



Vector dissipative light bullets in optical laser beam

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Received: 24 October 2019 / Accepted: 17 March 2020 / Published online: 3 April 2020
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Abstract

The dynamics of light bullets propagating in nonlinear media with linear/nonlinear, gain/loss and coupling described by the (2+1)-dimensional vectorial cubic–quintic complex Ginzburg–Landau (CGL) equations is considered. The evolution and the stability of the vector dissipative optical light bullets, generated from an asymmetric input with respect to two transverse coordinates x and y , are studied. We use the variational method to find a set of differential equations characterizing the variation of the light bullet parameters in the laser cavity. This approach allows us to analyze the influence of various physical parameters on the dynamics of the propagating beam and its relevant parameters. Then, we solve the original coupled (2+1)D cubic–quintic CGL equation using the split-step Fourier method. Numerical results and analytical predictions are confronted, and a good agreement between the two approaches is obtained.

1 Introduction

Solitons, which are fascinating nonlinear wave phenomena, are self-trapped light beams or pulses supported by the balance between diffraction and/or dispersion (in the spatial and/or temporal domain) and nonlinearities of various types [1–6]. The generation, propagation and interaction of optical vortices in nonlinear media have attracted much attention and have been the subject of extensive studies in recent years. In the single-mode fibers (SMFs), in which the soliton formation has drawn a lot of attention, the light propagation is governed by the nonlinear Schrödinger equation (NLS)

equation [7] and, when considering the vectorial nature of light, its propagation in the SMF is governed by a set of coupled NLS equations [8], whose characteristics and dynamics are richer than those of the scalar NLS equation.

In laser systems, the formation and dynamics of transverse light patterns in nonlinear resonator remains a field of intense research [9–12]. Assuming that the direction of the electric field is constant and applying the slowly varying envelope approximation to the Maxwell–Bloch (MB) equations helped to establish the relationship between nonlinear optics and the cubic complex Ginzburg–Landau (CGL) equation [12, 13]. Considering the interaction of an electromagnetic field with matter in a laser cavity without the assumption of a fixed direction of the transverse electric field, Gil [9], using the standard perturbative nonlinear analysis, exclusively proposed that the MB equations could be reduced to a vectorial cubic CGL equation. Although this later formulation has been widely adopted, soliton solutions in this context are usually not stable, both for anomalous [14] and normal dispersions [15], except if higher-order nonlinear (quintic) terms are introduced, leading to the so-called cubic–quintic CGL equation. The competition between the cubic and quintic nonlinearities may open new possibilities with a broad range of behaviors, characterized by different kinds of patterns formation in one-(localized structures) [16–18] and two-spatial dimensions (topological defects) [16, 19–24] for the vectorial cubic CGL equation. Motivated by these, we recently adopted the same procedure as Gil and rather derived a vectorial cubic–quintic CGL equation from which

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we obtained that the modulational instability (MI) mechanism may lead to localized modes with interesting and new behaviors, strongly related to the change in the laser cavity parameters [25]. However, the MI technique becomes obsolete when it comes to describe the main characteristics of individual pulse evolution, namely its amplitude, temporal and spatial widths, position of the maximum, unequal wavefront curvatures, chirp parameters and phase shift. Depending on the field of application, a number of treatments have been adopted, namely the method of moments [26], method of collective coordinates [27–29], time-dependent variational method [30], effective-particle method [31], averaged Lagrangian description [32], just to name a few. Particularly, the variational method has been successfully applied to address a variety of nonlinear problems through the use of the Lagrange density and its relation to the Euler–Lagrange equations [33]. Besides, the fundamental advantage of the variational approach is that it provides an outline under the field dynamics during propagation [34, 35]. A potential well is generated into a stationary point due to the exact balance between repulsive and attractive potentials, justifying the formation of stable optical solitons. In the present work, to investigate the generation of the vector light bullets of the laser coupled cubic–quintic CGL equation, and their stability for several dynamical regimes, we make use of the variational method to obtain physical insight in terms of a few important parameters and then propose numerical simulations that confirm qualitatively our analytical predictions.

The rest of the paper is organized as follows. In Sect. 2, we present the propagation model of the optical light bullet which includes the cubic and quintic coupling terms, diffraction terms, cubic and quintic nonlinearity terms. In addition, with the help of the variational method involving trial function, the set of variational equations resulting from the Euler–Lagrange equations is obtained. In Sect. 3, the average equations for the effective potential function is derived along with the numerical treatment of the analytical results. Section 4 is devoted to some concluding remarks.

2 Model and analytical treatment using variational approach

In the good cavity limit, when both the induced polarization and population inversion are enslaved by a slowly varying vector electromagnetic field of a multi-transverse-mode laser (MTM), a general form of the equations describing vector soliton process is reduced to the coupled (2+1)D cubic–quintic CGL equation [25]

$$i \frac{d\psi_{\pm}}{dt} + \psi_{\pm} + \Delta \psi_{\pm} + |\psi_{\pm}|^2 \psi_{\pm} + \gamma_r |\psi_{\mp}| \psi_{\pm} + \nu |\psi_{\pm}|^4 \psi_{\pm} + \delta_r (|\psi_{\mp}|^2 + 2|\psi_{\pm}|^2) |\psi_{\mp}|^2 \psi_{\pm} = Q_{\pm}, \quad (1)$$

with

$$Q_{\pm} = i(\delta \psi_{\pm} + \beta \Delta \psi_{\pm} + \varepsilon |\psi_{\pm}|^2 \psi_{\pm} + \gamma_i |\psi_{\mp}| \psi_{\pm} + \mu |\psi_{\pm}|^4 \psi_{\pm} + \delta_i (|\psi_{\mp}|^2 + 2|\psi_{\pm}|^2) |\psi_{\mp}|^2 \psi_{\pm}). \quad (2)$$

The left-hand-side of Eq. (1) contains the conservative terms, where $\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$. The optical envelope $\psi_{\pm}(x, y, t)$ is the normalized complex function of three real variables x , y and t , where x and y are the normalized two transverse coordinates and t is the normalized propagation time. All the parameters of Eqs. (1) and (2), i.e., ν , δ , ε , γ_r , γ_i , μ , δ_r and δ_i , are real constants, and the pump profile parameter related to $\psi_{\pm}$ is normalized to unity [36]. The negative coefficient ν is the saturation of the Kerr nonlinearity, and the quintic coefficient $\delta_r < 0$ is the saturation of the cubic coupling absorption γ_r . The right term $Q_{\pm}$ of Eq. (1), given by Eq. (2), stands for the dissipation, where δ is the linear loss/gain coefficient, β is the diffusion coefficient, ε is the cubic gain, μ is the quintic loss, γ_i and δ_i , respectively, denote the cubic and the quintic coupling coefficients.

