $\theta(\eta)$ Vs η for varied Y_E .

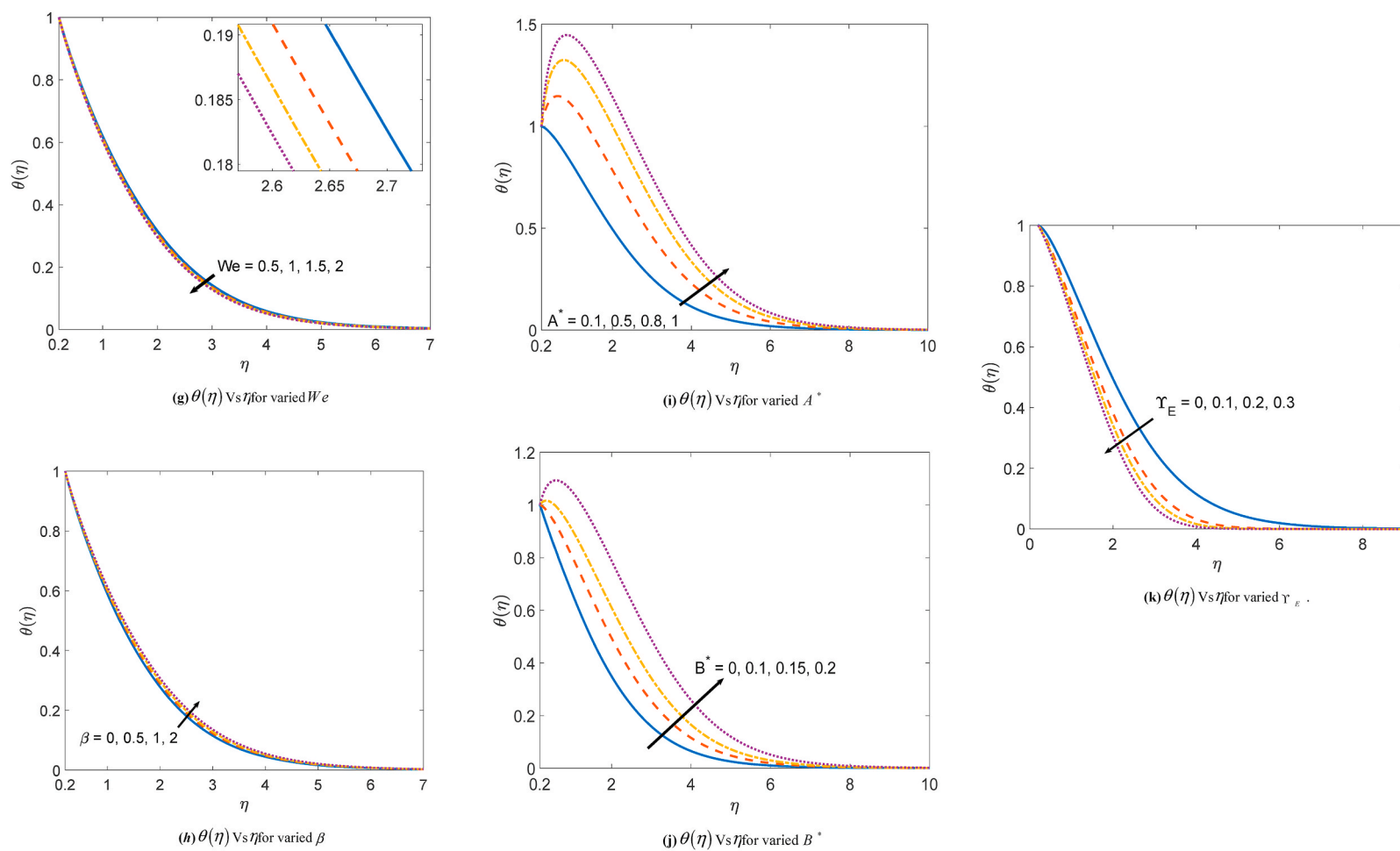


Fig. 3. (continued).

