



Faculty of Sciences
Department of Mathematics & Statistical Sciences

**OPTIMIZED PARAMETER ESTIMATION AND
INTEGRATED NEURAL NETWORK
FORECASTING OF DYNAMIC
PLANT-LIVESTOCK MODELS FOR
AGRO-ENVIRONMENTAL CONTROL SYSTEMS**

by

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October 19, 2023

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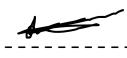
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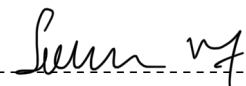
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Abstract

This body of work makes use of the dynamics of Plant-Herbivore interaction based on Lotka-Volterra's prey-predator model to study the relationship between traditional livestock production and vegetative conditions. This research focuses on developing advanced techniques to improve the accuracy and efficiency of parameter estimation in these models, as well as integrating neural networks for enhanced forecasting capabilities. An extension of the Plant-Herbivore models is derived in order to incorporate the existing information of Botswana's climate and livestock factors. Adequate approaches are taken to investigate novel optimization algorithms for parameter estimation, evaluate their effectiveness in dynamic plant-livestock models, and develop an integrated forecasting framework using neural networks. The utilization of the specialized Runge-Kutta method as an integrator is justified because of the inclusion of time series data for soil moisture in the model. The Runge-Kutta method paired with multi-start and local solver `fmincon` in MATLAB enabled efficient exploration of parameter spaces, to improve the accuracy of parameter estimation, with objective functions achieving set thresholds. The combination of these optimized parameter estimation techniques with the normalized nonlinear least squares, presented very robust results. Estimate weighting and time conversion are incorporated to reduce the error brought about by the assumption of homogeneity within the data. Systematic approaches were taken to estimate the extended Plant-Herbivore models parameters accurately and provide reliable predictions for system behavior despite the low observation points. The incorporation of nonlinear least squares is explored to further enhance the optimization process, allowing for the identification of parameters that best fit the observed livestock data, even in the presence of non-linearity. Neural Network Auto-Regressive (NNAR) forecasting, with its ability to learn complex patterns and relationships from historical data of livestock counts, was found to be a powerful tool for predicting soil moisture values which were then used for livestock count prediction. Based on numerous accuracy measures, ME, RMSE, MAE, and MPE, it was found out that NNAR models generally perform better for smaller numbers of predictions than ETS models. NNAR forecasts were found to have narrower confidence intervals than Exponential Smoothing (ETS) models for soil moisture forecasts. NNAR models were used to obtain soil moisture, death and harvest rates forecasts, which were then integrated within the extended Plant-Herbivore model to estimate future trajectories of the model estimates and draw relevant interpretations.

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1. Introduction

1.1 Background

In the last 100 years the world has undergone a never before seen rate of change due to climate change brought about by factors such as global warming. Climate change has been one of the leading causes of both environmental and animal migration changes. According to [36], temperature is one of several important factors affecting herbivore performance, this is derivative of the rising temperature due to global warming. Vegetative coverage and livestock production are areas of interest in coming up with Agro-environmental controls that are sustainable. It is imperative that these changes be studied and documented as in [5], so that applicable solutions and counter measures can be established. Botswana has been left behind in this regard, little-to-no environmental condition forecasting research has been undertaken.

In traditional livestock farming, the livelihood of animals depends highly on existing plant biomass, which is affected by the level and intensity of temperature, rainfall, humidity, bio-chemical and other meteorological variables. Researchers rely heavily on times series data based forecasts of these climatic variables as highlighted by [4]. Their findings are ultimately applied to agro-environmental control systems which are then optimized. [53] states that the development of smart agricultural approaches for transforming agricultural systems to guarantee ecological intensification and ensure food security under a changing climate, is the main motivator for use of modeling and forecasting techniques in ecology. This leads us to study how animals and plants interact within certain environmental conditions.

1.2 Introduction

Herbivore-plant interactions provide knowledge about fundamental life processes that are visible in ecological patterns. Since they first began to coexist, plant and animal species have been evolving in tandem. These two types of creatures have established mutually beneficial relationships at a variety of levels, particularly those pertaining to eating behavior [32]. Relationships of this type are founded on a variety of factors, from mutual benefits for both parties, to observed parasite interactions and the wild ruminants' control over plant growth. Because of the significance of plant-herbivore interactions, mathematical ecology has long focused on the dynamic interplay between these two groups of organisms. [4] Any event that could lead to any herbivore activity losing its equilibrium, such as environmental changes, species extinction or bio-invasions may result in significant change that affects more than just the affected species [11]. Key policy areas such as land management, environmental preservation, and animal husbandry all depend on an understanding of the interactions between herbivores and plants.

The creation of dynamical system models of populations has sparked considerable attention due to the astonishing diversity of dynamical behaviors displayed by plant and herbivore species. Recent works include the likes of [2], [13], [17], [19] and [20]. Predator-prey systems are analogous to plant herbivore systems in traditional models of these systems. Plant herbivore dynamics don't typically follow any particular pattern. Instead, the demographic characteristics of the populations of plants and herbivores as well as the timing, kind, and degree of density dependence that they exhibit substantially influence evolution [13]. Strong ecological evidence supports the idea that plant population dynamics affect plant-herbivore interactions more significantly than does herbivore population [17]. The objective of modeling

these interactions is fueled by the desire to understand the dynamics of the relations between animals and plants, and also predict how the systems would change over time. These models are characterized by parameters which are integral in forecasting.

In actuality, the parameters included in models are not absolute constants but rather oscillate about some average value due to ongoing environmental variations, such as variation in sunshine intensity, temperature, water level, etc. As a result, as time goes on, the population density never reaches a set value but instead constantly oscillates around an average figure. In this body of work we are interested in parameter estimation. In the field of environmental sciences, the concept of parameter estimation refers to the process of using data integration or other related methods to determine the optimal values for specific parameters in a numerical model [11]. As a result, the practice is closely related to fixing model flaws brought on by incorrect parameters.

In numerical models, parameters may be a component of explicitly resolved or sub grid-scale parameterized processes. This typically involves approximating the equation's derivatives or integrals in order to estimate the equation of interest. At a discrete set of points, the approximation equation has a solution, and this solution roughly resembles the original equation's solution. Finite difference methods are a common name for such numerical procedures. This method is used to resolve the majority of initial value problems for both ordinary and partial differential equations [65]. Approximation theory and the solution of relatively large linear and nonlinear systems of equations are frequently used in numerical methods for solving differential and integral equations. Numerical parameter estimation method make up a bulk of the work being done.

1.2.1 Research Gap

One significant research gap in the field of plant-herbivore systems in Botswana is the lack of real data for parameter estimation. While parameter simulation techniques have been widely employed in modeling plant-herbivore interactions as shown in [25], [2] and [3], the absence of real data limits the accuracy and applicability of these models in real-world settings. Consequently, there is a need for research that focuses on developing optimized parameter estimation techniques.

Another important research gap in the field of parameter estimation is the need to effectively bridge the variations in base data, often represented by the P-H model, and the observed data. This issue arises due to the inherent differences in the time indexes of collected data and model data. Resolving this gap is crucial for accurate parameter estimation and improving the reliability of modeling and analysis in various domains.

An area of estimation that we want to explore in detail is the estimation of parameters for dynamic model systems that exhibit non-convex parameter relationships. Since traditional optimization methods may struggle to find the global optimum or fail to identify multiple optimal solutions we saw it fit to take on this gap in research for Plant-herbivore models. Last we want to address the lack of forecasting for these type of models as challenges arise from the prediction of climatic factors embedded within the dynamics of plant-herbivore systems.

We intend to use the mathematical model given by [2] and developed by [3], to reasonably anticipate how the plant herbivore system will behave under certain conditions. We then proceed specify an objective function which calculates the difference between the estimated response and the data for a specific set of parameters. We then find a set of parameters for the given model so that the system's anticipated behavior replicates actual behavior (measurements/data) under the identical external conditions is the task at hand. Finally, we use our estimated parameters to come up with adequate forecasts and finally conclude on what the findings can be used in agricultural controls.

1.3 Motivation

Ecological systems are inherently complex, and plant-herbivore interactions are no exception. Plant-Herbivore models tend to have multiple parameters which are intricately related, non-convexity often arises when the relationships between these parameters are nonlinear. As the number of parameters in these models increases, the optimization landscape becomes more complex thus requiring new multi-dimensional global optimum solvers. Reliable parameter estimates are necessary to obtain accurate forecasting of these systems, which is essential for effective decision-making in various sectors, including livestock management, crop planning, and resource allocation. However, accurate forecasting of these systems remains a considerable challenge due to the complex interplay of parameters such as plant growth, livestock behavior, and external factors such as climate and resource availability. Moreover, the inherent difficulties in data collection, including limited availability, measurement errors, and incomplete records in Botswana, further exacerbate the accuracy and reliability of existing forecasting models.

This thesis seeks to tackle these challenges head-on by introducing an innovative approach to optimized parameter estimation, with emphasis on the non-convexity in the optimization problem brought about by the relationship of parameters in the Plant-Herbivore dynamic model. This body of work further aims to integrate machine learning techniques such as Neural Network forecasting models, to come up with reliable forecasts of soil moisture values that are incorporated within the dynamic model. By leveraging advanced global optimization algorithms, the proposed methodology aims to estimate the system parameters more accurately, capturing the dynamic relationships between plant and livestock components in Botswana. Furthermore, by combining the power of neural networks with auto-regressive modeling, the integrated NNAR models will enhance the forecasting accuracy by considering both historical data and the inherent dynamics of the system.

The significance of this research lies in its potential to revolutionize decision-making processes in plant-livestock systems, enabling stakeholders like the Ministry of Agriculture to make informed choices regarding resource allocation, production planning, and risk management. The proposed approach will provide more accurate and reliable forecasts, enabling farmers, policymakers, and agricultural practitioners to optimize their strategies, mitigate risks, and maximize productivity.

Moreover, the findings of this research will contribute to the broader field of agricultural forecasting by providing insights into overcoming data-related difficulties in dynamic systems. By addressing challenges related to limited data availability, measurement errors, and incomplete records, this thesis will offer practical solutions to improve forecasting techniques, not only in plant-livestock systems but also in other domains where data limitations are prevalent.

1.4 Statement of the Problem

Inadequate understanding of Botswana's climatic and environmental state is a problem that is getting more alarming with the progression of time. Climate change has altered precipitation patterns, leading to shifts in the availability of water resources, including soil moisture. This has affected the growth of grasses and other vegetation, impacting the carrying capacity of pastures. Rising temperatures due to climate change have increased evaporation rates, further depleting soil moisture content. Reduced soil moisture also contribute to increased soil erosion and the degradation of soil quality, leading to long-term negative impacts on plant biomass and livestock agricultural practices. All this has made it imperative that Government and other stake holders make policies and decisions based on accurate forecasts of plant-livestock interactions in ecological systems. However, the complexity of these systems, with intricately related parameters and non-linear relationships, poses significant challenges for parameter estimation and

forecasting. Additionally, data-related difficulties, including limited availability, measurement errors, and incomplete records, further hinder the accuracy and reliability of existing forecasting models, particularly in Botswana. Only a few models take into account environmental factors such as soil moisture to forecasting plant biomass and livestock density in Botswana. This has hindered preemptive action to combat or slow down the effects of climate change in the form of controls that can be taken in agro-environmental systems.

1.4.1 Research Questions

1. How can the complex interplay of parameters in plant-herbivore interactions be effectively modeled and optimized to overcome non-convexity in the optimization problem?
2. What is a suitable multi-dimensional global optimum solvers for accurately estimating the parameters in plant-herbivore dynamic models with a large number of intricately related parameters?
3. How can machine learning techniques, specifically Neural Network forecasting models, be integrated into the parameter estimation process to improve the reliability and accuracy of soil moisture value forecasts?
4. What are the dynamic relationships between plant and livestock components in Botswana's ecosystem, and how can they be accurately captured and incorporated into the optimization and forecasting models?

1.5 Objectives

1.5.1 Main Objective

The main goal of this project is to study the interaction between vegetation coverage, traditional livestock production and the corresponding environmental controls while developing a mathematical model that takes into account the dynamic nature of identified environmental factors that will then be used coming up with accurate forecasts.

1.5.2 Specific Objectives

- Identify relationship factors between livestock production and vegetative conditions in select areas.
- Build models that serve as extensions and compliment to already existing deterministic mathematical models.
- Identify an adequate estimation technique and use it to perform parameter estimation on differential model.
- Study and select most appropriate forecasting technique and use estimated parameters to forecast traditional livestock production.

The outcomes of this body of work will help with understanding the impact of the different parameters on the changes in Botswana's climatic and environmental state. It also helps with understanding the optimal control phenomena of the general model with periodic plant and Herbivore. Furthermore, the results from this thesis can help policy makers come up with strategies to optimize vegetation coverage and livestock production in the form of controls that can be taken in agro-environmental systems. In-depth analysis of the literature surrounding our problem statement is necessary for formulating an adequate approach to achieving the set objectives, as such in the next chapter we access multiple sources and methodologies related to vegetation and livestock in Botswana.