Due to its complexity, the coupled (2+1)D cubic–quintic CGL equation does not allow exact analytical solutions. Therefore, the use of an approximated analytical approach is needed, which includes for example the collective coordinate method [37], the moment method [38], the variational method [34, 35, 39, 40], and so on. Solutions are in general analyzed by considering trial Gaussian [34] functions with a few free parameters such as the amplitude, spatial pulse widths, the central position of the pulse maximum, the unequal wavefront curvatures, and the phase shift, which depend on the propagation time t . In this respect, the variational method for dissipative systems is used in the following, with the objective to obtain more physical insight in terms of a few relevant parameters and then present numerical simulations that confirm the analytical predictions. In so doing, the left-hand side of Eq. (1) is derived from the Lagrangian density given by

$$\mathcal{L} = \frac{i}{2} \left(\psi_{-}^{*} \frac{\partial \psi_{-}}{\partial t} - \psi_{-} \frac{\partial \psi_{-}^{*}}{\partial t} + \psi_{+}^{*} \frac{\partial \psi_{+}}{\partial t} - \psi_{+} \frac{\partial \psi_{+}^{*}}{\partial t} \right) + \left| \frac{\partial \psi_{\mp}}{\partial x} \right|^2 + \left| \frac{\partial \psi_{\mp}}{\partial y} \right|^2 + \frac{1}{2} |\psi_{\pm}|^4 + \frac{\nu}{3} |\psi_{\pm}|^6 + \gamma_r |\psi_{\mp}|^2 |\psi_{\pm}|^2 + \delta_r (|\psi_{\pm}|^4 |\psi_{\mp}|^2 + |\psi_{\pm}|^2 |\psi_{\mp}|^4), \quad (3)$$

where $\psi_{\pm}^{*}$ stands for the complex conjugate of $\psi_{\pm}$. To extend our study to the characterization of the evolution of

the single parameters of the dissipative light bullet during propagation in the medium, we adopt the following Gaussian trial function

$$\psi_{\pm} = A \exp \left\{ -\frac{(x \pm X_m)^2}{2X^2} - \frac{y^2}{2Y^2} + iC(x \pm X_m)^2 + iSy^2 + i\phi \right\}, \quad (4)$$

where A is the amplitude, X and Y are the unequal spatial widths, C and S are the unequal wavefront curvature, X_m is the central position, and ϕ is the phase of the light bullet, all of which are functions of the independent variable t .

To proceed to the analytical treatment of the model and characterize the solutions depending on the various dissipative parameters, the equations for the motions of the variational parameters η are obtained through the Euler–Lagrange equations [34, 35]

$$\begin{aligned} \frac{d}{dt} \left(\frac{\partial \mathbf{L}}{\partial \dot{\eta}} \right) - \frac{\partial \mathbf{L}}{\partial \eta} \\ = 2R_e \int \int dx dy \left(Q_- \frac{\partial \psi_-^*}{\partial \eta} + Q_+ \frac{\partial \psi_+^*}{\partial \eta} \right), \end{aligned} \quad (5)$$

where $\dot{\eta} = \frac{d\eta}{dt}$. Here, η represents the parameters A , X , Y , C , S , X_m and ϕ in the ansatz, while

$$\mathbf{L} = \int \int \mathcal{L} dx dy \quad (6)$$

is the average Lagrangian of the conservative system obtained by substituting Eqs. (4) into (3) and taking out integration over coordinates x and y . Thus, we make use of the Euler–Lagrange equation [40] and obtain the following set of differential equations describing the dynamics of the seven relevant parameters of the trial solution (4):

$$\begin{aligned} \frac{dA}{dt} = & 2A(S + C) - \delta A - \frac{3}{4}\epsilon A^3 - \frac{5}{9}\mu A^5 \\ & + \beta A \left(\frac{1}{X^2} + \frac{1}{Y^2} \right) \\ & + \gamma_i A^3 e^{-\frac{2X_m^2}{X^2}} \left(-1 + \frac{1}{4X^2} + \frac{1}{4Y^2} + \frac{X_m^2}{2X^2} \right) \\ & + A^5 \delta_i e^{-\frac{8X_m^2}{3X^2}} \left(-1 + \frac{1}{4X^2} + \frac{1}{4Y^2} + \frac{4X_m^2}{9X^2} \right), \end{aligned} \quad (7a)$$

$$\begin{aligned} \frac{dX}{dt} = & 4CX(\beta CX^2 - 1) - \frac{\beta}{X^2} + XA^2 \left(\frac{\epsilon}{4} + \frac{2\mu A^2}{9} \right) \\ & + \gamma_i A^2 e^{-\frac{2X_m^2}{X^2}} \left(-\frac{X_m^2}{X} - \frac{1}{X} + \frac{X}{2} \right) + \delta_i A^4 e^{-\frac{8X_m^2}{3X^2}}, \end{aligned} \quad (7b)$$

$$\begin{aligned} \frac{dY}{dt} = & 4SY(\beta SY^2 - 1) - \frac{\beta}{Y^2} + YA^2 \left(\frac{\epsilon}{4} + \frac{2\mu A^2}{9} \right) \\ & + \gamma_i A^2 e^{-\frac{2X_m^2}{X^2}} \left(-\frac{1}{Y} + \frac{Y}{2} \right) \\ & + \delta_i A^4 e^{-\frac{8X_m^2}{3X^2}} \left(-\frac{1}{2Y} + \frac{Y}{2} \right), \end{aligned} \quad (7c)$$