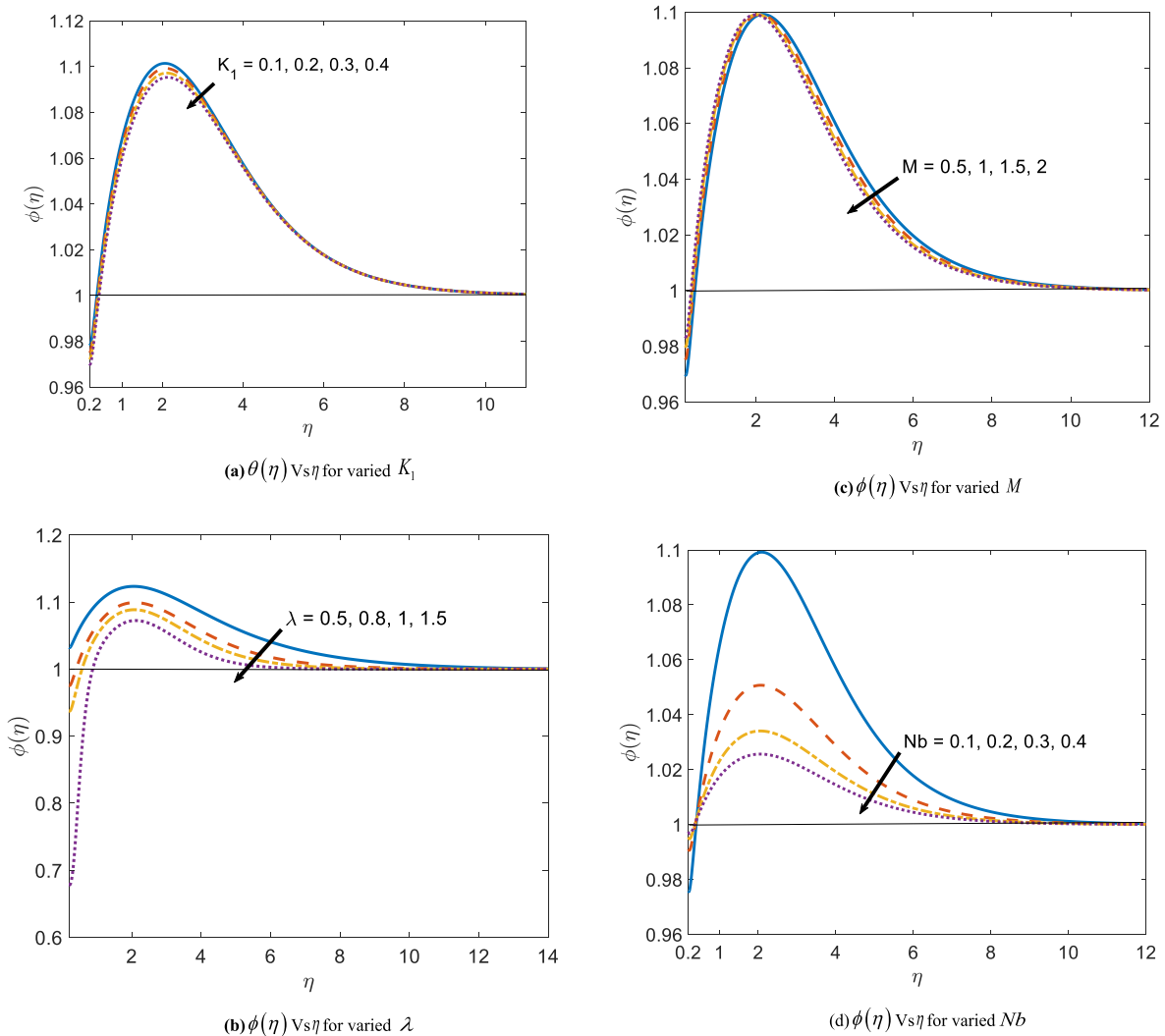
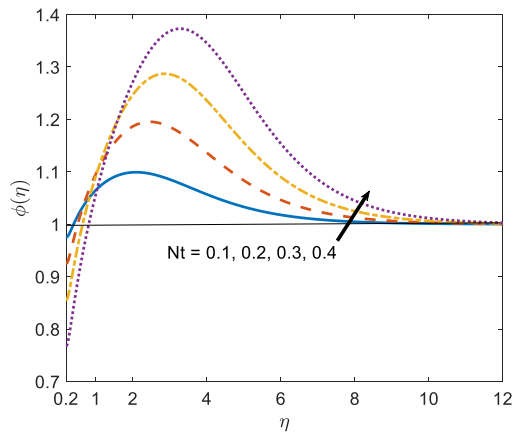
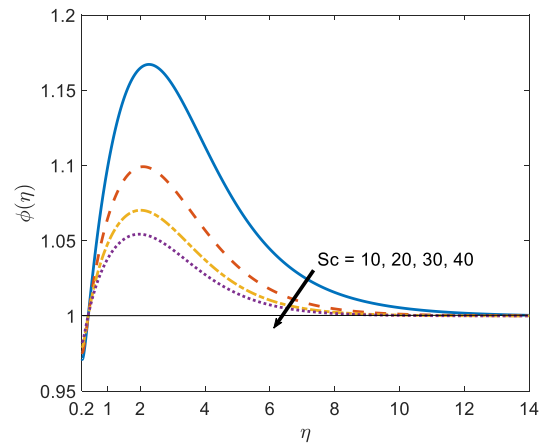
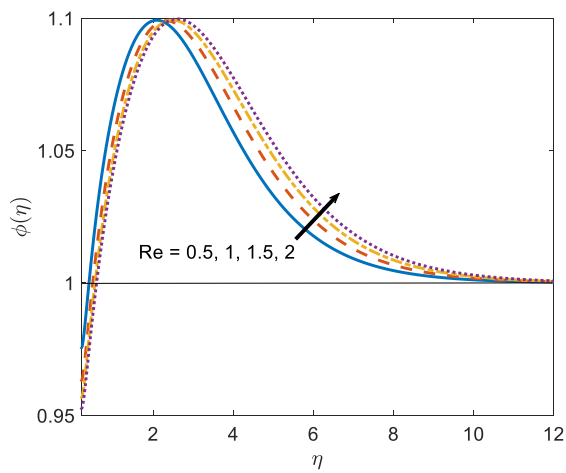
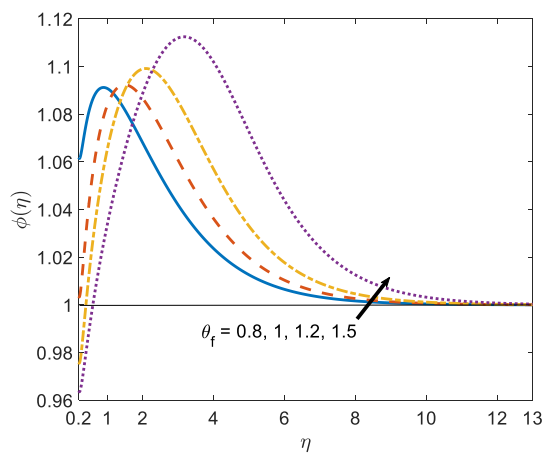
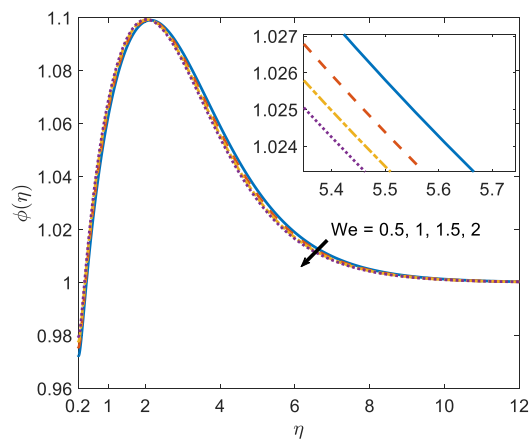