2. Literature Review

2.1 Vegetation and Livestock Dynamics in Botswana

2.1.1 Livestock Census

The commercial and traditional sectors of agriculture are two separate sub-sectors in Botswana. Because the commercial sector was not adequately represented in the agricultural survey conducted in 2019 and only the traditional sector was covered, no actionable information could be obtained to inform policy and decision-making about it.

As a result, the study by [57] solely includes the conventional sector's 2019 survey findings. Since there was no survey in 2018, the findings of the 2019 survey were compared to those of the 2017. The traditional sector's cattle population decreased from 1.1 million in 2017 to 935 000 in 2019. A rise in cow fatalities from 64,447 in 2017 to 102,255 in 2019 was blamed for the fall in the number of animals. At 53,571 in 2019, the number of cattle lost due to wandering or theft was lower than the 79,799 in 2017 but still significant. This suggests that between 2017 and 2019, a sizable number of cattle were lost owing to theft or wandering. Although the birth rate of cattle rose from 47.3% to 56.5%, the death rate doubled from 5.9% to 10.9 percent between 2017 and 2019, and the off-take rate climbed from 5.5 to 7.0 percent over the same time period. All this data can be visualised by Figure 2.1.

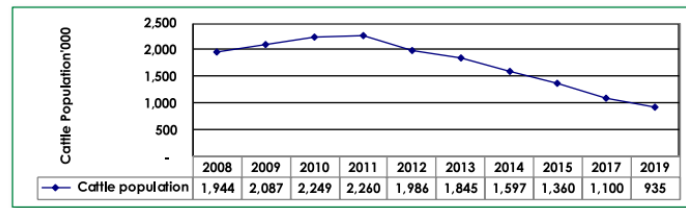


Figure 2.1: Cattle Population Trend ('000) from 2008 - 2019 : Statistics Botswana

As shown by the trend line of Figure (2.1), which depicts yearly cattle population statistics from 2008 to 2019. The harsh drought that devastated the nation in the seasons of 2017–2018 and 2018–2019 takes the blame for the high mortality. According to the Botswana Environment Statistics, Natural and Technological Disasters Digest 2019 [58], the years 2017/18 and 2018/19 were both deemed drought years for the whole nation.

As a result, drought is to blame for the fall in both agricultural and livestock output indices. The number of goats climbed from 1,200,000 in 2017 to 1,229,000 in 2019 over the time period under consideration. Goat birth rates climbed from 39.1 to 39.2 percent, an increase of 0.1 percentage points. In 2017 and 2019, the mortality rate for goats dropped from 23.3 to 18.9 percent [57]. The population of goats increased as a result of a little rise in birthrate and a slight decline in death rate. From 7.3 to 6.6 percent, the off-take rate decreased.

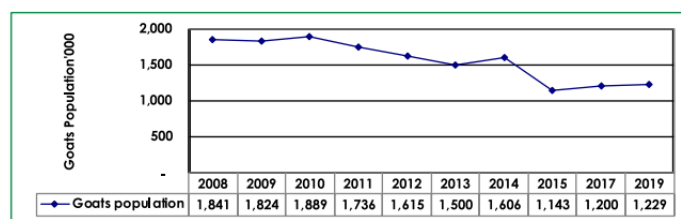


Figure 2.2: Goats Population Trend from 2008 - 2019: Statistics Botswana

The time series data for the goat production indicator data shown in Figure 2.2 above spans from 2008 to 2019. Figure 2.2's time series data shows that the number of goats decreased from 1.8 million in 2008, 2009, and 2010 to 1.5 million in 2013 [57]. But the number of goats rose to 1.6 million in 2014 before rapidly dropping to 1.1 million in 2015 and then rising to 1.2 million in the final two survey years of 2017 and 2019.

Between 2017 and 2019, there were 242,911 more sheep than there were in 2017 [57]. In the same time frame, the birth rate decreased slightly from 33.6 to 32.8 percent, while the death rate decreased from 16.1 percent in 2017 to 14.1 percent in 2019. Nevertheless, the off-take rate rose from between 3.9 and 4.5 percent. From the 2017 to 2019 survey years, the sheep industry showed a small improvement in performance. All this can be summarised in the Table 2.1.

INDICATORS	2017 Traditional	2019 Traditional
CATTLE:- Total Cattle Holdings	33,819	29,355
Total Cattle	1,100,375	934,732
Offtake rate (%)	5.5	7.0
Birth rate (%)	47.3	56.5
Death rate (%)	5.9	10.9
GOATS:- Total Goats Holdings	58,332	42,179
Total Goats	1,199,661	1,228,744
Offtake rate (%)	7.3	6.6
Birth rate (%)	39.1	39.2
Death rate (%)	23.3	18.9
SHEEP:- Total Sheep Holdings	12,992	11,905
Total Sheep	234,621	242,911
Offtake rate (%)	3.9	7.7
Birth rate (%)	33.6	32.8
Death rate (%)	16.1	14.1

Table 2.1: Livestock Holdings, Population & Indicators Between 2017 & 2019: Statistics Botswana

A decrease in the number of agricultural holdings made up of all kinds of livestock is shown in Table 2.1 above. The populations of goats and sheep had marginally grown by 2.4 and 3.5 percent, respectively, in contrast to the fall in total holdings of each animal. There were lower death rates in 2019 compared to the 2017 growing season, are responsible for this rise. Due to the high death rate in 2019, the population of cattle decreased. We can now move onto looking at Botswana's plant vegetation dynamics to better understand the plant livestock interactions within the country.

2.1.2 Vegetation Coverage Mapping

Botswana is a landlocked country located in Southern Africa, with a varied landscape including desert, grasslands, and savannas [52]. Vegetation coverage in Botswana varies depending on the region and environmental conditions. According to [45], the northern parts of Botswana, particularly the Okavango Delta, have a higher vegetation coverage due to the presence of seasonal flooding, which provides the necessary moisture for the growth of trees, grasses, and other vegetation. The Okavango Delta is home to a variety of flora and fauna, including reeds, papyrus, water lilies, and other aquatic plants.

In contrast, the central and southern regions of Botswana, including the Kalahari Desert, have lower vegetation coverage due to their arid and semi-arid environments [48]. The vegetation in these areas consists mainly of thorny shrubs and grasses adapted to the dry conditions. Overall, Botswana has a relatively low vegetation coverage compared to other African countries, with an estimated 22.2% of its land area covered by forests and woodlands. However, it is important to note that vegetation coverage in Botswana is essential for the country's wildlife and ecosystems, as it provides food and shelter for a variety of species. Below is Botswana's Soil and Vegetation Map from the 1991 survey conducted by the Botswana Ministry of Agriculture

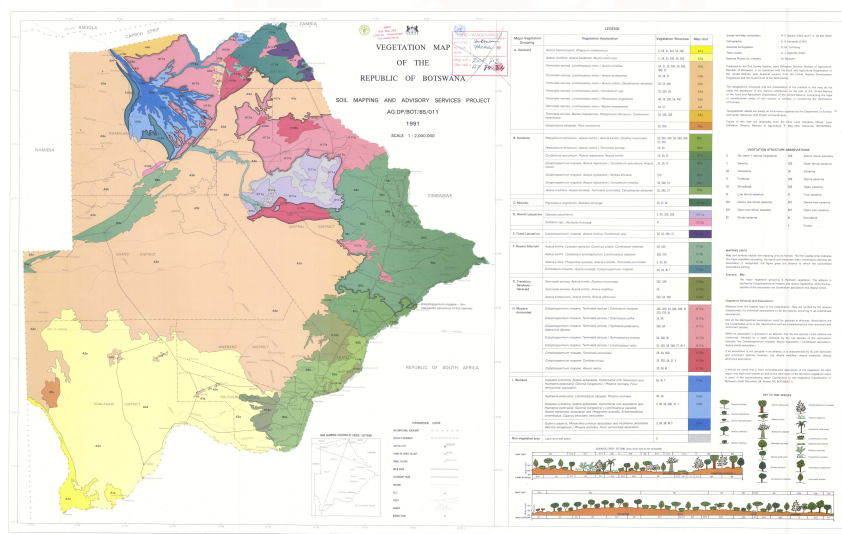


Figure 2.3: Botswana Soil and Vegetation Map (1991) : Ministry of Agriculture

The Botswana Soil and Vegetation Map was produced in 1991 by the Botswana Ministry of Agriculture. The map shown in Figure 2.3 provides a detailed overview of the soils and vegetation types found in Botswana. The map covers the entire country and is based on a classification system that takes into account factors such as soil type, climate, vegetation cover, and land use. Figure 2.3 distinguishes between different soil types, such as sandy soils, loamy soils, and clayey soils, and also provides information on the depth and texture of the soil. In terms of vegetation, the map identifies different types of vegetation zones, such as the Kalahari sandveld, the Mopane woodland, the Acacia savanna, and the Okavango

Delta. The map also provides information on the distribution of different plant species and their ecological characteristics.

The Botswana Soil and Vegetation Map has been widely used for agricultural planning, land use management, and conservation purposes [5]. It has helped to identify areas suitable for crop cultivation, grazing, and wildlife conservation, and has provided valuable information on the ecological processes and environmental conditions that shape the country's ecosystems. While the map is now over 30 years old and may not reflect current conditions, it remains a valuable resource for understanding the soil and vegetation patterns of Botswana. The next subsection will look at soil moisture in Botswana as it is a major component of the environmental condition under review.

2.1.3 Soil Moisture in Botswana

Botswana is landlocked with a semi-arid to arid climate. The country is predominantly covered by savanna grasslands, but it also includes some desert and scrubland areas [55]. Soil moisture in Botswana is a critical resource for agricultural activities, which are the mainstay of the country's economy and livelihoods. Soil moisture is affected by various factors such as precipitation, temperature, and evapotranspiration. In Botswana, rainfall is highly variable both spatially and temporally, with the northern parts of the country receiving more rainfall than the southern parts [60]. The country experiences two distinct rainfall seasons, with the main season occurring from November to March and the secondary season from June to August.

The soil moisture content in Botswana varies depending on the type of soil and the depth of the soil layer. Soils in Botswana are generally sandy with low water-holding capacity, making them susceptible to droughts and water stress [43]. The shallow-rooted vegetation in the savanna grasslands is adapted to these conditions, and many plant species have developed drought-resistant traits such as deep roots and water storage tissues. In recent years, the impacts of climate change and variability have affected soil moisture in Botswana, leading to increased water stress and reduced agricultural productivity [16]. Efforts are being made to improve soil moisture management through measures such as rainwater harvesting, soil conservation, and irrigation. These efforts are crucial for sustaining agricultural activities and ensuring food security in the country.

Recent literature on soil moisture in Botswana includes “Monitoring vegetation dynamics in semi-arid African rangelands: Use and limitations of Earth observation data to characterize vegetation structure” by [60]. This paper discusses the spatiotemporal variability of soil moisture in a semi-arid savanna ecosystem in Botswana using ground-based and satellite-based data. The study found that soil moisture varies greatly depending on the location and time of year, with the wet season showing higher soil moisture levels than the dry season. “Effect of planting date on yield of maize varieties grown in the north-east region of Botswana” by [43] which investigates the effects of soil texture on soil water retention characteristics and maize yield in a semi-arid area of Botswana. The study found that soils with higher clay content had higher soil water retention capacity and supported higher maize yields than sandy soils. [16] assesses the effects of climate change on crop production in Botswana using the CERES-maize model. The study found that climate change could lead to reduced soil moisture and crop yields in the country, highlighting the need for improved soil moisture management practices.

These papers provide a glimpse into the current state of research on soil moisture in Botswana, highlighting the importance of this resource for agricultural activities and the need for sustainable management practices to ensure food security in the country. A visualization of the dynamics of soil moisture in Botswana can be seen in Figure 2.4 below.