$$\begin{aligned} \frac{dC}{dt} = & 4C^2 + \frac{1}{X^2} \left(4\beta C - \frac{A^2}{4} - \frac{2\nu A^4}{9} - \frac{1}{X^2} \right) \\ & + \frac{\gamma_i A^2}{4X^2} e^{-\frac{2X_m^2}{X^2}} \left(\frac{4X_m^2}{X^2} - 1 \right) \\ & + \frac{2\delta_i A^4}{3X^2} e^{-\frac{8X_m^2}{3X^2}} \left(\frac{8X_m^2}{3X^2} - 1 \right), \end{aligned} \quad (7d)$$

$$\begin{aligned} \frac{dS}{dt} = & 4S^2 + \frac{1}{Y^2} \left(4\beta S - \frac{A^2}{4} - \frac{2\nu A^4}{9} - \frac{1}{Y^2} \right) \\ & - \frac{\gamma_i A^2}{4Y^2} e^{-\frac{2X_m^2}{X^2}} - \frac{2\delta_i A^4}{3Y^2} e^{-\frac{8X_m^2}{3X^2}}, \end{aligned} \quad (7e)$$

$$\frac{dX_m}{dt} = A^2 X_m \left(\gamma_i e^{-\frac{2X_m^2}{X^2}} + \frac{8\delta_i A^2}{9} e^{-\frac{8X_m^2}{3X^2}} \right), \quad (7f)$$

$$\begin{aligned} \frac{d\phi}{dt} = & -1 - 2\beta(C + S) + \frac{1}{X^2} + \frac{1}{Y^2} + A^2 \left(\frac{3}{4} + \frac{5\nu A^2}{9} \right) \\ & + \frac{\gamma_i A^2}{2} e^{-\frac{2X_m^2}{X^2}} \left(-\frac{X_m^2}{X^2} + \frac{3}{2} \right) \\ & + \frac{\delta_i A^4}{3} \left(-\frac{8X_m^2}{3X^4} + 5 \right) e^{-\frac{8X_m^2}{3X^2}}. \end{aligned} \quad (7g)$$

Eqs. (7a–7g) represent the coupled equations that show how pulse parameters change during propagation. Looking for steady state solutions of the system of Eqs. (7a–7g) after vanishing derivatives of amplitude, width, curvature, and central position, we focus our study on the symmetric steady state solutions, with $X_m = 0$, equal widths ($X = Y$) and spatial chirps ($C = S$). For convenience, all dissipative parameters are considered as small quantities $\theta = \max(\beta, \delta, \epsilon, \mu)$. The corresponding steady state amplitude has two discrete values A^+ and A^- given by

$$A^{\pm} = \sqrt{\frac{9\beta(-4\beta(1+\beta\epsilon+\gamma_r)\pm\sqrt{\Delta})}{2(\beta\mu+4\nu+4\delta_r)}}, \quad (8)$$

with $\Delta = \frac{(1+\beta\epsilon+\gamma_r)^2}{16\beta^2} - \frac{4\delta(\beta\mu+4\nu+4\delta_r)}{9\beta}$. We also obtain the width,

$$X = \left(\left(\left(\frac{\mu}{3} + \frac{\delta_i}{2} \right) A^4 + \frac{1}{2} (\epsilon + \gamma_i) A^2 + \delta \right) / \beta \right)^{-1/2} + 0(\theta^2), \quad (9)$$

and the chirp

$$C = \frac{C_4 A^4 + C_2 A^2 + \delta}{4\beta^2}, \quad (10)$$

with $C_4 = \frac{1}{3}(2\mu + 3\delta_i + 4\beta(\delta_r + \frac{\nu}{3}))$, and $C_2 = \frac{1}{4}(2(\epsilon + \gamma_i) + \beta(1 + \gamma_r))$.

3 Stability analysis and numerical results

To study the dynamical stability of the vector light bullet solution given above, we first calculate the effective potential. An important criterion for the stability of our stationary solution is to investigate the possibility of bullets to be trapped in the well. We examine the dynamical behaviors of a light bullet using the initial conditions given by Eqs. (8–10), from its symmetric equilibrium. Integration of variational Eqs. (7a–7g) gives [7, 35, 39, 41]

$$\frac{1}{4} \left(\frac{dX}{dt} \right)^2 + \frac{1}{4} \left(\frac{dY}{dt} \right)^2 + U(X, Y) = U(X_0, Y_0), \quad (11)$$

where $U(X, Y)$ is the effective potential given in "Appendix". Equation. (11) describes the dynamics of the widths and shows a competition among diffraction and nonlinearity, and can be adopted to study the stability of the optical light bullet through the medium. The existing stationary solution describes the motion of a particle located at the bottom of the potential well. Given the effective potential $U(X, Y)$, we have the equations

$$\frac{d^2 X}{dt^2} = -2 \frac{\partial U(X, Y)}{\partial X}, \quad (12a)$$

$$\frac{d^2 Y}{dt^2} = -2 \frac{\partial U(X, Y)}{\partial Y}, \quad (12b)$$

which are equivalent to those describing the dynamics of a particle in two-dimensional potential well. When the nonlinearity exactly balances both the diffraction, the curve of the potential shows an extremum, which is an equilibrium solution. In such a direction, the obtained effective potential U , in the plane of nonlinear dissipative parameters, is depicted in Fig. 1, where panel (a) describes the effective potential versus (ϵ, δ_i) , while panels (b) and (c) show U versus (ϵ, μ) , confirming that the effective potential has a global minimum. This reveals the existence of a stable stationary light bullet. This confirms as well the existence of the stationary soliton solution corresponding to the optical dissipative soliton located at the bottom of the effective potential well. From Fig. 1, we observe that the shape of the effective potential depend on both the dissipative and soliton parameters. However, from Fig. 1a–c, the areas corresponding to the bottom of the effective potential well (minima) are the domains of parameters for which the soliton can be trapped, leading to a stable propagation. These confirm the analytical predictions from Eqs. (7a–7g). Figure 1 presents two extrema, minima and maxima. Therefore, depending on the dissipative parameters and the central position, the beam will be either trapped or diffracted. The beam with initial condition corresponding to the point lying on the potential curve below the maximum will always be trapped, therefore, generating a light bullet [7]. Using the variational method, it can be predicted that all beams initially around the minimum of the potential well will form light bullets pulsating around