Fig. 4. (a) $\theta(\eta)$ Vs η for varied K_1 (b) $\phi(\eta)$ Vs η for varied λ (c) $\phi(\eta)$ Vs η for varied M (d) $\phi(\eta)$ Vs η for varied Nb (e) $\phi(\eta)$ Vs η for varied Nt (f) $\phi(\eta)$ Vs η for varied Re (g) $\phi(\eta)$ Vs η for varied Sc (h) $\phi(\eta)$ Vs η for varied θ_f (i) $\phi(\eta)$ Vs η for varied We .

Reescalates N_G and B while that of N_R makes both to fall significantly.

Fig. 2(a) demonstrates the strategic feature of velocity field $f'(\eta)$ due to varied λ ($\lambda = 0, 0.4, 0.8, 1$). For $\lambda = 1$, a Blasius type flow of cross nfs (characterized by stationary needle in moving cross nf) takes place. In such flow regime, $f'(\eta)$ attains minimum at $\eta = 0.2$ i.e., $f'_{\min}(\eta)|_{\eta=0.2} = 0$. Thereafter, $f'(\eta)$ steps up forward looking growth as we move away from the needle surface and finally gets saturation towards the ambient. Equally important, there is a dramatic change when $\lambda = 0$, the flow of cross nf is a Sakiadis flow type. It is characterized by a moving needle in a stationary cross nf. In such environment $f'(\eta)$ attains maximum at $\eta = 0.2$ i.e., $f'_{\max}(\eta)|_{\eta=0.2} = 0.5$ and beyond $\eta = 0.2$ it falls very sharply and finally decays to zero in the ambient fluid. Interesting is here that there is an obvious in flow pattern in the sense that $f'_{\text{Sakiadis flow}}$ is a mirror image of $f'_{\text{Blasius flow}}$ cross nfs over thin needle. Further, $f'(\eta)$ profile for $\lambda = 0.4$ is also a mirror image to $f'(\eta)$ profile for $\lambda = 0.8$. This implicates that lower values of λ ($\lambda = 0, 0.4$) yield reverse flow in contrast to the flow due to higher values of λ ($\lambda = 0.8, 1$). Such scenario of flow pattern is due to the relative velocity of thin needle and the associated cross nf. This is well agreed with the result reported by Khan et al. [49].

Fig. 2(b) represents the velocity variation for varied M ($M = 0.5, 1, 1.5, 2$). Hike in M declines the flow. Higher M ($M = 1.5, 2$) provides significant decay in $f'(\eta)$ and the related VBLT. Augmented M yields greater Lorentz force which in turn uplifts the fluid resistance thereby imparts flow restraint. This well understood from Hina et al. [50]. The $f'(\eta)$ is found to upgrade for different estimations n ($n = 0.5, 1, 1.2, 1.5$) in few layers of cross nf. But this scenario is veered significantly after a few layers of cross nfs in the BLR

(e) $\phi(\eta)$ Vs η for varied Nt (g) $\phi(\eta)$ Vs η for varied Sc (f) $\phi(\eta)$ Vs η for varied Re (h) $\phi(\eta)$ Vs η for varied θ_f (i) $\phi(\eta)$ Vs η for varied We **Fig. 4.** (continued).