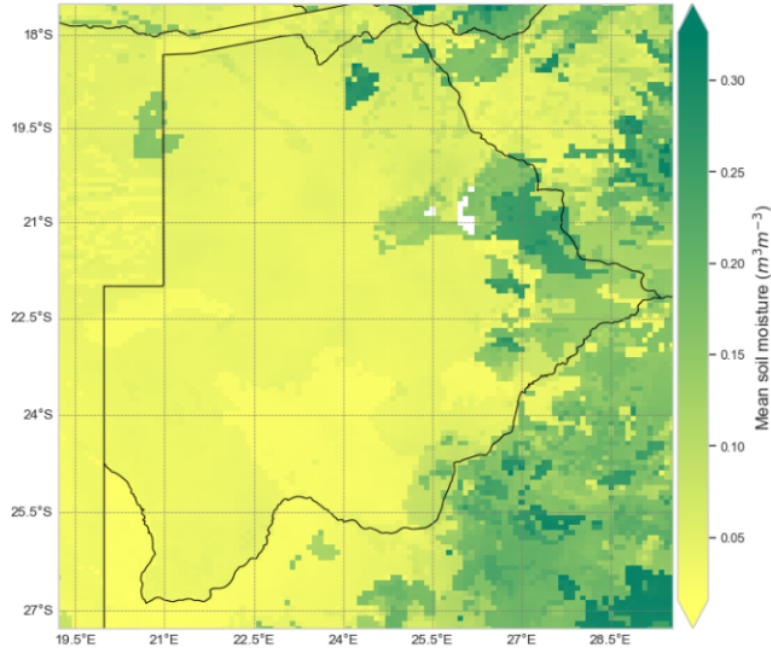


Figure 2.4: Distribution of Daily Mean Soil Moisture (SMAP observation) Data, in the Years 2016 -2020, for Botswana as described in [25].

In this study, the SMAP Level 4 soil moisture data with spatial resolution of 9km is used by combining the coarse resolution data with a land surface model. The NASA Soil Moisture Active Passive (SMAP) project is a satellite mission launched by NASA in 2015 and continues to provide soil moisture data globally. The data is publicly available with online registration on the web at <https://nsidc.org/data/smap/smap-data.html>. A major portion of Botswana has a moisture content of $0.1 \text{ m}^3/\text{m}^3$ or less mean soil moisture, which will be referred to as dry regions; few regions between 0.1 and $0.2 \text{ m}^3/\text{m}^3$ mean soil moisture; few regions with more than $0.2 \text{ m}^3/\text{m}^3$ mean soil moisture; and few regions that are wet areas [25]. Incorporating soil moisture information in plant growth modelling is more informative as soil moisture itself is a result of interaction of more meteorological variables and soil type, in addition to temperature and rainfall. Mathematical modeling of vegetation and livestock dynamics through the use of plant-herbivore systems of equations is a topic that is the focus of the next section.

2.2 Mathematical Modeling of Plant-Herbivore Interactions

It's a given, that in perfect conditions, plants grow well. Creating those circumstances requires preparation and expertise. Depending on the stage of growth a plant is in, different varieties of plants need varied surroundings. By taking into account these various factors, there will be a chance to improve plant health and promote stronger yields [28]. One would have a higher chance of creating the ideal environment for themselves and their plants to flourish if they are aware of each of these aspects and can manage them separately. The main areas of application of Agro-environmental controls are agricultural monitoring, pesticide reduction, precision agriculture and adaptation strategies to climate change in agriculture and horticulture [54]. Factors such as pollinator decline, vegetation response to global change; water cycle-vegetation feedback, mitigation strategies to enhance soil carbon sequestration and reduce greenhouse gas emissions from agricultural soils are some of the hot button topics researchers today are looking into. Mathematical models are some of the potent tools used by researchers in this regard.

Mathematical modeling is a key area in analysis of dynamic relationships in ecology such as how vegetative covering influences livestock production in areas across the country. In order to produce forecasts and guide strategic decision-making, time series forecasting involves examining time series data using statistics and modeling. Forecast probabilities might vary greatly—especially when dealing with the variables that frequently fluctuate in time series data and with outside influences [2]. Frequently, the projections might be more accurate the more complete the data is available to the researchers. While prediction refers to future data generally, forecasting in some industries may relate to data at a specific future time point. Recent works under forecasting of climate data are [63], [34] and [1].

A system of differential equations comprising of functions of only one independent variable and one or more of their derivatives with respect to that variable is known as an ODE (ordinary differential equation) model. In science and engineering, ODEs are the most popular formalism for modeling dynamical systems [38]. It is difficult to incorporate environmental effects into the dynamics of the plant-herbivore relationship. To explore the impact of climatic conditions in predator-prey models, several researchers have instead used periodic mathematical models [66], [22], [24], [23], [30], [21]. These models use ordinary differential equations to represent the interactions between predators and their prey. The Lotka-Volterra model is considered the earliest such differential equation system.

2.2.1 Lotka-Volterra Model

When attempts were initially made to identify ecological rules of nature, the Lotka-Volterra equations system, the first differential equation model and the most basic of the predator-prey types, was created [66]. Discrete and continuous predator-prey models have formed a key component of ecological theory. The Lotka-Volterra equations, often known as predator-prey equations, are a set of nonlinear differential equations of the first order that were intended to model the dynamics of biological systems [8]. As they disregard aspects like competition, sickness, and mutualism and concentrate on the predator-prey relationship, they can be characterized as a more generic form of the Kolmogorov model.

The Lotka-Volterra equations can be written as:

$$\frac{dx}{dt} = \alpha x - \beta xy \quad (2.1)$$

$$\frac{dy}{dt} = \delta x - \gamma y, \quad (2.2)$$

where, x is the prey population, $\frac{dx}{dt}$ is instantaneous rates for the prey, y is predator population, $\frac{dy}{dt}$ is instantaneous rates for the predators and t is time, [10], α, β, γ and δ are positive real constants that define the rates and interactions between the prey and population compartments. Numerous assertions are made about the environment and the evolution of the predator and prey population in the Lotka-Volterra model (see [30] and [66]).

Equation (2.1) can be understood as the change in the prey density which is given by the difference in its own growth and the rate at which it is preyed upon. As for equation (2.2) it can be interpreted as the growth of the predator population minus the loss rate of the predator due to either natural death or emigration and can be interpreted as the change in the predator population as growth fueled by the food supply minus natural death.

Dynamical systems have been used to analyze plant-herbivore interactions in great detail, going much beyond the original Lotka-Volterra model in terms of both theory and modeling strategy. The predator-prey system is the foundation for traditional methods used to create plant-herbivore models [26]. Some

extensions of the Lotka-Volterra include the population growth model that factors in the carrying capacity formulation is the logistic equation known as Verhulst model [56] and the Lotka-Volterra equation with logistic growth equation was expanded by [49] to incorporate density dependent growth and a functional response. Functional response incorporating models termed by [37] as Holling type I, II and III .

Focusing on the type II functional response, predation rates rise quickly in response to an initial rise in prey density in a more complex functional response than the type I, and then gradually grow until they reach a stable level as taken room [37].

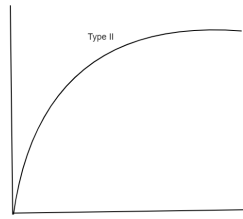


Figure 2.5: Holling Type II Function Response on a Prey Versus Predator

An initial almost linear increase in a type II functional response may be seen on the graph of prey killed versus prey density, [2]. However, it finally asymptotes at a maximum feeding rate as it gradually slows down. The mathematical form of the Type II functional response replaces the constant predation rate in β in (2.1) with the following function ;

$$\frac{\beta}{1 + \beta T_h P},$$

where T_h is the handling time and P is the plant biomass.

2.2.2 Plant-Herbivore Mathematical Model with Constant Coefficients

The standard predator-prey ecological model in its version as provided in [59], which takes into account the Allee effects for the herbivore, is the fundamental system in which this model is built on. By taking into account the logistic growth rate of the plant population and the Holling type II functional response for the feeding interaction, [2] analyzed the relationship between plants and herbivores. The number of herbivores was calculated using a constant multiplier of the Allee effect and feeding relationship (efficacy) together with birth rate. Additionally, the model incorporates a variable herbivore death rate that is based on plant density.

Our working model is directly taken from [3]. The following assumptions are used in formulating the mathematical model:

- The growth of plants follow a logistic pattern.
- No alternative source of food for Animals (Herbivores).

- Reproduction of herbivores is dependent on the availability and amount of food source (plants).
- Herbivore reproduction is also dependent on the existence of enough mate (Allee effect of herbivores).
- The death rate of animals is non-constant.

With the density of plants, P , and herbivore density as H , the nonlinear ordinary differential equations that represent the dynamics are given as follows:

$$\begin{aligned}\frac{dP}{dt} &= rP \left[1 - \frac{P}{K} \right] - \left(\frac{a}{b+P} \right) PH \\ \frac{dH}{dt} &= H \left[c \left(\frac{a}{b+P} \right) P \left(\frac{H}{h+H} \right) - D \left(\frac{a}{b+P} \right) - \mu \right],\end{aligned}\tag{2.3}$$

where $0 \leq P(0), 0 \leq H(0)$.

Table 2.2 below shows descriptions of the parameters found in (2.3)

Parameter	Description
r	Plant growth rate
a	Feeding rate
K	Carrying capacity(for plant population)
b	half saturation constant of herbivore feeding on the plant biomass
c	Conversion rate of plant biomass to herbivore offspring
h	Allee constant
μ	Harvest rate

Table 2.2: The names of the compartments and their descriptions in Constant coefficient model as stated in [2]

Equations (2.3) consider that each parameter will remain constant. The amount of food available and the state of the weather, however, affect how quickly plants grow and how quickly herbivores eat [3]. Therefore, it is crucial to take into account these environmental fluctuations in order to make the analysis more accurate. The impact of the environment on the interactions between plants and herbivores is taken into account.

2.2.3 General Model with Periodic Plant-Herbivore feeding rates

This subsection considers an extension to the mathematical model seen in equation (2.3), by incorporating the periodic variation of several of the key climatic characteristics and not assuming them to be constant as in equation (2.3). The state variables of our model are the plant biomass, represented by P , and the herbivore density, represented by H . The following assumptions on the interaction between plants and herbivores were made by [2] while formulating and analyzing the model.

A1. Weather conditions and environmental changes (such as temperature, rainfall, etc.) have a substantial impact on the growth of plants. We therefore assume that the logistic equation, in which the coefficients are periodic functions to allow for seasonal fluctuations, governs the growth rate of the plant $r(t)$.

A2. Based on assumption A1, we assume that because of the periodic growth rate of plants, herbivores' feeding or consumption rate of plants $a(t)$ is also periodic and continuous with respect to time. Let's take into account the following predator-prey type model for the plant-herbivore interaction under these presumptions.

$$\begin{aligned}\frac{dP}{dt} &= r(t)P \left[1 - \frac{P}{K} \right] - \left(\frac{a(t)}{b+P} \right) PH \\ \frac{dH}{dt} &= H \left[c \left(\frac{a(t)}{b+P} \right) P \left(\frac{H}{h+H} \right) - D(F(P)) \right] - \mu(H),\end{aligned}\tag{2.4}$$

where $0 \leq P(0), 0 \leq H(0)$.

Table 2.3 below shows descriptions of the parameters found in (2.4)

Parameter	Description
$r(t)$	Time-dependent intrinsic growth rate of plant population
$a(t)$	Time-dependent herbivore feeding rate
K	Carrying capacity(for plant population)
b	Half saturation constant of herbivore feeding on the plant biomass
c	Conversion rate of plant biomass to herbivore offspring
h	Allee constant
$\mu(H)$	Harvesting (off-take) function

Table 2.3: The names of the compartments and their descriptions in Periodic model

where $r(t)$ is the plant growth rate at time t , which is defined by:

$$r(t) = r(\theta, m) = m(t) + [1 - m(t)] U_r(\theta) \beta_r(\theta),$$

here θ is the soil moisture and $m(t)$ is the minimum plant growth rate at time t . $U_r(\theta)$ and $\beta_r(\theta)$ are given by

$$U_r(\theta) = \exp \left[-(\theta - \theta_c)^2 / 2\pi\sigma(\theta)^2 \right],$$

and

$$\beta_r(\theta) = \begin{cases} 1, & \theta \geq \theta_c \\ \frac{\theta - \theta_w}{\theta_c - \theta_w}, & \theta_w < \theta < \theta_c \\ 0, & \theta \leq \theta_w, \end{cases}$$

respectively. With θ_c being the critical soil moisture, θ_w wilting point soil moisture and $\sigma(\theta)^2$ is the variance of the soil moisture data.

The expression of the feeding rate as a Holling type II functional response is $F(P) = \frac{a(t)P}{b+P}$. The density dependent herbivore death rate, which is a decreasing function of the feeding response and is bounded by the minimum and maximum death rates d_1 and d_2 , is represented by the term $D(F(P))$ in the second equation of the model system (2.4) [2]. It comes from:

$$D(F(P)) = d_2 - \frac{F(P)}{g + F(P)},$$

after taking into account $D(F(0)) = d_2$ and $D(F(K)) = d_1$ to find the constant g , reduces to:

$$D(F(P)) = \frac{d_2 K (1 + d_1 - d_2) (b + P) + P (b + K) (d_2 - d_1) (d_2 - 1)}{K (1 + d_1 - d_2) (b + P) + (d_2 - d_1) (b + K) P}. \quad (2.5)$$

Consequently, we state $d_1 \leq D(F(P)) \leq d_2$ as in [2]. The theory behind this plant-density dependent herbivore death rate also includes the notion that because it is dependent on the availability of plants, herbivore dynamics follow plant dynamics and so it will never achieve its own carrying capacity.