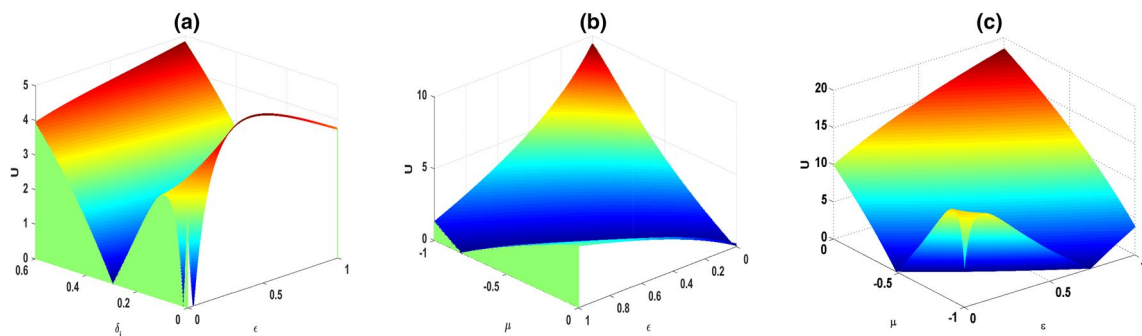


Fig. 1 Effective potential U versus (ϵ, δ_i) in panel **a**, and versus (ϵ, μ) in panels **b**, **c** for the parameters $\delta = -0.01059$, $\beta = 0.49$, $\gamma_i = 0.25118$, $\nu = -1$, and: **a** $\gamma_r = 1.1087$, $\delta_r = -0.5074$, $\mu = -9$,

$X_m = 1.3$, **b** $\gamma_r = 0.9$, $\delta_r = -0.5074$, $\delta_i = 0.04289$, $X_m = 1.3$, **c** $\gamma_r = 0.9$, $\delta_r = -0.35074$, $\delta_i = 0.04289$, $X_m = 0.05$

its equilibrium condition, with initial condition focusing until the turning point at the bottom region. The analysis of the potential well reveals the existence of a stable equilibrium which plays the role of an attractor [35, 39].

To confirm the analytical predictions, the coupled (2+1) D cubic–quintic CGL Eqs. (1) have been numerically solved by means of split-step Fourier method (SSFM) with a time-step $\Delta t = 10^{-3}$, on a mesh of size 100×100 , with space-steps $\Delta x = \Delta y = 0.01$. The used initial conditions are Gaussian trial function given by Eq. (4), with amplitude $A = A^-$ (see Eq. (8)), the unequal spatial widths X given by Eq. (9) and $Y = X/3$, the unequal wavefront curvature C given by Eq. (10) and $S = C$. For SSFM, the propagation of the wave solution from t to $t + \Delta t$ is carried out in two steps. In fact, the main idea behind this method is to obtain an approximate solution by assuming that for the wave solution over a small time Δt , the dispersive and nonlinear effects act independently. In the first step, the nonlinearity acts alone, and in the second step dispersion acts alone [39, 42]. In this context, Eqs. (1) are written as

$$\frac{\partial \psi_+}{\partial t} = (\hat{D} + \hat{\mathcal{N}}_+) \psi_+, \quad (13a)$$

$$\frac{\partial \psi_-}{\partial t} = (\hat{D} + \hat{\mathcal{N}}_-) \psi_-, \quad (13b)$$

where $\hat{D}$ is the operator for dispersion and absorption in the linear medium,

$$\hat{D} = (\delta + i) + (\beta + i) \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right), \quad (14)$$

while $\hat{\mathcal{N}}_+$ and $\hat{\mathcal{N}}_-$ are the nonlinear operators for pulse propagation defined by

$$\begin{aligned} \hat{\mathcal{N}}_+ = & (\epsilon + i) |\psi_+|^2 + (\gamma_i + i\gamma_r) |\psi_-|^2 + (\mu + i\nu) |\psi_+|^4 \\ & + (\delta_i + i\delta_r) \left(|\psi_-|^2 + 2|\psi_+|^2 \right) |\psi_-|^2, \end{aligned} \quad (15a)$$

$$\begin{aligned} \hat{\mathcal{N}}_- = & (\epsilon + i) |\psi_-|^2 + (\gamma_i + i\gamma_r) |\psi_+|^2 + (\mu + i\nu) |\psi_-|^4 \\ & + (\delta_i + i\delta_r) \left(2|\psi_-|^2 + |\psi_+|^2 \right) |\psi_+|^2. \end{aligned} \quad (15b)$$

Then, to improve the accuracy of the standard SSFM over a small interval of time, i.e., $[t, t + \Delta t]$, we adopt the following procedure to propagate the wave:

$$\begin{aligned} \psi_+(x, y, t + \Delta t) \approx & \exp \left(\Delta t \int_t^{t+\Delta t} \hat{D}(x, y, t') dt' \right) \\ & \times \exp \left(\Delta t \hat{\mathcal{N}}_+ \right) \psi_+(x, y, t), \end{aligned} \quad (16)$$

and similarly for $\psi_-(x, y, t + \Delta t)$.

The corresponding results are summarized in Figs. 2, 3, 4, 5. For a good choice of dissipative parameters in the vicinity of the effective potential of Figs. 1, 2a, d show the spatial transverse profiles of the initial asymmetric dissipative light bullets with corresponding values of parameters taken around the bottom of the effective potential of Fig. 1c, with $\delta_r = -0.35074$, $\epsilon = 0.456$, and $\mu = -0.98047$. During the evolution, we note a regular change from an asymmetrical to a symmetrical light bullet for each solution ψ_+ and ψ_- (see Fig. 2c, f). We also observe a small shift of the central position, accompanied by a reduction of the intensity of the solution during a short time of propagation. Thereafter, a stable propagation for longer time is observed. Fig. 2b, e have been recorded at $t = 100$, and Fig. 2c, f correspond to $t = 50000$. The results of Fig. 2 are in very good agreement with analytical results, concerning the soliton trapped at the bottom of effective potential.