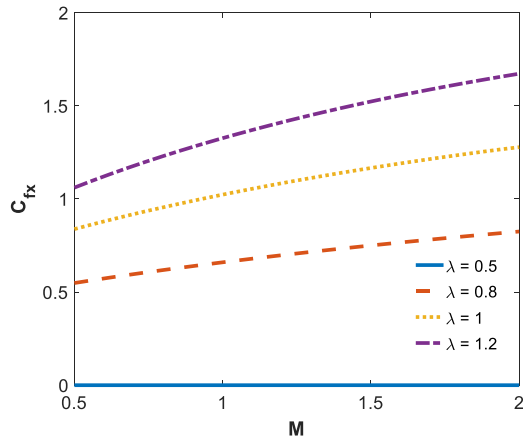
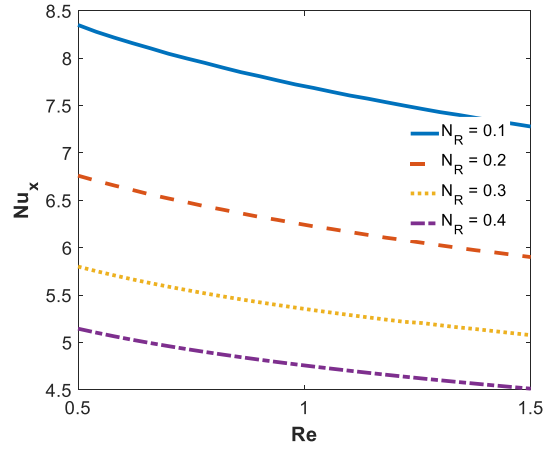
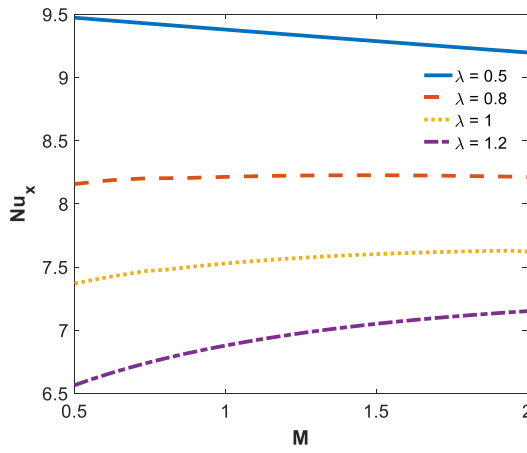
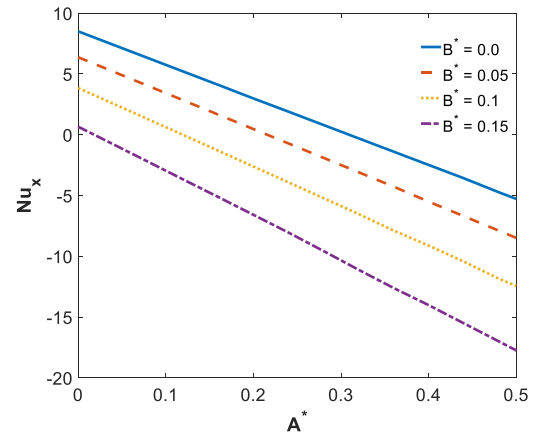
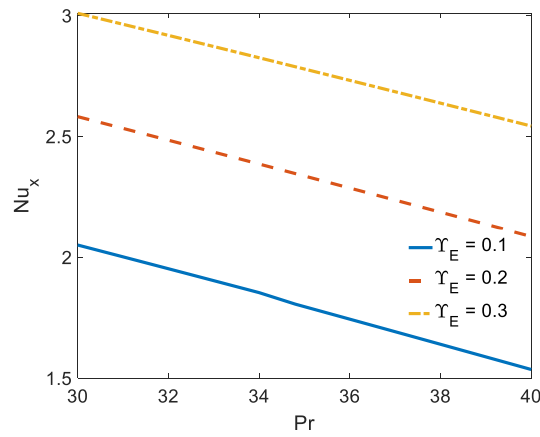
(a) C_{fx} Vs M for varied λ (c) Nu_x Vs Re for varied Nr (b) Nu_x Vs M for varied λ (d) Nu_x Vs A^* for varied B^* (e) Nu_x Vs Pr for varied γ_E

Fig. 5. (a) C_{fx} Vs M for varied λ (b) Nu_x Vs M for varied λ (c) Nu_x Vs Re for varied Nr (d) Nu_x Vs A^* for varied B^* (e) Nu_x Vs Pr for varied γ_E .