Additionally, since the population of herbivores reproduces sexually, it is essential that there be an adequate number of these animals to reproduce or give birth to new ones [2]. This is handled by the term $\left(\frac{H}{h+H}\right)$, where h is the Allee constant, because $cF(P) \left(\frac{H}{h+H}\right)$ disappears as H becomes extinct [2]. That is, as the herbivore density decreases to zero, individual female herbivores will find it difficult to reproduce, either because they cannot mate or because there is a higher danger of inbreeding.

The function $\mu(H)$ is known as the harvesting (off-take) and is given by

$$\mu(H) = \begin{cases} 0, & 0 \leq H \leq H_{min}; \\ \mu_0, & H_{min} \leq H \leq H_0; \\ \mu H, & H > H_0, \end{cases}$$

We must deal with positive values for population or biomass variables in order for the system to make sense mathematically and practically. Therefore, we establish the domain space as

$$\Omega = \{(P, H) \in \mathbb{R}_+^2 | P \geq 0, h \geq 0\}.$$

When only numbers from the first quadrant, or the positive, are used to represent the population of both plants and herbivores. The existence and singularity of a periodic solution for the model system are theoretically demonstrated in the mathematical analysis of model system (2.4), positiveness of the solutions, and persistence and permanence of the system [3]

We discretize the time-interval of interest using finite difference techniques and use a difference equation to approximate the derivative in the chosen system of equations. We iteratively approximate the value of the solution at the discretization points using the initial conditions as a starting point. All finite difference techniques have the same general structure, however we may categorize particular finite difference methods depending on the specifics of their derivations [47]. One-step versus multi-step approaches and explicit versus implicit methods are two categorization categories.

Multi-step approaches employ information about several discretization points to estimate the solution at a discretization point, in contrast to one-step methods, which use information about a single discretization point. Implicit techniques might require the solution of a nonlinear system of algebraic equations at each iteration whereas explicit methods depend on information at the current discretization point and knowledge about prior discretization points.

Many differential equations are difficult to precisely solve. But for practical purposes, a numerical approximation of the solution is frequently enough, as in engineering. There are various numerical solvers available for solving mathematical problems and simulations. Some commonly used solvers include the Newton-Raphson method, bisection method, secant method, Runge-Kutta methods, finite difference

method, finite element method, Monte Carlo method, genetic algorithms, simulated annealing, and linear programming solvers. The Newton-Raphson [46] method and bisection method [14] are iterative techniques used for finding roots of functions, while the secant method [62] approximates derivatives. Runge-Kutta methods [51] are effective for solving ordinary differential equations, while the finite difference method and finite element method are used for differential equations and partial differential equations, respectively. We focus on two methods used to numerically estimate solutions of systems of ODEs.

The Euler method and the Runge-Kutta method are two numerical methods used to approximate the solutions of ordinary differential equations (ODEs). The Euler method is a first-order method that uses the tangent line to approximate the solution at each time step, while the Runge-Kutta method is a higher-order method that uses a weighted sum of function evaluations at different time steps to approximate the solution. The first method studied is the Euler method discussed in the next section.

2.2.4 Euler Method

Euler's technique is the most basic numerical approach for resolving initial value problem [35]. Although this approach is rudimentary that it is rarely utilized in practice, its simplicity makes it impractical to more complex ODE's [33]. The foundation of Euler's technique is the idea that the integral curve of an ODE at $(x_i, y(x_i))$ roughly approximates the integral curve across the range $[x_i, x_{i+1}]$. The equation of the tangent line to the integral curve at $(x_i, y(x_i))$ is because the slope of the integral curve of an at $(x_i, y(x_i))$ is $y'(x_i)=f(x_i, y(x_i))$ the equation of the tangent line to the integral curve at $(x_i, y(x_i))$ is

$$y = y(x_i) + f(x_i, y(x_i))(x - x_i),$$

setting $x_i = h + x_{i+1}$, we get the following

$$y_{i+1} = y(x_i) + hf(x_i, y(x_i)), \quad 0 \leq i \leq (n - 1). \quad (2.6)$$

In general, Euler's method starts with the known value $y(x_0) = y_0$ and computes $y_1, y_2, \dots, y_n$ successively by using equation 2.6. To demonstrate how the forward Euler methods, we use it solve (2.3), the solution curve of this system of equations is shown below:

Numerical solution for PH model using Forward Euler:

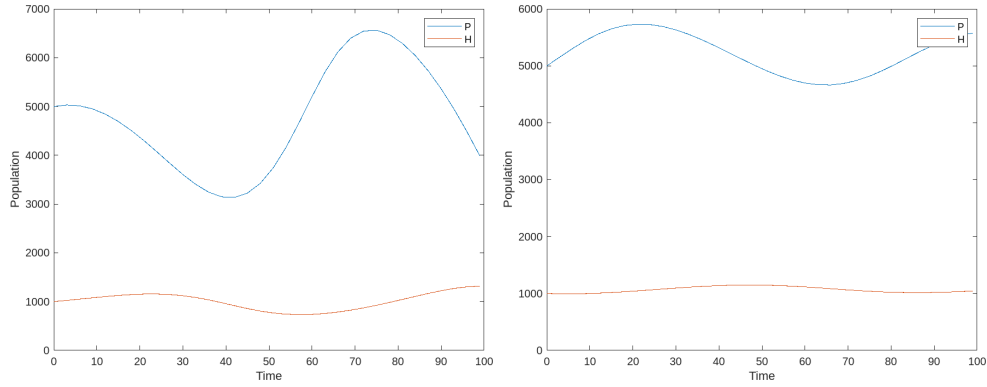


Figure 2.6: PH Model Euler Solution for $a = 0.75$ and $r = 0.25$ $r = 0.5$

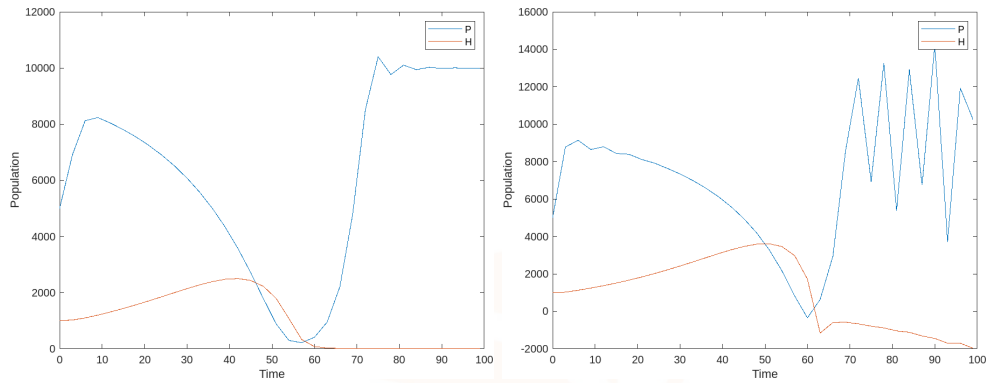


Figure 2.7: PH Model Euler Solution for $r = 0.25$ and $a = 0.75$ $a = 0.7$

Parameter	value	Description
K	10000	Carrying capacity(for plant population)
b	1100	Half saturation constant of herbivore feeding on the plant biomass
c	0.25	Conversion rate of plant biomass to herbivore offspring
h	40	Allee constant
d1	0.1	Minimum death rate
d2	0.3	Maximum death rate
μ	0.025	Harvesting rate

Table 2.4: List for Values of Parameters in P-H Model taken from [3]

2.2.5 Runge-Kutta Technique

The Runge-Kutta (RK) method is a reliable and popular approach for solving differential equation initial-value ODE. Without the necessity for high order derivatives of functions, the Runge-Kutta method can be utilized to build high order accurate numerical methods by functions alone. The general form of the RK method is given by

$$y_{n+1} = y_n + h \sum_{i=1}^m c_i f(\tau_i, y(\tau_i)), \quad (2.7)$$

where $h = x_{n+1} - x_n$, m is the number of linear approximation values $f(\tau_i, y(\tau_i))$ of $f(x, y(x))$ on the interval $[x_n, x_{n+1}]$. c_i, τ_i are different values chosen to obtain different forms of computation formula for RK. [65] The most widely used Runge–Kutta formula is

$$\begin{aligned} y_{n+1} &= y_n + \frac{h}{6}(K_1 + 2K_2 + 2K_3 + K_4), \\ K_1 &= f(x_n, y_n), \\ K_2 &= f(x_n + \frac{1}{2}h, y_n + \frac{1}{2}K_1), \\ K_3 &= f(x_n + \frac{1}{2}h, y_n + \frac{1}{2}K_2), \\ K_4 &= f(x_n + h, y_n + K_3). \end{aligned} \tag{2.8}$$

It is also known as the fourth-order Runge-Kutta method and requires four function values for each step iteration. There are numerous papers and resources available on the Runge-Kutta method. Some examples of papers on the Runge-Kutta method: [7] presents a numerical study of the fractional order Burgers' equation using a Runge-Kutta method. The study compares different orders of Runge-Kutta methods and shows that higher order methods provide better accuracy and convergence rates. [33] presents a class of embedded Runge-Kutta methods that adaptively choose the time step to achieve a given level of accuracy. “High-order explicit Runge-Kutta methods for solving ordinary differential equations” by [12] is an overview of high-order explicit Runge-Kutta methods and their properties, including their stability and order of convergence. Lastly, [29] proposes a new fourth-order two-stage explicit Runge-Kutta method for solving stiff systems. The method is shown to be more efficient and accurate than other existing Runge-Kutta methods for stiff problems.

These are just a few recent examples of the many papers available on the Runge-Kutta method. To demonstrate the Runge-Kutta method, we use it to solve (2.3), the solution curve of this system of equations is shown below.

Numerical solution for PH model using Runge-Kutta

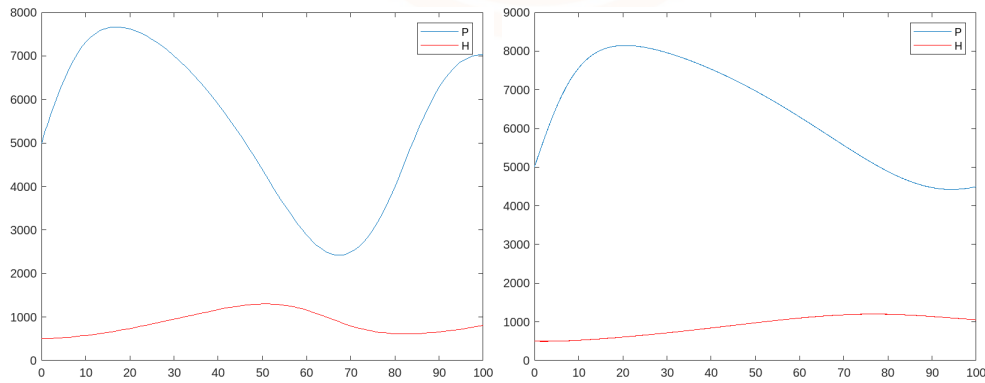
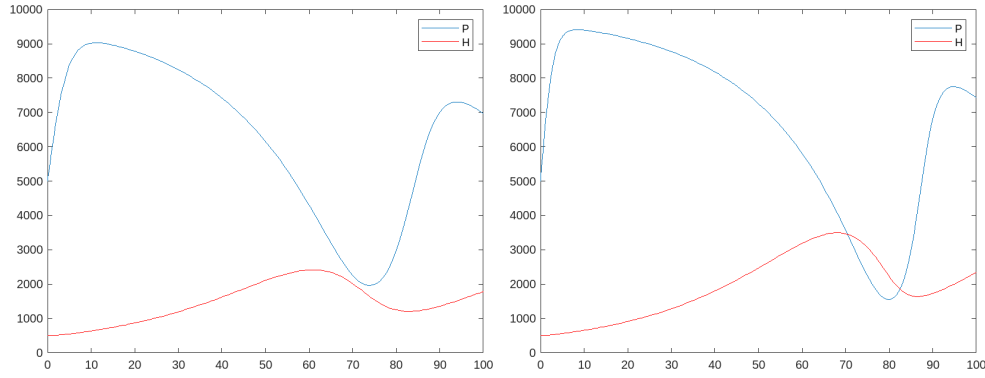


Figure 2.8: PH model RK solution for $a = 0.75$ and $r = 0.25$ $a = 0.5$

Figure 2.9: PH model RK solution for $r = 0.25$ and $a = 0.75$ $a = 0.7$

The list of parameter values is shown in Table 2.4.