To evaluate the influence of dissipative parameters on the stable dynamics of our predicted soliton solutions, the results of Fig. 3a, b have been obtained for $\gamma_r = 0.9$, $\delta_r = -0.3074$, $\epsilon = 0.456$, $\mu = -0.98047$, $X_m = 1.5$ (values of other parameters are the same as in Fig. 2). Similarly to Fig. 2, we observe a small shift of the central position, a decreasing of the widths and also a small reduction of the intensity during a short-time of propagation, which shows that stable dynamics for longer time of propagation. To better appreciate the influence of the coupled terms on the light bullet stability, Fig. 3c, d have been obtained for $\gamma_r = 1.109$, (values of other parameters are the same as in Fig. 3a, b). A regime of stable evolution of solutions ψ_+ (see Fig. 3c) and ψ_- (see Fig. 3d) is obtained in comparison to Fig. 3a, b for this value of the coupling coefficient.

Second, we evaluated the influence of the quintic-coupling term, and the results are summarized in Fig. 4. Figure 4a, b display the evolution of the cross-section of the dissipative light bullets obtained from direct numerical simulations of the full coupled (2+1)D cubic–quintic CGL equation for $\delta_r = -0.15074$, $\epsilon = 0.49$, $\mu = -0.9$, $X_m = 10$, while the other parameters remain the same as in Fig. 3c, d. Figure 4a, b show the generation of the second couple of

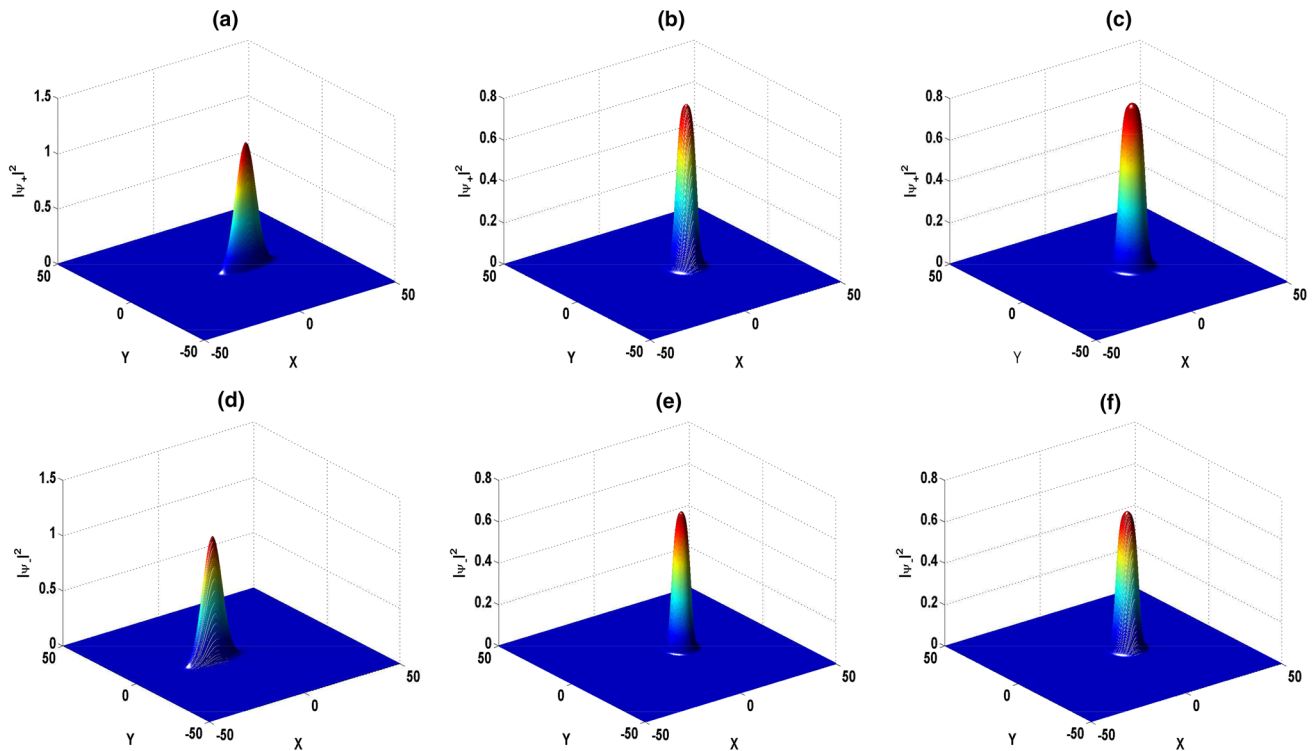


Fig. 2 Spatial profile of $\psi_+(x, y)$ (panels **a** to **c**) and $\psi_-(x, y)$ (panels **d** to **f**). **a**, **d** Correspond to $t = 0$, **b**, **e** correspond to $t = 100$, while **c**, **f** have been recorded at time $t = 50000$, with the param-

eters: $\delta = -0.01059$, $\beta = 0.45$, $\varepsilon = 0.456$, $\gamma_r = 0.9$, $\gamma_i = 0.25118$, $\delta_i = 0.04289$, $\delta_r = -0.35074$, $\nu = -1$, $\mu = -0.98047$, and $X_m = 10$

light bullets with stable evolution. To have a stable evolution of the coupled dissipative light bullets, we need the balance between gain/loss, diffraction/nonlinearity and, as shown in our analysis, the balance between the coupled effects [35].

In Fig. 5, we have summarized the stable evolution the dissipative light bullets. The obtained (2+1)D stable coupled spatial dissipative light bullets are among new interesting dynamical aspects of laser optics. We have realized through

numerical simulations that the intensity of the initially generated asymmetric light bullets (see Fig. 5a, b) decreases in the beginning (see Fig. 5c, d), which is followed thereafter by the reconstitution of the coupled light bullets when the coupling changes (see Fig. 5e, f). This gets more pronounced when time increases and remains very sensitive to the coupling parameters, leading to a stable evolution of the light bullets over longtime propagation (see Fig. 5g, h).