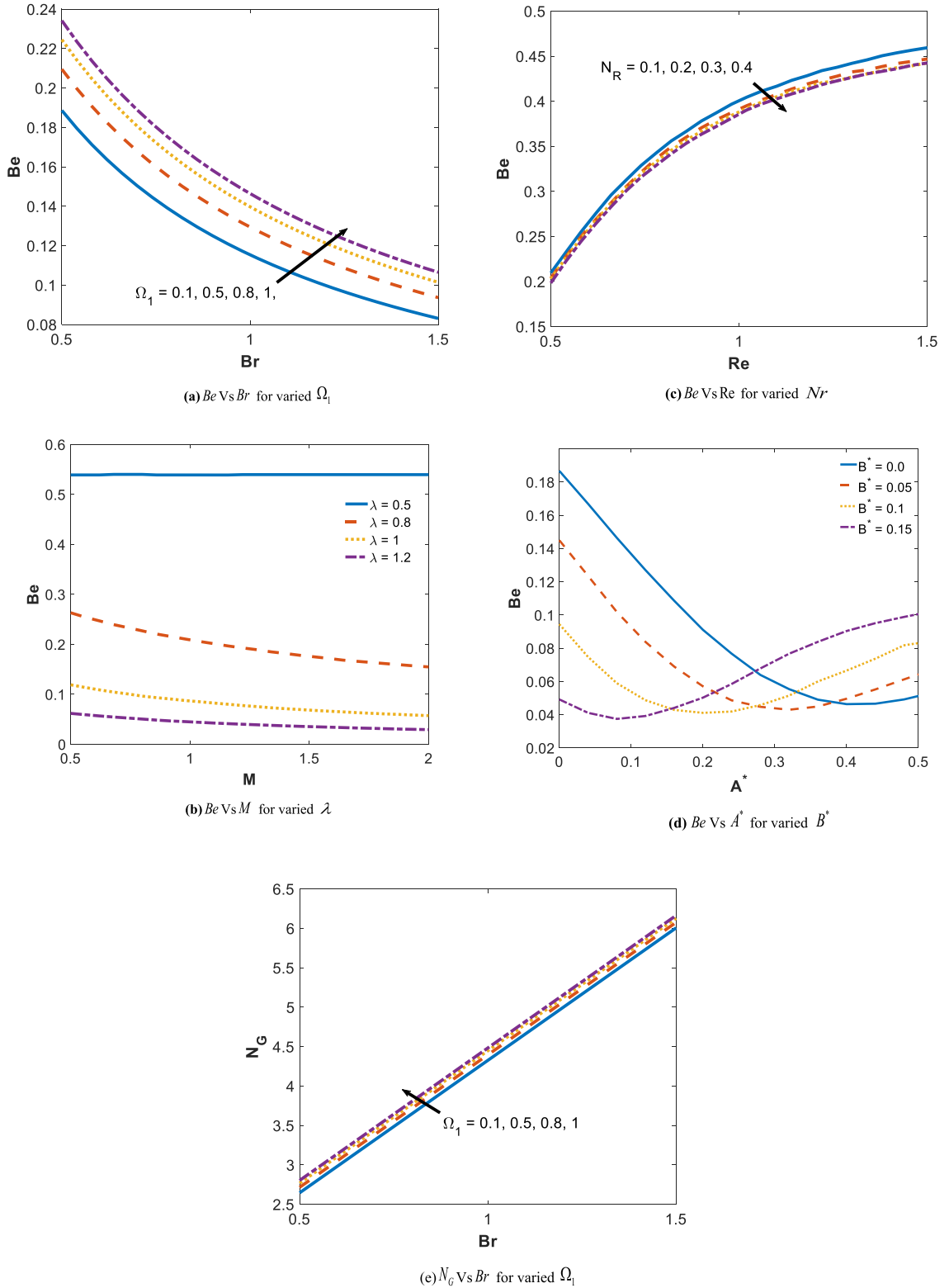


Fig. 6. (a) Be Vs Br for varied Ω_1 (b) Be Vs M for varied λ (c) Be Vs Re for varied Nr (d) Be Vs A^* for varied B^* (e) N_G Vs Br for varied Ω_1 (f) N_G Vs M for varied λ (g) N_G Vs Re for varied N_R (h) N_G Vs We for varied Ω_2 (i) N_G Vs A^* for varied B^* (j) N_G Vs Pr for varied γ_E .

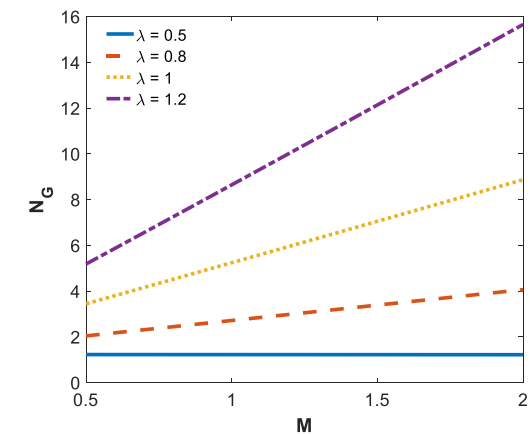
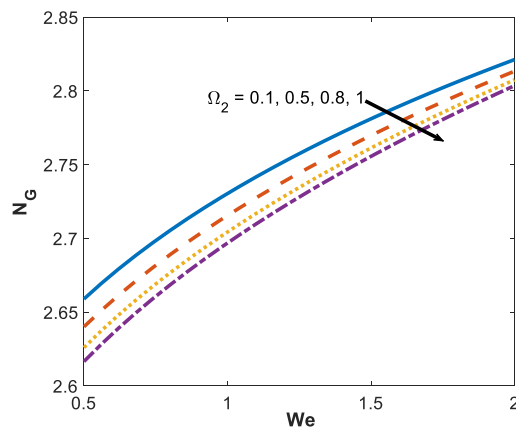
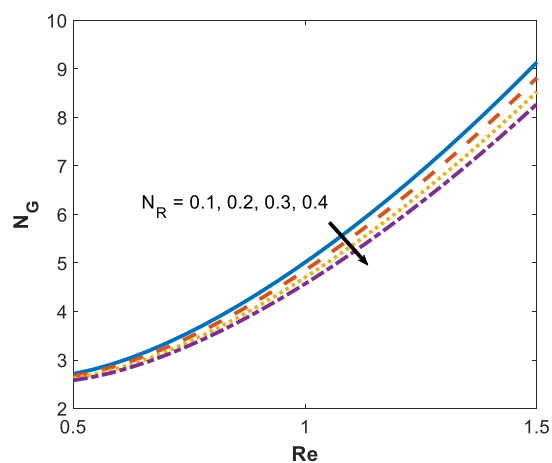
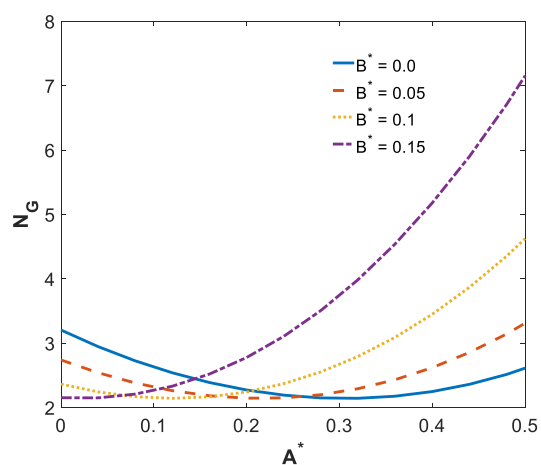
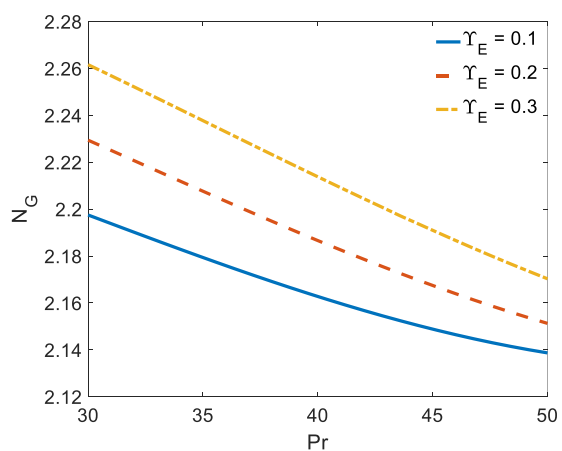
(f) N_G Vs M for varied λ (h) N_G Vs We for varied Ω_2 (g) N_G Vs Re for varied N_R (i) N_G Vs A^* for varied B^* (j) N_G Vs Pr for varied γ_E .