When comparing the solution curves obtained using the Euler method (Figures 2.6, 2.7) and the Runge-Kutta method (Figures 2.8 and 2.9) we see that the Euler methods produces solutions curves that are not smooth as compared to those of Runge-Kutta. The solutions curves of the Euler behave quite erratically when slight changes are made o the parameters as shown by Figure 2.7 while the Figure 2.9 is more stable compared to the Euler method when the parameters of the systems are varied. However, the Euler method is handy when dealing with stochastic differential equations models because it is then extended to Euler-Maruyama technique.

It is generally expected that the Runge-Kutta method provides more accurate results than the Euler method. This is because the Runge-Kutta method uses more information about the solution at each time step, which allows it to capture more accurately the dynamics of the system. As we can see, the solution curves obtained using the Runge-Kutta method are much closer to the true solution than those obtained using the Euler method. The errors in the Euler method accumulate quickly and lead to a significant deviation from the true solution, while the errors in the Runge-Kutta method are much smaller and the approximation is much closer to the true solution. We discuss different methods for parameter estimation in the following section.

2.3 Methods of Parameter Estimation

The experimental selection of a model's optimal coefficients using measured empirical data with a random component is known as parameter estimation [10]. The coefficients/parameters indicate how much of an impact each variable has on the model; the closer a variable is to zero, the less impact it has; the further away, the more impact it has. The prevalent coefficients that may be calculated with ease are often those using linear or nonlinear regression models. This is due to the model's simplicity and the fact that the function of the curve is simple to find for data fitting. The analytical system based on ODEs gives the best alternative for determining the curve and is generally the answer. In the case where the analytical solution doesn't exist, then the system's numerical solution can be discovered using numerical solving techniques such as Euler, Newton-Rapson and Runge-Kutta Method as mentioned in the previous section. There are several methods for parameter estimation of ODE.

2.3.1 Gradient-Based Optimization

This method involves minimizing an objective function that measures the discrepancy between the model predictions and the observed data. This is typically done using gradient-based optimization algorithms such as gradient descent, conjugate gradient, or quasi-Newton methods.

2.3.2 Markov Chain Monte Carlo (MCMC) Methods

This method involves simulating a sequence of parameter values using a Markov chain and accepting or rejecting proposed parameter values based on their likelihood. It can be used to generate a sequence of samples from the posterior distribution using MCMC algorithms such as the Metropolis-Hastings algorithm or Gibbs sampling.

2.3.3 Bayesian Inference

This method involves using Bayes' theorem to compute the posterior distribution of the parameters given the observed data. The method provides a probabilistic framework for parameter estimation and can incorporate prior knowledge about the parameters. This can be given as

$$P(\theta|y_i, t_i) \propto P(\theta) \prod_{i=1}^n p(y_i|t_i, \theta), \quad (2.9)$$

where $P(\theta)$ is the prior distribution of the parameters, and y_i and t_i are the observed values of a random variable Y from a sample of n and their corresponding time points, respectively.

2.3.4 Maximum Likelihood Estimation

This method involves finding the set of parameters that maximize the likelihood $L(\theta)$ of the observed data. The method is widely used in statistics and provides a simple and efficient approach to parameter estimation. We usually have the likelihood function:

$$L(\theta) = \prod_{i=1}^n p(y_i|t_i, \theta), \quad (2.10)$$

where $p(y_i|t_i, \theta)$ is the probability density function of the ODE solution at time t_i with parameters θ .

2.3.5 Nonlinear Least Squares

This method involves minimizing the sum of the squared differences between the predicted and observed data using an optimization algorithm. The method is widely used and is suitable for both small and large datasets. It is a popular estimation techniques because it can handle complex models and noisy data, while still providing estimates of the uncertainty in the parameter estimates. Some common algorithms used for nonlinear least squares include Levenberg-Marquardt, Gauss-Newton, and Powell's method. The objective function to be minimized is given by:

$$\min_{\theta} \sum_{i=1}^n [y_i - f(t_i, \theta)]^2, \quad (2.11)$$

where θ represents the unknown parameters of the ODE system, n is the number of observations, y_i are the observed values at time t_i , and $f(t_i, \theta)$ is the ODE solution at time t_i . This method can be extended to a system of ODE's similar to equation (2.4). Let

$$\frac{dx}{dt} = f_1(x, y, \lambda), \quad (2.12)$$

$$\frac{dy}{dt} = f_2(x, y, \theta), \quad (2.13)$$

be system of equations where x and y are the dependent variables, t is time, f_1 and f_2 are nonlinear functions of x , y , and the vectors of unknown parameters are λ , and θ . To estimate the values of the parameters, an extension of equation (2.11) can be used. The objective function to minimize is:

$$\operatorname{argmin}_{\lambda, \theta} \sum_{i=1}^n (x_i - x(t_i, \lambda))^2 + (y_i - y(t_i, \theta))^2 \quad (2.14)$$

where x_i and y_i are the i th data points, t_i is the corresponding time, and $x(t_i, \lambda)$ and $y(t_i, \theta)$ are the solutions of the ODEs at time t_i with parameter vector values λ , and θ , respectively. The choice of the method will depend on the specific ODE model, the nature of the data, and the computational resources available. The goal of parameter estimation is to obtain the best-fit parameter values, which can be used to predict the behavior of the system over time or under different conditions. Heterogeneous approaches have been developed which incorporate 2 or more of these parameter estimation techniques.

2.4 Model Forecasting Techniques

Probabilistic and trajectory model forecasting are two approaches used in predicting future events or outcomes based on historical data and mathematical models. Probabilistic forecasting involves estimating the likelihood of different outcomes or events occurring. It takes into account the inherent uncertainty and variability in the data and provides a range of possible future scenarios along with their associated probabilities. Trajectory model forecasting, on the other hand, focuses on predicting the path or trajectory that a system or variable will follow over time. Under probabilistic forecasting we start off by looking at time series forecasting.

2.4.1 Time Series Forecasting

Time series forecasting is a technique used in statistics and econometrics to make predictions about future values of a variable based on historical data. Time series data are characterized by the presence of serial correlation, where observations in a sequence are dependent on previous observations [18]. The objective of time series forecasting is to estimate the future values of a variable, based on past observations, with a certain degree of confidence.

Autoregressive Integrated Moving Average (ARIMA) ARIMA is a differenced series of AR and MA. This model uses an equation formulated from its own lags and the lagged forecast errors in order to forecast future values. The equation for ARIMA is given by;

$$Y_t = \alpha + \beta_1 Y_{t-1} + \beta_2 Y_{t-2} + \dots + \beta_p Y_{t-p} + \epsilon_t + \phi_1 \epsilon_{t-1} + \phi_2 \epsilon_{t-2} + \dots + \phi_q \epsilon_{t-q},$$

whereby

Y_t is the predicted value,

α is the constant,

$\beta_n Y_{t-n}$ is the linear combination Lags of Y (upto p lags),

$\phi_i \epsilon_{t-i}$ linear combination of lagged forecast errors (up-to q lags).

There are various techniques available for time series forecasting, such as classical time series methods, ARIMA models mentioned above, and machine learning algorithms. Classical time series methods include exponential smoothing, moving averages, and trend analysis [42]. ARIMA models are a popular choice for forecasting stationary time series, while machine learning algorithms such as random forests and neural

networks are widely used for non-linear time series forecasting. One of the most important aspects of time series forecasting is selecting the appropriate model for the data. This involves identifying the type of trend and seasonality present in the data, as well as any other factors that may affect the variable being forecasted [31]. The accuracy of the forecast is highly dependent on the quality of the data and the appropriateness of the model selected.

Box and Jenkins Approach

Box - Jenkins Analysis refers to a systematic method of identifying, fitting, checking, and using integrated autoregressive, moving average (ARIMA) time series models [9]. The method is appropriate for time series of medium to long length of observations. Using plots of the data, autocorrelations, partial autocorrelations, and other information, a class of simple ARIMA models is selected. This amounts to estimating appropriate values for p , d and q .

Seasonal Autoregressive Integrated Moving Average (SARIMA)- When there is seasonality in the data, the seasonal ARIMA is used. A seasonal ARIMA model is represented as an $ARIMA(p,d,q)(P,D,Q)m$ model, whereby P is the number of seasonal autoregressive (SAR) terms, D is the number of seasonal differences, Q is the number of seasonal moving average (SMA) terms and m is the number of observations per year.

Neural Network Auto-regression (NNAR)

NNAR models, also known as auto-regressive neural networks, combine the power of neural networks with auto-regressive processes to capture complex temporal patterns in data. Several studies have demonstrated the effectiveness of NNAR models in forecasting various time series data. For instance, [64] proposed a NNAR model for financial time series forecasting, achieving superior performance compared to traditional methods such as ARIMA. Similarly, [6] used a NNAR model to forecast short-term electricity load, outperforming other forecasting techniques in terms of accuracy and flexibility.

The general formulation of NNAR can be represented as follows:

$$y_t = f(y_{t-1}, y_{t-2}, \dots, y_{t-p}),$$

where y_t is the target variable at time t , and p represents the order of auto-regression. The function $f(\cdot)$ is typically represented by a feed-forward neural network with appropriate activation functions.

Several studies have shown the effectiveness of NNAR in various forecasting tasks. For instance, applied NNAR to predict stock prices and achieved improved forecasting accuracy compared to traditional linear auto-regressive models. Another study by [50] employed NNAR to forecast electricity demand, demonstrating its superior performance over benchmark models.

Exponential Smoothing (ETS)

ETS models are widely employed in time series forecasting due to their ability to capture trend, seasonality, and error components. The ETS framework includes multiple variations, which differ in the way they handle trend, seasonality, and error terms. [39] introduced an automatic ETS algorithm that selects the appropriate model based on the data characteristics, demonstrating its effectiveness in forecasting various time series. Moreover, [61] conducted an extensive evaluation of forecasting methods and found

that ETS models consistently performed well across different forecasting horizons and data patterns. The general formulation of ETS can be expressed as follows:

$$y_t = l_{t-1} + b_{t-1} + s_{t-m} + \varepsilon_t,$$

where y_t represents the target variable at time t , l_{t-1} and b_{t-1} denote the level and trend components, s_{t-m} represents the seasonal component with a periodicity of m , and ε_t is the error term.

In recent years, researchers have also explored the combination of NNAR and ETS models to leverage the strengths of both approaches. For example, [40] proposed a hybrid NNAR-ETS model that combined the nonlinear modeling capability of neural networks with the flexible trend and seasonality handling of ETS models. Their experiments on real-world data demonstrated improved forecasting accuracy compared to standalone NNAR and ETS models.

This leads us to a more appropriate method for forecasting ODE's, which is Trajectory Forecasting.

2.4.2 Trajectory Forecasting

Trajectory forecasting of ordinary differential equations (ODEs) is a mathematical technique used to predict the future behavior of a dynamical system described by ODEs. The technique involves solving the ODEs numerically to obtain a trajectory that describes the system's motion over time. In trajectory forecasting of ODEs, the initial conditions of the system are specified, and the ODEs are used to determine how the system evolves over time [44]. This evolution can be visualized as a trajectory in the phase space of the system, where each point in the space corresponds to a particular configuration of the system.

To perform trajectory forecasting, numerical methods such as Euler's method, Runge-Kutta methods, or other more sophisticated algorithms can be used to solve the ODEs numerically [35]. These methods allow the system's trajectory to be computed at discrete time steps, enabling predictions to be made about the system's future behavior. Trajectory forecasting of ODEs is used in a variety of scientific fields, including physics, engineering, and biology [47]. It can be used to model and predict the motion of physical systems such as particles, pendulums, and springs, as well as the behavior of more complex systems such as chemical reactions, ecological systems, and population dynamics.

There are several methods of trajectory forecasting of ordinary differential equations (ODEs). These methods differ in their numerical algorithms and their accuracy, stability, and computational complexity. Here are some of the most common methods used for trajectory forecasting of ODEs:

1. Euler's method: This is a simple and straightforward method that uses first-order approximations to estimate the solution to an ODE. It involves approximating the solution by a straight line, which is determined by the derivative of the function at the current time step.
2. Runge-Kutta methods: These are a family of numerical methods that use a weighted average of several estimates of the derivative of the function to compute the solution to an ODE [12]. These methods are more accurate than Euler's method and come in different orders, such as the 2nd-order, 4th-order, or higher.
3. Backward differentiation formulas: These methods use a backward difference approximation to estimate the derivative of the function, resulting in a more stable and accurate approximation of the solution.

4. Multistep methods: These methods use previous and current values of the solution to approximate the derivative and compute the next value of the solution. These methods can be more accurate than one-step methods but are generally more complex to implement.
5. Adaptive step size methods: These methods adjust the size of the time step used in the numerical algorithm based on the rate of change of the solution. This can improve the accuracy of the solution while reducing the computational cost.

ODE models can often be more computationally efficient than time series models, especially for systems with many variables and complex interactions. ODE models can often be solved using numerical methods, such as the Runge-Kutta method, which can provide accurate solutions without the need for complex statistical techniques. Time series analysis and ODEs are both valuable approaches for modeling time-dependent systems, the use of time series analysis on ODEs is typically not necessary or practical, as ODE models already capture the underlying dynamics of the system and provide accurate predictions.