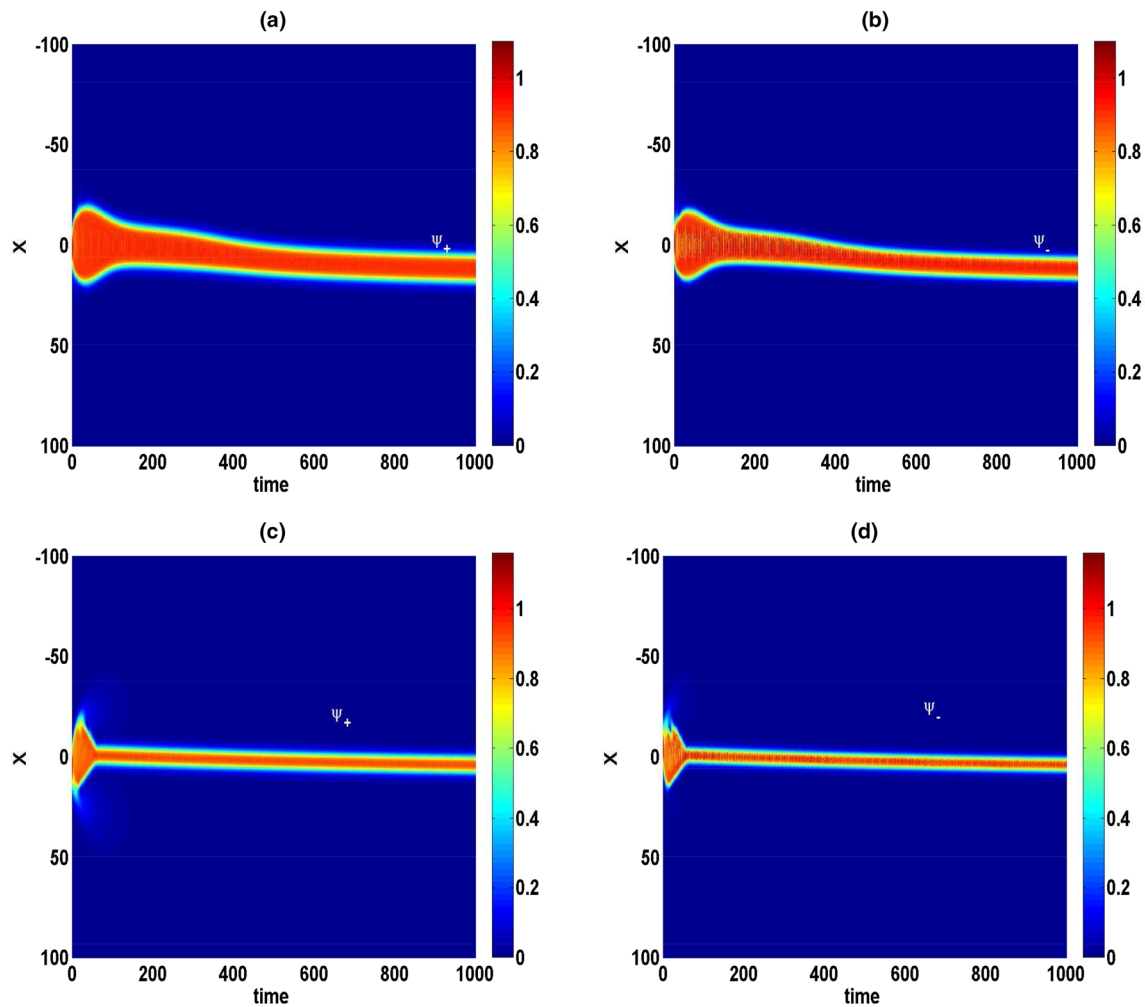


Fig. 3 The density profiles of the cross section of the space-time behavior of ψ_+ (panels **a** and **c**) and ψ_- (panels **b** and **d**). **a, b** Correspond to $X_m = 0.5$ and $\gamma_r = 0.9$, while **c, d** give the results related to $X_m = 5$ and $\gamma_r = 1.109$. The rest of parameters remains the same as in Fig. 2

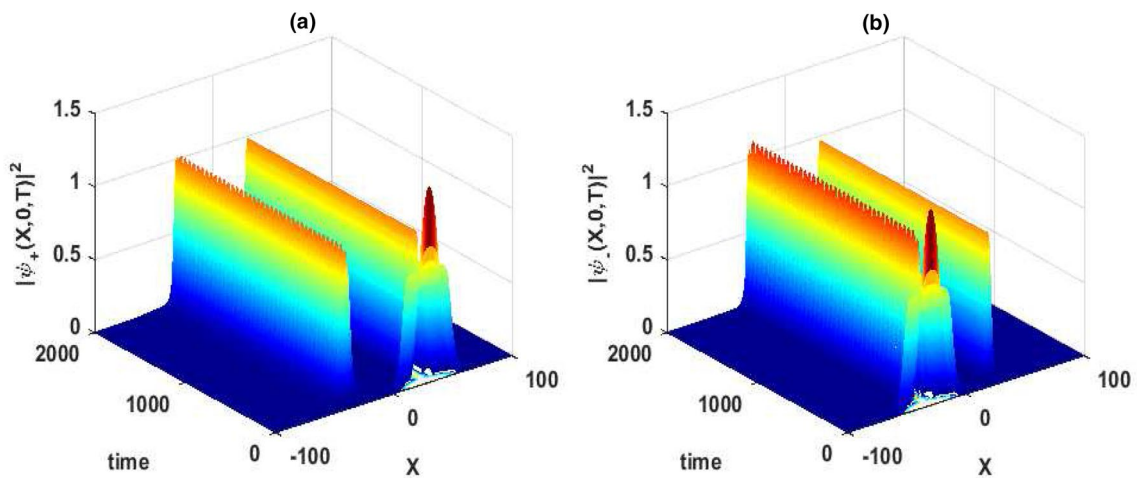


Fig. 4 Cross sections of the wave space-time dynamics, which corroborates the stability of the dissipative light bullets **a** $\psi_+(x, 0, t)$ and **b** $\psi_-(x, 0, t)$ obtained for the $\gamma_r = 1.109$ and $\delta_r = -0.15074$, with the other parameters keeping the similar values as in Fig. 2