Fig. 6. (continued).

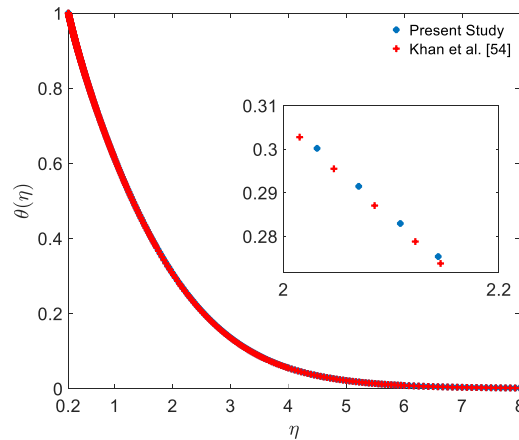


Fig. 7. Comparison of the present results with Khan et al. [46] with $We = \beta = M = Nb = 0, Nt \rightarrow 0, Re = 1$.

(Fig. 2 (c)). Fig. 2(d) reveals the impression of $Re f'(\eta)$. Rise in Reynolds augmented $f'(\eta)$ and the ascending VBL. Obviously, greater Re indicates less viscous force resulting into greater $f'(\eta)$ as seen in Faisal [51]. Similar nature of $f'(\eta)$ is the outcome of different We ($We = 0.5, 1, 1.5, 2$). As a consequence, VBL expands (Fig. 2(e)). Fig. 2(f) highlights that the rise in β influences $f'(\eta)$ in the sense that $f'(\eta)$ peters out for increment in β . In fact, increasing permeability parameter provides greater shear stress and hence high resistance due to porous medium opposes the flow.

Fig. 3(a) conveys the variation of temperature field $\theta(\eta)$ in respect to varied λ ($\lambda = 0, 0.4, 0.8, 1$). The value $\lambda = 0$ indicates the moving needle in a stationary cross nf. In such environment (Sakiadis flow of cross nfs), $\theta(\eta)$ deepens but very slowly with respect to distance from the needle surface. Further, $\lambda = 1$ indicates the situation where the needle is stationary but the cross nf moves. In such environment (Blasius flow of cross nfs) $\theta(\eta)$ decays sharply. In a comparative study of above two cases we may ascertain that the related TBLT is prominent for Sakiadis flow of cross nfs compared to Blasius flow of cross nfs. For intermediate values of λ ($\lambda = 0.4, 0.8$), $\theta(\eta)|_{\text{Sakiadis flows of cross nfs}} > \theta(\eta)|_{\text{Blasius flows of cross nfs}}$. This is valid for $\eta \geq 2$. The well information regarding the variation of $\theta(\eta)$ for varied estimations of M ($M = 0.5, 1, 1.5, 2$) is shown in Fig. 3(b). Incremented M uplifted $\theta(\eta)$ and the related BLT. This is obvious from Alsaadi et al. [52]. Rise in N_R contributes to diminishing $\theta(\eta)$ and the related squeezed TBLT (Fig. 3(c)). This is in consent with Shaw et al. [53]. Increase Nt boosts up $\theta(\eta)$. Consequently, the related TBLT gets enlarged (Fig. 3(d)). For larger Nt indicating greater thermophoretic force drags the hot fluid particles from hot surface to the cold region and contribute to the augmentation of fluid temperature. This is clear from Alsaadi et al. [53]. Simply, rise in Pr led to the diminution of $\theta(\eta)$ and the squeezed TBL (Fig. 3(e)). Higher Prandtl fluids having less thermal conductivities cause lower diffusion of heat through it yield low fluid temperature. This has been acknowledged from Hina et al. [50]. Rise in θ_f escalates $\theta(\eta)$ and grows the related TBL in the flow of cross nfs in the BLR (Fig. 3(f)). The basic cause behind such temperature escalation is that surface temperature is more than that of free stream. This is noticed from Shaw et al. [53]. Incremented We yielded diminished $\theta(\eta)$ and shrinkage of TBL. But the variation of $\theta(\eta)$ is marginal (Fig. 3(g)). Rise in β leads to the growth of $\theta(\eta)$ and the related BLT (See Fig. 3(h)). This denotes that the existence of porous medium increases resistance to fluid movement and hence caused more heat generation leading to enhancement of thermal resistance (fluid temperature). Fig. 3(i) indicates that $\theta(\eta)$ and the related BLT upsurge due to rise in A^* . The enhancement of $\theta(\eta)$ is prominent in the fluid zone near the solid boundary than that of the ambient. Similar nature of $\theta(\eta)$ profiles is obtained for hike in B^* [See Fig. 3(j)]. The profiles in the form of overshoots appear near the solid boundary in both situations. The cause behind it is that more heat is propagated into the region near the solid boundary than that into the ambient fluid. In addition, Fig. 3(k) explains that rise in r_E ($r_E = 0, 0.1, 0.2, 0.3$) whittles down $\theta(\eta)$ and the related layer thickness. In the absence of CCHF ($r_E = 0$), $\theta(\eta)$ attained maximum value, however, with the imposition of CCHF, $\theta(\eta)$ declines prominently.