3. Methodology & Study Approach

3.1 Data Set Descriptions

3.1.1 Livestock Data

The 2019 Livestock Survey data from Statistics Botswana provides information on the livestock population in Botswana as of August 2019. The data is presented in various tables, which can be accessed on the Statistics Botswana website. The survey covered all districts of Botswana and collected data from both communal and commercial livestock farmers. The survey collected data on various aspects of livestock farming, including the number and types of livestock, animal health, breeding practices, and sales and marketing of livestock products. The main types of livestock covered in the survey included cattle, goats, sheep, horses, donkeys, pigs, and poultry. The variables in each table include administrative district where the cattle, goat, sheep were located, sex of the cattle, goat, sheep (male or female), age group of the animal (e.g., 0-12 months, 12-24 months, etc.) breed of the animal, primary purpose of the animal (e.g., milk, beef, draught, etc.), number of animal in each category. The takeoff variable refers to the number of animals sold or removed from the herd during the survey period. In the cattle data table, this variable is labelled as “Cattle sold or removed from herd” and in the goat and sheep data tables, it is labelled as “Goats/Sheep sold or removed from herd”. The death rate variable refers to the number of animals that died during the survey period. In the cattle data table, this variable is labelled as “Cattle deaths” and in the goat and sheep data tables, it is labelled as “Goats/Sheep deaths”. The count variable refers to the total number of animals in the herd at the time of the survey. In the cattle data table, this variable is labelled as “Total cattle in herd” and in the goat and sheep data tables, it is labelled as “Total goats/sheep in herd”.

3.1.2 Soil Moisture Data

On March 31, 2020, the Soil Moisture Active Passive (SMAP) Level 4 soil moisture data was made public. SMAP is a NASA satellite project that offers measurements of the freeze-thaw state and soil moisture on a global scale, the SMAP satellite was launched on January 31, 2015. The Level 4 soil moisture product, which contains estimates of soil moisture at a resolution of 9 km and a temporal resolution of 3 hours per day, is the highest level of processing for the SMAP soil moisture data. The SMAP Level 3 soil moisture product, which is created by merging the data from the SMAP L-band radiometer and synthetic aperture radar (SAR), is the source of the Level 4 soil moisture product. For the purpose of increasing the precision of the soil moisture values, the Level 4 product also incorporates additional data sources, such as land surface models. The SMAP mission, a NASA mission that employs an L-band radiometer and an L-band radar to measure soil moisture and the freeze/thaw state of the Earth’s surface, produced SMAP (Soil Moisture Active Passive) Level 4 soil moisture data. The Level 3 soil moisture data product, which offers daily, global soil moisture maps with a spatial precision of about 36 km, is the source of the SMAP Level 4 soil moisture data product. The Level 4 product outperforms the Level 3 product by offering greater accuracy, longer time, and higher spatial resolution. The data taken from the SMAP for use in this project is recorded as daily observations for 11 sampled spatial points running from April 2015 to December 2021. These 11 spatial points are :

- Borolong - Boteti sub-district, located in the North-West District

- Bobonong - Bobirwa sub-district, located in the Central District
- Boteti - Boteti sub-district, located in the North-West District
- Mahalapye - Mahalapye sub-district, located in the Central District
- Serowe - Serowe-Palapye sub-district, located in the Central District
- Kgatleng East - Kgatleng District
- Kgatleng West - Kgatleng District
- Ngwaketse West- Ngwaketse District
- Ngwaketse East- Ngwaketse District
- South East - Southern District
- South West - Kweneng District

3.2 Model Specifications

3.2.1 Modified Plant and Herbivore Model

Before parameter estimation, it is important to simplify the dynamic systems of ODE to eliminate any unnecessary complexity. This will make it easier to estimate the parameters accurately and efficiently. The simplified ODE's can also provide insight into the behavior of the system and help to identify important parameters that should be estimated. Dynamic model simplification is technique used to reduce the complexity of our P-H model, which will make it easier to estimate parameters. This technique involves simplifying the model by reducing the number of parameters, simplifying the functional form of the model equations, or both. Below is the simplified P-H model (See chapter 2 for additional details):

$$\begin{aligned}\frac{dP}{dt} &= r(t)P \left[1 - \frac{P}{K} \right] - \left(\frac{a}{b+P} \right) PH \\ \frac{dH}{dt} &= H \left[c \left(\frac{a}{b+P} \right) P - D(H) - \mu(H) \right],\end{aligned}\tag{3.1}$$

where $P(0), H(0) > 0$ and $r(t)$ is the plant growth rate at time t , which is defined by:

$$r(t) = r(\theta(t), m) = m + [1 - m] U_r(\theta(t)) \beta_r(\theta(t)),$$

here $\theta(t)$ is the soil moisture at time t and m is the minimum plant growth rate, $U_r(\theta(t))$ and $\beta_r(\theta(t))$ are given by

$$U_r(\theta(t)) = \exp \left[-(\theta(t) - \theta_c)^2 / 2\pi\sigma(\theta)^2 \right],$$

and

$$\beta_r(\theta(t)) = \begin{cases} 1, & \theta(t) \geq \theta_c \\ \frac{\theta(t) - \theta_w}{\theta_c - \theta_w}, & \theta_w < \theta < \theta_c \\ 0, & \theta(t) \leq \theta_w, \end{cases}$$

respectively. With θ_c being the critical soil moisture, θ_w wilting point soil moisture and $\sigma(\theta)^2$ is the variance of the soil moisture data.

The model depicts the interplay between a prey population (plant) and a predator population (livestock), whereby the prey population's growth rate is constrained by its carrying capacity (K) and predation by

the livestock population. The availability of prey and the rate of harvesting both constrain the rate of growth of the predator population. The model outlines a number of parameters, including the intrinsic growth rate of the prey population ($r(t)$), the constant livestock population's predation feeding rate (a), the carrying capacity (K), the harvesting rate ($\mu(H)$), the rate at which prey biomass is converted into predator biomass (c), and the livestock population's mortality rate due to natural causes, ($D(H)$).

The first parameter to be modified is the livestock population's predation feeding rate (a) which is assumed to be constant. The intrinsic plant growth rate ($r(t)$) which has three impeded functions within it, these function are $U_r(\theta)$, $\beta_r(\theta)$ and m (which is assumed to be constant) are all stated above. Focusing on $U_r(\theta)$, $\beta_r(\theta)$ which are both functions of the soil moisture (θ), this variable can be taken from the satellite data from the SMAP Level 4 soil moisture data with spatial resolution of $36 \times 36 \text{ km}^2$.

The density dependent herbivore death rate, is given as a function stated in equation (2.5) and it is bounded as follows $d_1 \leq D(F(P)) \leq d_2$, where d_1, d_2 are the minimum and maximum death rates respectfully. We propose the modification of this parameter to be a variable rather than a function as the death rate data for livestock (cattle, goats and sheep) exists and dates as far back as 2017 to 2019. This change would reduce the number of parameters to be estimated by dropping both d_1 and d_2 thus reducing the complexity of the P-H model stated in equation (2.4). When formulating the the death rate for herbivores, it is important to note that there are three groups of animals that make up herbivores. Thus, the death rate should account for the death rate of each of these groups, below is the density weighted average herbivore death rate :

Let $S(t), C(t), G(t), Hr(t), D(t)$ be the sheep, cattle, goat, horse and donkey population density's from our data for a particular year such that $H(t) = [S(t) + C(t) + G(t) + Hr(t) + D(t)]$.

$$D(t) = [\alpha_1 d(t)_{cattle} + \alpha_2 d(t)_{goats} + \alpha_3 d(t)_{sheep} + \alpha_4 d(t)_{horse} + \alpha_5 d(t)_{donkey}],$$

where $\alpha_1 = \frac{C(t)}{H(t)}$, $\alpha_2 = \frac{S(t)}{H(t)}$, $\alpha_3 = \frac{G(t)}{H(t)}$, $\alpha_4 = \frac{Hr(t)}{H(t)}$ and $\alpha_5 = \frac{D(t)}{H(t)}$ are the weights assigned to each animal type such that $\sum_{r=1}^5 \alpha_r = 1$. The individual death rates for each of these groups are represented by $d(t)_{cattle}$, $d(t)_{goats}$ and $d(t)_{sheep}$.

The harvest rate is given as a function of two parameters μ and μ_0 in equation (2.4). Applying the same weighting method used above to this parameter and using the take-off values found in the data set, we can change the harvest rate into a variable as follows:

$$\mu(t) = [\alpha_1 \mu^*(t)_{cattle} + \alpha_2 \mu^*(t)_{goats} + \alpha_3 \mu^*(t)_{sheep} + \alpha_4 \mu^*(t)_{horse} + \alpha_5 \mu^*(t)_{donkey}],$$

with each individual take-off rates for each of these groups represented by $\mu^*(t)_{cattle}$, $\mu^*(t)_{goats}$, $\mu^*(t)_{sheep}$, $\mu^*(t)_{horse}$ and $\mu^*(t)_{donkey}$. The weights α_r are the same as the ones used for the death rates.

Livestock was assumed to reproduce naturally without human intervention in equation (2.4), and was accounted for by the term $\left(\frac{H}{h+H}\right)$, where h is the allee constant. In our case, we assume farmer intervention is present in the forms of artificial insemination and bull swapping, therefore h is taken to be 0. Setting the allee effect to 0 results in $\left(\frac{H}{h+H}\right) = 1$. Thus further reducing the complexity by eliminating a parameter.

Thus the parameters to be estimated as follows m, a, c and b with K being assumed to be the same as in [3].

3.2.2 Numerical ODE Solver

The fourth order Runge-Kutta (RK4) algorithm is the chosen numerical solver and the justifications for using the RK method to solve the system of ODEs is that the RK method is a higher-order method that can provide accurate solutions for systems of ODEs, this was showcased in chapter 2 when comparing it to the Euler method. The accuracy of the method can be controlled by adjusting the step size and the order of the method.

The RK method is a stable method, which means that the numerical solution does not become unstable or divergent as the simulation progresses. This is important for the long-term stability of the simulation. This method can be used to solve a wide range of systems of ODEs, including stiff systems that are difficult to solve using other numerical methods.

The RK4 algorithm can be computationally efficient, especially when combined with adaptive step size control. The run time of the solutions generated in chapter 2 showed that the RK4 was computationally quicker than the Euler method. This allows for efficient use of computational resources and faster simulation times. It is a robust method that can handle a wide range of initial conditions and parameters. Overall, the RK method is a reliable and flexible numerical method for solving systems of ODEs thus it is the ideal solver for our P-H dynamic model.

3.3 Optimization Procedure

Optimization of our research problem involves the process of finding the best solution (minimum) for a given problem, subject to a set of constraints.

3.3.1 Objective Function

Firstly, we define the objective function which is the nonlinear least squares equation that needs to be optimized below

$$e = \sum_{t=1}^n \left[\frac{(P(t)_{obs} - P(t)_{est})^2}{P(t)_{obs}^2} + \frac{(H(t)_{obs} - H(t)_{est})^2}{H(t)_{obs}^2} \right],$$

where $P(t)_{obs}$ and $P(t)_{est}$ are the observed and estimated plant biomass, respectively. Similarly $H(t)_{obs}$ and $H(t)_{est}$ are the livestock observed and estimated population weights.

The nonlinear least squares equation above has been modified by dividing each contributing group by the square of its observed value. This normalization is useful as it can help to avoid numerical instability or overflow issues that can arise when working with equations or data that have very large or small values which is the case with our data. It also makes it easier to compare the relative importance or contribution of different variables or coefficients in an equation, and to interpret the results in a meaningful way.

According to the Food and Agriculture Organization (FAO), an agency of the United Nations [27], the average live weight of mature cattle in Botswana is around 400-450 kg, with some breeds such as the Tswana and Ngamiland being larger and heavier, (62). For goats the average live weight of mature goat in Botswana is around 25-30 kg, although this can vary depending on the breed and gender of the goat. While the average live weight of mature sheep, donkey and horse in Botswana is 35-40, 200-400 and 700-1000 kg respectfully. It should be noted that these weights are averages, and actual weights can vary depending on factors such as age, gender, breed, and feeding practices. Additionally, there may be regional or local variations in weight due to differences in environmental conditions and management practices. According to [15], Botswana has a land area of approximately 581,730 square kilometers.

In order to convert the different yearly animal counts into yearly livestock data with the units in (kg), we simply multiply each animal groups yearly count by their corresponding lower bound average weight. The sum of these values then gives the observed yearly livestock weights which are referred to as $H(t)_{obs}$.