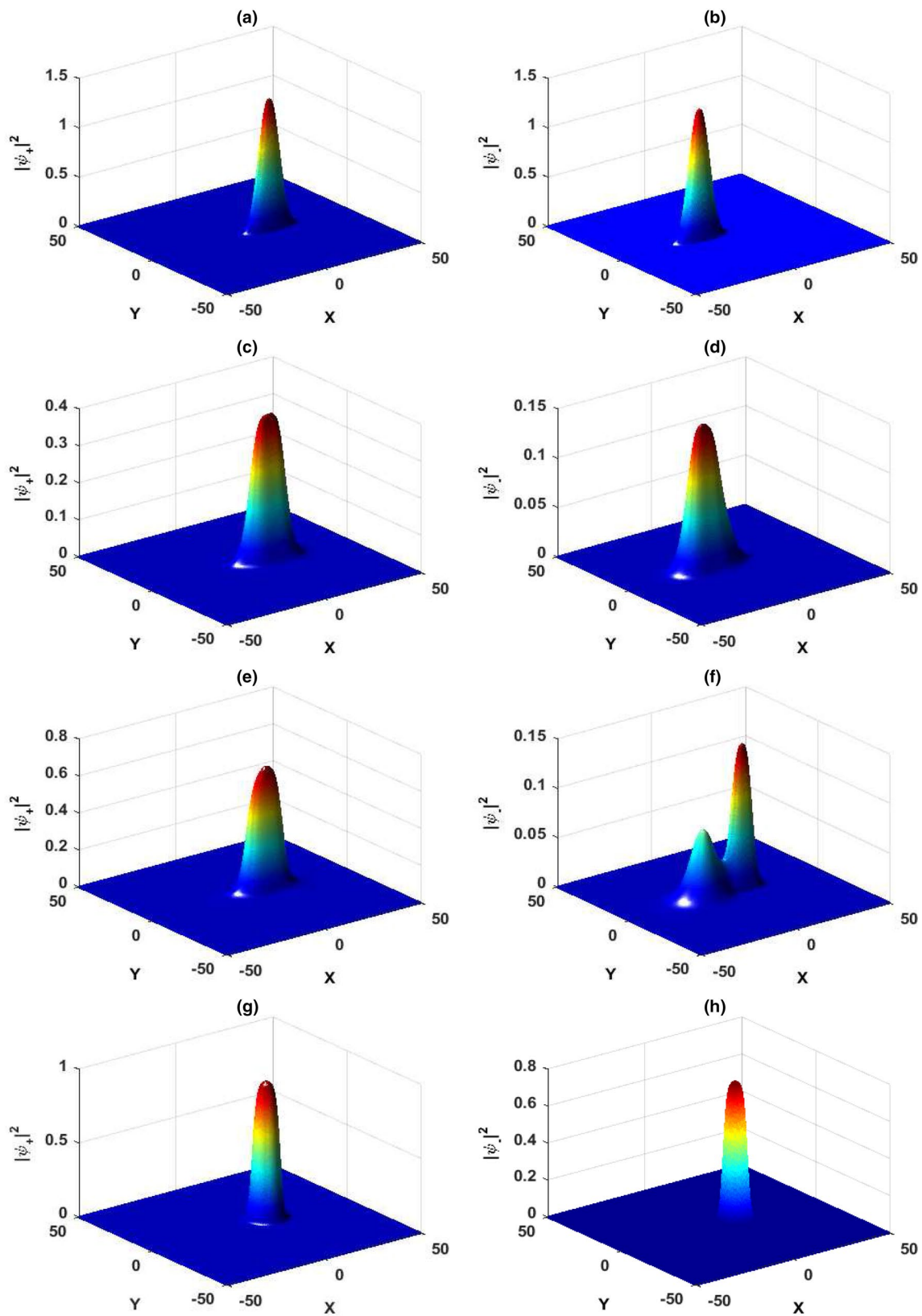


Fig. 5 (2+1)D stable spatial profiles of the light bullet intensities $|\psi_+(x,y)|^2$ (left column) and $|\psi_-(x,y)|^2$ (right column) obtained from direct simulation of Eq. (1) at times $t = 0$ [a, b], $t = 10$ [c, d], $t = 25$

[e, f], and $t = 50000$ [g, h], with the set of parameters: $\delta = -0.01059$, $\beta = 0.45$, $\varepsilon = 0.49$, $\gamma_r = 0.9$, $\gamma_i = 0.25118$, $\delta_i = 0.04289$, $\delta_r = -0.25074$, $\nu = -1$, $\mu = -0.9$, and $X_m = 10$

4 Conclusion

We have reported a comprehensive analysis on the formation of vector light bullets in an optical nonlinear medium described by a set of (2+1)D cubic–quintic CGL equations. Through the variational method, we have performed the theoretical analysis using a Gaussian trial function as solution, and we have obtained seven coupled first-order differential equations whose solutions have been discussed. Such analytical results have been confronted to direct numerical simulations via the the SSFM, where we have successfully shown that the system is able to interchange energy to keep both light bullets bounded, this under the balance between gain/losses, dispersion/diffraction and nonlinearities. Under such conditions, we have noticed the emergence of asymmetric dissipative light bullets and their disintegration on one hand, and a stable evolution of the coupled dissipative light bullets on the other hand. Moreover, the formation of (2+1)D stable coupled spatial dissipative light bullets has been detected. They have been found to be very sensitive to coupling parameters, whose impact on the long-time dynamics was pronounced. However, this implies new interesting dynamical aspects of laser optics, since the involved coupling mechanism can be adopted as a tool to generate appropriate stabilized vector light bullets which could be otherwise difficult to obtain. Interestingly, no collapse phenomena are observed in simulations when input focussing, chirping, as well as noise both in x and y , are introduced, leading to robust generation of new vector light bullets.