Fig. 4(a) illustrates that rising K_1 declines the nanoparticles concentration $\varphi(\eta)$ and the related BLT. This agrees well with Faisal et al. [51]. Incremented λ produces diminishing $\varphi(\eta)$ and showing a wide range of concentration distribution near the needle surface (Fig. 4(b)). Increase in M (indicating more magnetic field strength) upgrades $\varphi(\eta)$ in the fluid zone contiguous to the needle surface ($\eta \leq 2$). This is because more M causes to decline $f'(\eta)$. Consequently, nanoparticles stay more time in the range $\eta \leq 2$. This leads to more $\varphi(\eta)$ in that zone (Fig. 4(c)). After $\eta > 2$, the present situation reverses. This is due to active chemical reactions (HHRs) in the flow domain. Fig. 4(d) describes the $\varphi(\eta)$ behavior for varied estimations of Nb ($Nb = 0.1, 0.2, 0.3, 0.4$). As Nb enhances $\varphi(\eta)$ belittles. Consequently, the CBL squeezes. Larger Nb upgrades the rate of random diffusion of nanoparticles with different velocities due to the large number of collisions among fluid particles. Consequently, the concentration field decreases. It is well evident from Alsaadi et al. [52]. It seems clear that for lower Nb ($Nb = 0.1, 0.2$), the rate at which $\varphi(\eta)$ decays is prominent while that is less for higher Nb ($Nb = 0.3, 0.4$). Multiple overshoots are attained near the needle surface in respect of different estimations of Nb . At certain Nb ($Nb = 0.1$), as η increases, $\varphi(\eta)$ elevates and attains maximum value at $\eta = 2$ and after such transition $\varphi(\eta)$ decays sharply with increase in η . Peaks of different concentration curves represent $[\varphi(\eta)]_{\max}$ in respect to different Nb . Area under each curve is the measure of the total particles concentration for the particular

value of Nb . We have,

$$\varphi_{\max}(\eta)|_{Nb=0.1} > \varphi_{\max}(\eta)|_{Nb=0.2} > \varphi_{\max}(\eta)|_{Nb=0.3} > \varphi_{\max}(\eta)|_{Nb=0.4}$$

Fig. 4(e) emphasizes the behavior of $\varphi(\eta)$ for different Nt ($Nt = 0.1, 0.2, 0.3, 0.4$). Hiking Nt escalates $\varphi(\eta)$ and the related BL. As η increases towards the ambient, the peaks of the $\varphi(\eta)$ curves tilt towards right. These indicate that the values of η at which $[\varphi(\eta)]_{\max}$ occurs gradually enhance. Also, as Nt increases, accordingly the area under the corresponding curves increase indicating the augmentation of $[\varphi(\eta)]_{\max}$. For this,

$$\varphi_{\max}(\eta)|_{Nt=0.1} < \varphi_{\max}(\eta)|_{Nt=0.2} < \varphi_{\max}(\eta)|_{Nt=0.3} < \varphi_{\max}(\eta)|_{Nt=0.4}$$

Fig. 4(f) informs that hiked Re led to the uplift of $\varphi(\eta)$ and the related CBLT. Fig. 4(g) reveals the impression of Sc on $\varphi(\eta)$. Increase in Sc lessens $\varphi(\eta)$ and the related BLT. Greater Sc indicating less mass diffusion thereby yields less $\varphi(\eta)$. It is in consent with Alsaadi et al. [52]. Multiple overshoots (convex concentration curves) of gradually decreasing area corresponding to increased value of Sc appear near the needle surface. Peaks of the overshoots get lowered in respect of rising Sc . This provides mathematical implication as

$$\varphi_{\max}(\eta)|_{Sc=10} > \varphi_{\max}(\eta)|_{Sc=20} > \varphi_{\max}(\eta)|_{Sc=30} > \varphi_{\max}(\eta)|_{Sc=40}$$

Hiked θ_f ($\theta_f = 0.8, 1, 1.2, 1.5$) enlarges $\varphi(\eta)$ and the CBLT (Fig. 4(h)). Nevertheless θ_f , $\varphi(\eta)$ accumulation is prominent near the needle surface compared to the far apart region in the BLF of cross nfs. Increased We produced marginal decaying of $\varphi(\eta)$ (Fig. 4(i)).