3.3.2 Optimization Algorithm

After choosing the objective function above, the next step is to choose an appropriate optimization algorithm that can be used to find the solution. We have chosen the **function minimization with constraints** also known as *fmincon*. It is a popular optimization algorithm in MATLAB that performs constrained optimization using nonlinear programming and is widely used for solving a variety of optimization problems. It is based on the interior-point method, which is an iterative optimization method that works by solving a sequence of linearized subproblems. It can handle both linear and nonlinear constraints, as well as nonlinear objective functions which makes it ideal for handling our nonlinear least squares objective function.

We utilized interior point algorithms within *fmincon* to solve the optimization problem by moving from the interior of the feasible region towards the optimal solution, rather than by following the boundaries of the feasible region as is done in other algorithms. This is accomplished by adding a barrier function to the objective function, which creates a penalty for being near the boundaries of the feasible region. The optimization algorithm then minimizes this modified objective function, subject to the constraints we set out during parameter estimation.

The algorithm starts at an initial point that lies in the interior of the feasible region, and then moves towards the optimal solution by following a path of feasible points that satisfies the constraints. At each iteration, the algorithm finds the search direction that minimizes the barrier function subject to the constraints. The step size is then determined using a line search method, and the algorithm moves to the next point on the feasible path.

In the context of our nonlinear optimization problems, line search methods will be used to minimize the objective function subject to constraints set out. The objective function above, is evaluated at the current iteration point, and the algorithm will determine the search direction based on the gradient or Hessian of the function. The line search procedure will then find the optimal step size along this direction, which is used to update the current point to a new point that is closer to the optimal solution.

fmincon is subject to the restriction of only identifying a local minimum solution to the optimization function, therefore Multistart optimization technique is used to find the global minimum for the nonlinear least squares function that has multiple local minima. It involves running the optimization algorithm multiple times with different starting points, in order to increase the chances of finding the global minimum. Multistart optimization involves randomly selecting a set of starting points from the domain of the objective function and running the optimization algorithm (*fmincon*) from each of them. The algorithm then returns the solution with the lowest objective function value among all the runs. By running the optimization algorithm from different starting points, the chances of finding the global minimum are increased. To insure that the random sample is spread out evenly across the domain of the objective function then we can set a loop of re-sampling for multiple multistats runs and compare the lowest values from each iteration of multistart sample.

3.4 Forecasting Technique

The proposed methodology for time series forecasting is based on a Neural Network Autoregressive (NNAR) model. The NNAR model combines the power of artificial neural networks (ANNs) with the

autoregressive concept. The autoregressive component allows the model to capture the temporal dependencies in the time series data, while the neural network component enables the model to learn complex patterns and non-linear relationships.

The NNAR model consists of two main components: an autoregressive layer and a neural network layer. The autoregressive layer takes into account the previous observations of the time series to predict the next value. The neural network layer then processes the autoregressive outputs and learns the non-linear relationships between the predictors and the target variable. The architecture of the NNAR model can be represented mathematically as follows:

$$y_t = f \left(\phi_1 y_{t-1} + \phi_2 y_{t-2} + \dots + \phi_p y_{t-p} + \sum_{i=1}^n w_i x_{t,i} \right),$$

where y_t is the target variable at time t , y_{t-1} represents the lagged values of the target variable, ϕ_i are the autoregressive coefficients, $x_{t,i}$ denotes the input predictors at time t , w_i are the weights associated with the input predictors, and $f(\cdot)$ is the activation function of the neural network layer. To train the NNAR model, the data is divided into a training set and a validation set. The training set is used to optimize the model's parameters, including the auto-regressive coefficients and the weights of the neural network layer. The validation set is employed to monitor the model's performance and prevent over-fitting. The model parameters are updated iteratively using an optimization algorithm, such as gradient descent, to minimize a loss function, such as mean squared error (MSE). The optimal parameters are determined based on the performance of the model on the validation set.

After training the NNAR model, it can be used for time series forecasting. Given a set of future predictor values, the model predicts the corresponding target variable using the learned autoregressive coefficients and neural network weights. The forecast values can be obtained by recursively applying the model equation with the lagged values of the target variable and the future predictor values. The accuracy of the forecasts can be evaluated using various metrics, such as mean absolute error (MAE) or root mean squared error (RMSE). The performance of the NNAR model can be compared with exponential smoothing (ETS) forecasting methods to assess its effectiveness in capturing the underlying patterns and predicting future values accurately.

4. Parameter Estimation & Optimization

This chapter focuses on the topic of parameter estimation and optimization. The chapter begins by addressing the issue of limited availability, measurement errors, and incomplete records in Botswana. These issues are tackled in the discussion on the re-characterization of the available livestock count data, specifically addressing two important techniques: imputation of missing values and time indexing manipulation. These techniques are essential for preparing the data for parameter estimation and optimization processes.

The next section of the chapter delves into the details of a specialized Runge-Kutta algorithm for the 4th order of this method. The utilization of the specialized Runge-Kutta method as an integrator is justified because of the inclusion of time series data for soil moisture in the function $r(t)$ in the model. The documentation of the algorithm covers various aspects of this method, including its application in first, second, third, and fourth order scenarios. Additionally, the chapter explores time conversions related to the codes outputs which are converted from weekly to yearly values through a sequence of weighting factors.

Due the non-convexity in the optimization problem brought about by the relationship of parameters in the Plant-Herbivore dynamic model, the chapter shows how the global optimization algorithms (multi-start) paired with the numerical integrator (Runge-Kutta order 4) performed. As discussed in the methodology the Non-Linear Least squares (NLLS) is used as the error function and *fmincon* is the local solver that will be initialized from various random points by the multi-start algorithm. The results of each iteration of Multi-start are discussed in two sections namely, initial and refined iteration results. The first run is described, followed by the second run. These results provide insights into the performance of the parameter estimation and optimization techniques applied in the study.

The chapter then progresses to the refined iteration results, which focus on the refinement and improvement of the initial outcomes. The first run of the refined iteration results is discussed, followed by the second run. Furthermore, the chapter introduces a third run of the refined iteration results, providing a comprehensive view of the iterative process and its impact on parameter estimation and optimization.

4.1 Re-characterization of Data

In the Methodology, three data sets are listed as relevant sources for observation. It is however necessary to reconstruct each data set addressing issues such as missing values, unit conversions and variable transformations. The first issue to tackle is that of missing values, from the livestock data the livestock counts for 1991,1992, 1994, 2016 and 2018 are missing. This represents are only 17% of the overall data thus using the 83% available data a learning imputation method was developed to impute the missing values.

4.1.1 Imputation of Missing Values

This choice of imputation approach is known as “model-based imputation”. By training the model on the available livestock data, including the rows with missing values, and then using the trained model to forecast the missing values, the fitnsemble algorithm was used to impute missing values in the livestock dataset. The data was first preprocessed to indicate which values are missing by replacing

them with NaNs before using `fitrensemble` for imputation. The `fitrensemble` method then trains a model on the preprocessed livestock data. Finally, it generates forecasts for the missing numbers in your initial dataset using the trained model.

To integrate the imputed livestock data with MATLAB's `fitrensemble` function, the imputed data was exported from R and imported into MATLAB. The imputed data is exported to a format like CSV or Excel, which can be easily read by MATLAB. Once imported into MATLAB, the imputed data was used as input for the `fitrensemble` function to perform ensemble learning and predictive modeling tasks. MATLAB's `fitrensemble` can utilize the imputed data alongside other relevant variables to build an ensemble model and make predictions.

By combining the strengths of R's imputation techniques and MATLAB's `fitrensemble` function, we leveraged the advanced imputation capabilities of R and the predictive modeling capabilities of MATLAB to handle missing data and conduct sophisticated analyses on livestock data, leading to more robust insights and informed decision-making. Figure 4.1, shows the imputation code that was developed.

```
datamiss=readtable('herbivoresdata.xlsx', ...
    'Range','N1:Q38',...
    'ReadVariableNames',true);
dataToImpute=datamiss;
rng('default'); %random number generator
tmp = templateTree('Surrogate','on','Reproducible',true);% decision tree template
missingN = ismissing(dataToImpute.N);
% Fit ensemble of regression learners for N
rfN = fitrensemble(dataToImpute,'N','Method','Bag',...
    'NumLearningCycles',200,'Learners',tmp);
datamiss.N(missingN) = predict(rfN,...
    dataToImpute(missingN,:));
%%
missingO = ismissing(dataToImpute.O);
% Fit ensemble of regression learners for O
rfO = fitrensemble(dataToImpute,'O','Method','Bag',...
    'NumLearningCycles',200,'Learners',tmp);
datamiss.O(missingO) = predict(rfO,...
    dataToImpute(missingO,:));
%%
missingP = ismissing(dataToImpute.P);
% Fit ensemble of regression learners for P
rfP = fitrensemble(dataToImpute,'P','Method','Bag',...
    'NumLearningCycles',200,'Learners',tmp);
datamiss.P(missingP) = predict(rfP,...
    dataToImpute(missingP,:));
%%
missingQ = ismissing(dataToImpute.Q);
% Fit ensemble of regression learners for Q
rfQ = fitrensemble(dataToImpute,'Q','Method','Bag',...
    'NumLearningCycles',200,'Learners',tmp);
datamiss.Q(missingQ) = predict(rfQ,...
```

Figure 4.1: Imputation Code

4.1.2 Time Indexing Manipulation

The soil moisture data has 16870 missing readings throughout the 11 spatial points (districts). Rather than imputing these values, an approach of re-indexing the data from daily readings to weekly values yielded more preferential results. From the 2716 daily soil moisture values in each district, a total of 388 weekly readings were attained. The values were partitioned into groups of 7 and then averaged, while taking special care not to include the missing values.

```

%%readtable from Agolame_Sample.csv as provided originally
Alldata=readtable('Agolame_Sample', ...
    'Range','H1:H29876',...
    'ReadVariableNames',false);
%%reshaping the data
Alldata=Alldata{:,~};
smalldata=reshape(Alldata,[],7);
%%writing the data to table which comes out in Agolame_Dat2.csv
writematrix(smalldata,'Agolame_Dat2.csv');
%Every row represents weekly data for all regions
%Then we use =average(if(isnumber(range),(range))) to calculate weekly
%average. The Matlab code >> mean(A(isnan(A)==0)) works for any matrix A
%but not very useful in loops.

```

Figure 4.2: Time Indexing Code

The weekly average for each column is then determined by the code using the algorithm `=average(if(isnumber(range),(range)))`. The reshape function is then used to restructure the data into an matrix named “smalldata.” The writematrix function is then used to write the reshaped data to a new CSV file named “Agolame_Dat2.csv.” The CSV file’s information includes weekly data for various regions, with each row corresponding to a week’s amount of information for all regions. The data is then re-index in the format shown below:

	A	B	C	D	. . .
1	0.0557617	0.0651032	0.0661276	0.0686237	. . .
2	0.0880485	0.0725798	0.0560839	0.0614499	. . .
3	0.0496233	0.0635856	0.0588851	0.0372145	. . .
4	0.0688853	0.0565657	0.0624410	0.0739867	. . .
5	0.1120696	0.1162331	0.0559293	0.0476429	. . .
6	0.0642203	0.0837301	0.1175658	0.0638972	. . .
7	0.0552320	0.0674029	0.0477435	0.0694954	. . .
8	0.0739140	0.1148915	0.0509841	0.0533123	. . .
9	0.0824388	0.0561601	0.0697920	0.0761459	. . .

4.2 Specialized Runge-Kutta Order 4 Algorithm

When solving the systems of equations in (3.1), it was stated that the Runge-Kutta method was the most adequate numerical solver. However, the specialized RK4 is impeded in Matlab packages such as ODE45 which normally offer a black box approach to dealing with such problems, we opted for building an equation based RK4 code specifically to solve our P-H differential equations.

Coding the Runge-Kutta 4th order method is preferable since we aim to understand the specifics of the numerical method implementation and the P-H model has functional coefficients in the form of $r(t)$ which needs to be put into a specific form that can be exploited for computational efficiency. Additionally, by coding the method, more control over the numerical solution procedure is made possible, including the ability to select the step size and convergence tolerance as well as to modify the solver for this particular

problem. Below is a break down of the 4 orders in the code.

4.2.1 First Order

```

if (theta(i) >= theta_c)
    beta1 = 1;
elseif (theta_w < theta(i)) && (theta(i) < theta_c)
    beta1 = ((theta(i)-theta_w)/(theta_c-theta_w));
else
    beta1 = 0;
end %if statement

r1 = (p(1)+ (1-p(1))*exp(-(theta(i)-theta_c)^2/(2*pi*var(theta))))*beta1 ;

k11 = r1*P(i)*(1-P(i)/K)-(p(2)/(p(3)+P(1)))*P(i)*L(i);
k12 = L(i)*(p(4)*(p(2)/(p(3)+P(i)))*P(i)-D)- u;

```

Figure 4.3: First Order Code

There first level of the RK4 is represented as the k_{11} and k_{12} for the plant biomass and the livestock change rates as shown above in Figure 4.3. Importantly, the intrinsic plant growth rate is subject to condition checking for the i^{th} θ so as to capture the correct $\beta(\theta)$ function. The parameters slated for estimation have been defined as indexes ($P(i)$) from the parameter space P . During this first stage the RK function makes use of the i^{th} θ for computations as it is the first division (right hand side) of the space of estimation. This segment of the code directly links to the second level which is discussed in the next section.