Acknowledgements A. Djazet would like to thank the CETIC (University of Yaoundé I, Cameroon) for their helpful support. The work by CBT is supported by the Botswana International University of Science and Technology under the Grant DVC/RDI/2/1/16I (25). I thank the Kavli Institute for Theoretical Physics (KITP), University of California Santa Barbara (USA), where this work was supported in part by the National Science Foundation Grant no. NSF PHY-1748958

Compliance with ethical standards

Conflict of interest The authors declare that they have no conflict of interest.

Appendix

The effective potential obtained for the stability analysis is given by

$$\begin{aligned}
 U = & \left(\left(\left(5 - \frac{3}{2Y^2} \right) X^2 + \frac{55}{336} + \frac{9}{224X_m^2} + \frac{43}{252} X_m^2 \right. \right. \\
 & \left. \left. - \frac{\frac{4X_m^4}{9} + \frac{17X_m^2}{36} + \frac{1}{8}}{X^2} \right) \delta_i \gamma_i A^6 - \frac{1}{2} A^2 \ln(X) \right) e^{-\frac{14X_m^2}{3X^2}} \\
 & + \left(\frac{1}{8} \left(\left(\frac{7}{2} - \frac{1}{Y^2} \right) X^2 + \frac{3}{16X_m^2} + \frac{7}{9} \right) \right. \\
 & \left. + \frac{8X_m^2}{81} - \frac{\frac{1}{16} + \frac{16}{81} X_m^4 + \frac{2X_m^2}{9}}{X^2} \right) \delta_i^2 A^8 e^{-\frac{16X_m^2}{X^2}} \\
 & + \left(\left(\left(4 - \frac{1}{2Y^2} \right) X^2 + \frac{8X_m^2}{9} + \frac{1}{2} \right) \frac{\mu \delta_i A^8}{9} \right. \\
 & + \left(\left(7 - \frac{1}{2Y^2} \right) \frac{X^2}{8} + \frac{X_m^2}{9} + \frac{1}{16} \right) \epsilon \delta_i A^6 \\
 & + \left(\left(\frac{\delta}{2} - S + \left(C^2 - \frac{1}{2Y^2} + \frac{16C^2 X_m^2}{9} \right. \right. \right. \\
 & \left. \left. \left. + \frac{4}{3} C \right) \beta \right) \delta_i X^2 + \delta_i \left(\frac{2}{3} - \frac{16CX_m^2}{9} - C \right) \right. \\
 & \left. + \beta \delta_i \left(\frac{7}{12} + \frac{3}{16X_m^2} - C^2 X^4 - \frac{\frac{1}{4} + \frac{4X_m^2}{9}}{X^2} \right) \right) A^4 \right) e^{-\frac{8X_m^2}{3X^2}} \\
 & + \left(\left(\left(\left(3 - \frac{1}{Y^2} \right) X^2 + 1 + X_m^2 + \frac{1}{4X_m^2} \right) \right. \right. \\
 & \left. \left. - \frac{\frac{1}{4} + X_m^2 + X_m^4}{2X^2} \right) \gamma_i^2 A^4 + \frac{\beta \gamma_i A^2}{2} \right) e^{-\frac{4X_m^2}{X^2}} \\
 & + \left(\left(\left(\frac{11}{2} - \frac{1}{Y^2} \right) \frac{X^2}{18} + \frac{X_m^2}{9} + \frac{1}{18} \right) \mu \gamma_i A^6 \right. \\
 & + \left(\left(1 - \frac{1}{32Y^2} \right) \frac{X^2}{4} + \frac{X_m^2}{8} + \frac{1}{16} \right) \epsilon \gamma_i A^4 \\
 & + \left(\left(-C^2 X^4 + \frac{1}{8X_m^2} - \frac{1 + 2X_m^2}{4X^2} \right) \beta \gamma_i \right. \\
 & \left. - \gamma_i C(1 + 2X_m^2) \right. \\
 & + \frac{\gamma_i}{2} + \left(\gamma_i \frac{\delta - 2S - 2C}{4} + \beta \gamma_i \left(\frac{C}{2} + 2C^2 X_m^2 \right. \right. \\
 & \left. \left. + C^2 + \frac{1}{4Y^2} \right) \right) A^2 + \frac{1}{X^2} \right) e^{-\frac{2X_m^2}{X^2}} \\
 & + (X^2 + Y^2) \left(\frac{\mu^2 A^8}{9} + \frac{5\epsilon \mu A^6}{24} \right) \\
 & + \left(-\frac{4}{9} \{ (\nu + \beta \mu)(\ln(X) + \ln(Y)) + \beta \mu (C^2 X^4 + S^2 Y^4) \} \right. \\
 & + \frac{5\epsilon^2}{64} (X^2 + Y^2) + \left(4\beta C \nu + 2\mu \delta - 4\mu S - \frac{2\beta \mu}{Y^2} \right) \left(\frac{X^2}{9} \right) \\
 & + \left(4\beta S \nu + 2\mu \delta - 4\mu C - \frac{2\beta \mu}{X^2} \right) \left(\frac{Y^2}{9} \right) \Big) A^4 \\
 & + \left(-\frac{\beta \epsilon}{2} (C^2 X^4 + S^2 Y^4) - (\ln(X) + \ln(Y)) \left(\frac{2 + \beta \epsilon}{4} \right) \right. \\
 & + \left(-\frac{\beta \epsilon}{Y^2} + 2\epsilon C + 4\delta \epsilon - 2\epsilon S \right) \frac{X^2}{8} \\
 & + (4\beta S + 2\epsilon S - 2\epsilon C + \delta \epsilon \\
 & \left. - \frac{\beta \epsilon}{X^2} \right) Y^2 \Big) A^2 \\
 & - 4\beta^2 (C^4 X^6 + S^4 Y^6) + 4\beta (S^3 Y^4 + C^3 X^4) \\
 & - 6\beta^2 (C^2 X^2 + S^2 Y^2) + 12\beta (S \ln(Y) \\
 & + C \ln(X)) - \frac{\beta^2}{4} \left(\frac{1}{X^2} + \frac{1}{Y^2} \right)
 \end{aligned}$$

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