Fig. 5(a) provides the information regarding the variation of skin friction (C_{fx}) for varied estimations of λ ($\lambda = 0.5, 0.8, 1, 1.2$) against M . As λ grows, C_{fx} declines which is obvious. One thing observed here is that lower λ ($\lambda = 0.5, 0.8$) has marginal effect on viscous drag while that of higher λ contributes prominent variation (decaying nature) of viscous drag of cross nfs. Further, the varied estimations of λ ($\lambda = 0.5, 0.8, 1, 1.2$) has higher influence on HTR from the needle surface (Fig. 5(b)). Hike in λ reduces the HTR appreciably under the action of external magnetic field. Fig. 5(c) portrays the impression of N_R on Nu_x against Re . Incremented N_R led to reduction of HTR from the needle surface at each value of Re in the entire BLR. Fig. 5(d) conveys that in the absence of B^* , HTR was higher, however, with rise of B^* HTR gets reduced at each fixed value of A^* . Fig. 5(e) demonstrates that rate of heat transportation gets intensified because of rise in Y_E at each fixed value of Pr ($30 \leq Pr \leq 40$).

The alteration of Be for diverse estimations of Ω_1 ($\Omega_1 = 0.1, 0.5, 0.8, 1$) against Br is understood from Fig. 6(a). Rise in Ω_1 upsurgers Be at each value of Br . At certain Ω_1 (for instance $\Omega_1 = 0.1$), rise in Br provides monotonically decrement of Be . Fig. 6(b) conveys the variation of Be for varied values of λ ($\lambda = 0.5, 0.8, 1, 1.2$) against M . Increase in λ led to the diminution of Be in the presence of magnetic field. However, the fall in Be is prominent for lower λ ($\lambda = 0.5, 0.8$) and it is less for higher λ ($\lambda = 1, 1.2$). The Be variation in respect of different N_R ($N_R = 0.1, 0.2, 0.3, 0.4$) against Re is visualized from Fig. 6(c). Hiking N_R demanded and resulted in the decline of Be appreciably. Fig. 6(d) reveals that Be whittles down for rise in B^* for low values of A^* , however, the reverse behavior is attained for higher values of A^* . Irrespective the values of B^* , Be has higher value for low values of A^* than that of higher values of A^* .

The variation of EG number (N_G) for varied values of Ω_1 ($\Omega_1 = 0.1, 0.5, 0.8, 1$) against Br is availed from Fig. 6(e). Increase in Ω_1 uplifts N_G . Also, at certain Ω_1 ($\Omega_1 = 0.1$, say), there is a linear variation of N_G and Br throughout the flow of cross nfs within the BLR. Fig. 6(f) implicates the behavior of N_G for varied λ in the existence of magnetic field. Interestingly, for $\lambda = 0.5$ (i.e., for needle velocity U_w equal to free stream velocity U_∞) the EG rate curve is linear and parallel to magnetic field axis. Irrespective of M , the EG rate (N_G) is constant throughout the flow of cross nfs. It is a better result for the entropy minimization rather augmenting N_G due to rising λ in presence of magnetic field of given strength. Fig. 6(g) provides N_G variation for different N_R ($N_R = 0.1, 0.2, 0.3, 0.4$) against Re . Increase in N_R led to the decaying of N_G irrespective the value of Re . At certain N_R ($N_R = 0.1$, say) N_G increases monotonically with rising Re . Fig. 6(h) represents N_G variation in respect of different Ω_2 ($\Omega_2 = 0.1, 0.5, 0.8, 1$) against We . Incremented Ω_2 yielded a decline of N_G whatever the value of We may be. At certain Ω_2 ($\Omega_2 = 0.1$, say), what actually happens is that N_G upsurgers monotonically due to rise in We . In this case, N_G is an increasing function of We and decreasing function of Ω_2 . Fig. 6(i) informs that N_G augments for rise in B^* . Further, it is observed that N_G attains high value for comparatively larger A^* irrespective the value of B^* . Rate of EG upsurgers due to amplified Y_E at fixed Pr ($30 \leq Pr \leq 50$) as is seen in Fig. 6(j).

6. Validation

We have validated the present work with the existing work of Khan et al. [46] as shown in Fig. 7. It shows a very good agreement with the present work and imparts a confidence of our results.

7. Concluding remarks

The realistic assessment of the present study imparts the outcomes as:

- > The value $\lambda = 0$ and $\lambda = 1$ contribute to Sakiadis and Blasius type of flows. Its intermediate values yield opposite fashioned flow pattern of cross nfs.
- > Incremented magnetic field strength generated controlled flow of cross nfs.
- > Improved n , Re and We led to the uplift of $f'(\eta)$ and the related BLT.
- > Hiked Nt and θ_f upgraded $\theta(\eta)$ and the wall temperature gradient while that of N_R belittles it.
- > Mounted A^* and B^* contributed to escalate $\theta(\eta)$ and expand TBLT.

- > Increased K_1 , λ , Nb , S resulted in declined $\varphi(\eta)$ and with squeezed CBL while that of N and Re upsurged $\varphi(\eta)$ and with expanded CBL.
- > Higher λ contributes to diminution of viscous drag in the flow of cross nfs and HTR from the needle surface.
- > N_G and rate of heat transportation gets intensified due to rise in γ_E at each fixed value of Pr .
- > Augmented Ω_1 led to the rise in Be and N_G .

Author contribution statement

S. Shaw: Methodology, Software, Writing - Original Draft. **M.K. Nayak:** Conceptualization, Methodology, Software, Writing - Original Draft. **A.S. Dogonchi:** Formal analysis, Methodology, Software, Validation. **Ali J. Chamkha:** Supervision. **Yasser Elmasry:** Writing - Review & Editing, Investigation. **Radi Alsulami:** Writing - Review & Editing, Formal analysis.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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