4.2.2 Second & Third Order

```

if (0.5*(theta(i) + theta(i+1)) >= theta_c)
    beta2 = 1;
elseif (theta_w < 0.5*(theta(i) +theta(i+1))) && (0.5*(theta(i) +theta(i+1)) < theta_c)
    beta2 = ((0.5*(theta(i) +theta(i+1))-theta_w)/(theta_c-theta_w));
else
    beta2 = 0;
end

r2 = (p(1)+ (1-p(1))*exp(-(0.5*(theta(i) +theta(i+1))-theta_c)^2/(2*pi*var(theta))))*beta2;

k21 = r2*(P(i)+h2*k11)*(1-(P(i)+h2*k11)/K)-(p(2)/(p(3)+(P(i)+h2*k11)))*(P(i)+h2*k11)*(L(i)+h2*k12);
k22 = (L(i)+h2*k12)*(p(4)*(p(2)/(p(3)+(P(i)+h2*k11)))*(P(i)+h2*k11)-D)- u;

if (0.5*(theta(i) +theta(i+1)) >= theta_c)
    beta3 = 1;
elseif (theta_w < 0.5*(theta(i) +theta(i+1))) && (0.5*(theta(i) +theta(i+1)) < theta_c)
    beta3 = ((0.5*(theta(i) +theta(i+1))-theta_w)/(theta_c-theta_w));
else
    beta3 = 0;
end % if

r3 = (p(1)+ (1-p(1))*exp(-(0.5*(theta(i) +theta(i+1))-theta_c)^2/(2*pi*var(theta))))*beta3;

k31 = r3*(P(i)+h2*k21)*(1-(P(i)+h2*k21)/K)-(p(2)/(p(3)+(P(i)+h2*k21)))*(P(i)+h2*k21)*(L(i)+h2*k22);
k32 = (L(i)+h2*k22)*(p(4)*(p(2)/(p(3)+(P(i)+h2*k21)))*(P(i)+h2*k21)-D)- u;

```

Figure 4.4: Second and Third Order Code

Figure 4.4 shows the second level of the RK code, in which $K21$ and $K22$ are the second order of the plant biomass and the livestock change rates. For the third order of the plant biomass and the livestock change rates we have the variables $k31$ and $k32$. It can be noted that $K11$ and $K12$ are embedded within the $k21$ and $k22$ respectfully, this is also true for the second and third order equations showing each level is related to its preceding level. The equality of weights of these orders is apparent in the definitions of the $r2$ and $r3$ functions which are equivalent. These functions use an average of the i^{th} and $(i+1)^{th}$ θ value for both selection of the $\beta(\theta)$ and computation of r in each iteration. This differentiates $r1$ from $r2$ and $r3$. This two levels lead us to the fourth and last order.

4.2.3 Fourth Order

```

if (theta(i+1) >= theta_c)
    beta4 = 1;
elseif ((theta_w < theta(i+1)) && (theta(i+1) < theta_c))
    beta4 = (theta(i+1)-theta_w)/(theta_c-theta_w);
else
    beta4 = 0;
end % if

r4 = (p(1)+ (1-p(1))*exp(-(theta(i+1)-theta_c)^2/(2*pi*var(theta))))*beta4;

k41 = r*(P(i)+h2*k31)*(1-(P(i)+h2*k31)/K)-(p(2)/(p(3)+(P(i)+h2*k31)))*(P(i)+h2*k32)*(L(i)+h2*k32);
k42 = (L(i)+h2*k32)*(p(4)*(p(2)/(p(3)+(P(i)+h2*k31)))*(P(i)+h2*k31)-D)- u;

P(i+1) = P(i) + (h/6)*(k11+ 2*k21+2*k31+k41);
L(i+1) = L(i) + (h/6)*(k12+ 2*k22+2*k32+k42);

```

Figure 4.5: Fourth Order Code

The fourth level of the RK4 code is displayed in Figure 4.5 above, $K41$ and $K42$ are the fourth order of the plant biomass and the livestock change rates. With $r4$ using the left hand side $(i+1)^{th}$ θ value for computation and selection of the $\beta(\theta)$. This level has the same weighting as that of the first level in the computation of the plant biomass and livestock populations. With this code the weekly plant and livestock populations can be numerically estimated.

4.2.4 Time Conversions

The solutions of our modified P-H model are 388 weekly plant biomass and livestock densities for a 36×36 square km spatial segment in each of the 11 spatial points in chapter 3. This presents a problem when comparing this data to the observed livestock densities which are yearly and are country wide values. As such, it is necessary to convert the solutions from weekly regional estimates to yearly country wide estimates. This task has been achieved using the following procedures.

Each of the 11 spatial points has an area segment of 36×36 square km, therefore to get a livestock density estimate for the entire region we calculate the multiplying coefficient (a_i) of each region as the stated below

$$a_i = \frac{A_i}{n_i}, \quad \text{for } i = 1, 2, \dots, 11,$$

where A_i is the land mass of the district containing the i^{th} spatial point and n_i the number of spatial points that fall within the same district as the i^{th} spatial point. Therefore the estimated i^{th} region's t^{th} weekly livestock weight in kg is given by

$$H_i(t)_{esti} = a_i H_i(t),$$

where $H_i(t)$ is the i^{th} region's weekly model livestock density estimate for the t^{th} week measured in (kg/m^2). This equation can be used to create a weighted yearly country wide livestock estimate by

For the general case this includes 2016, 2017 and 2018:

$$\bar{H}(k)_{esti} = \frac{1}{52} \sum_{i=1}^{11} \sum_{t=(k-1)52+1}^{k \times 52} a_i H_i(t).$$

Since the satellite data runs from March 2015, we have adjusted for the year to be averaged by 35 weeks that make up 2015. Therefore for $k = 1$ we have :

$$\bar{H}(1)_{esti} = \frac{1}{35} \sum_{i=1}^{11} \sum_{t=1}^{35} a_i H_i(t).$$

Another factor to consider is the relative contribution factor of each region to the overall country livestock population density. This factor can be calculated using information from the 2019 annual livestock census, in which animal counts for each region was given for the year 2019. Thus the formulation of the contribution fact is

$$w_i = \frac{H(i)_{observed}}{H_{2019}}, \quad \text{for } i = 1, 2, \dots, 11, \quad (4.1)$$

where $H(i)_{observed}$ is the observed livestock density of the i 'th region in 2019 and H_{2019} is the total country wide livestock density for 2019. For the 2019 ($k=4$) case:

$$\bar{H}(5)_{esti} = \frac{1}{52} \sum_{i=1}^{11} \sum_{t=209}^{260} a_i w_i H_i(t).$$

```
%subsetting the 11 regions livestock matrix into years
LR15 = z(:,1:35);
LR16 = z(:,36:88);
LR17 = z(:,89:140);
LR18 = z(:,141:193);
LR19 = z(:,194:245);

% getting the livestock estimate for the specific year using land weights
% and livestock density weights

L2015 = mean(0.078*50*LR15(1,:) + 0.058*37*LR15(2,:) + 0.068*50*LR15(3,:) + 0.078*37*LR15(4,:) + 0.1*38*LR15(5,:) + 0.064*3.5*LR15(6,:) + 0.064*3.5*LR15(7,:) + 0.018*2*LR15(8)
L2016 = mean(0.078*50*LR16(1,:) + 0.058*37*LR16(2,:) + 0.068*50*LR16(3,:) + 0.078*37*LR16(4,:) + 0.1*38*LR16(5,:) + 0.064*3.5*LR16(6,:) + 0.064*3.5*LR16(7,:) + 0.018*2*LR16(8)
L2017 = mean(0.078*50*LR17(1,:) + 0.058*37*LR17(2,:) + 0.068*50*LR17(3,:) + 0.078*37*LR17(4,:) + 0.1*38*LR17(5,:) + 0.064*3.5*LR17(6,:) + 0.064*3.5*LR17(7,:) + 0.018*2*LR17(8)
L2018 = mean(0.078*50*LR18(1,:) + 0.058*37*LR18(2,:) + 0.068*50*LR18(3,:) + 0.078*37*LR18(4,:) + 0.1*38*LR18(5,:) + 0.064*3.5*LR18(6,:) + 0.064*3.5*LR18(7,:) + 0.018*2*LR18(8)
L2019 = mean(0.078*50*LR19(1,:) + 0.058*37*LR19(2,:) + 0.068*50*LR19(3,:) + 0.078*37*LR19(4,:) + 0.1*38*LR19(5,:) + 0.064*3.5*LR19(6,:) + 0.064*3.5*LR19(7,:) + 0.018*2*LR19(8)

%subsetting the 11 regions plant biomass matrix into years
PR15 = y(:,1:35);
PR16 = y(:,36:88);
PR17 = y(:,89:140);
PR18 = y(:,141:193);
PR19 = y(:,194:245);

% getting the plant estimate for the specific year

P2015 = mean(100*PR15(1,:) + 113*PR15(2,:) + 100*PR15(3,:) + 113*PR15(4,:) + 113*PR15(5,:) + 7*PR15(6,:) + 7*PR15(7,:) + 4*PR15(8,:) + 4*PR15(9,:) + 13*PR15(10,:)+ 13*PR15(11)
P2016 = mean(100*PR16(1,:) + 113*PR16(2,:) + 100*PR16(3,:) + 113*PR16(4,:) + 113*PR16(5,:) + 7*PR16(6,:) + 7*PR16(7,:) + 4*PR16(8,:) + 4*PR16(9,:) + 13*PR16(10,:)+ 13*PR16(11)
P2017 = mean(100*PR17(1,:) + 113*PR17(2,:) + 100*PR17(3,:) + 113*PR17(4,:) + 113*PR17(5,:) + 7*PR17(6,:) + 7*PR17(7,:) + 4*PR17(8,:) + 4*PR17(9,:) + 13*PR17(10,:)+ 13*PR17(11)
P2018 = mean(100*PR18(1,:) + 113*PR18(2,:) + 100*PR18(3,:) + 113*PR18(4,:) + 113*PR18(5,:) + 7*PR18(6,:) + 7*PR18(7,:) + 4*PR18(8,:) + 4*PR18(9,:) + 13*PR18(10,:)+ 13*PR18(11)
P2019 = mean(100*PR19(1,:) + 113*PR19(2,:) + 100*PR19(3,:) + 113*PR19(4,:) + 113*PR19(5,:) + 7*PR19(6,:) + 7*PR19(7,:) + 4*PR19(8,:) + 4*PR19(9,:) + 13*PR19(10,:)+ 13*PR19(11)
```

Figure 4.6: Time Conversion Code

The values for plant biomass for each region and year are contained in the matrix y , whereas the values for

livestock density for each region and year are contained in the matrix z . The livestock density and plant biomass numbers are then filtered for each specific year (2015–2019) using indexing, and the mean for each region is calculated using weighted averages. The code contains hard-coded values for each region's weights and the biomass of animals and plants. L2015-L2019 and P2015-P2019, respectively, contain the derived estimations for the biomass of livestock and plants for each year.

4.3 Initial Iterations Results

This sections deals with the actual estimation of parameters from the code developed. We start of with the solution. Each of the 4 parameters for estimation have been initialized from 0.5 ,0.75, 20.0 and 0.25 for m, a, c and b , respectively. The first iteration of the parameter estimation code with 10 random starts for `fmincon.m` is given in the next Subsection.

4.3.1 First run

```
Initial Objective: 1563.4521
```

MultiStart completed some of the runs from the start points. 6 out of 10 local solver runs converged with a positive local solver exit flag.

```
Final Objective: 81.3626
mstar: 0.79109
astar: 258.9894
bstar: 190.1992
cstar: 0.62262
```

The result the optimization problem is shown above, the nonlinear least squares objective function has an initial value of 1563.4521. MultiStart, the solver that was utilized, completed some of the runs from the starting locations. Six of the 10 iterations of the local solver (`fmincon`) converged with an acceptable local solver exit flag. After optimization, The final objective obtained after optimization was 81.3626. This represents a factor decrease of 19.22. The optimization results further give the values for the parameters `mstar`, `astar`, `bstar` and `cstar`, which are bounded with the constraints. To further validate these results the number of initialization points was increased from 10 to 100 and then 1000.

4.3.2 Second run

```
Initial Objective: 1563.4521
```

MultiStart completed some of the runs from the start points. 47 out of 100 local solver runs converged with a positive local solver exit flag.

```
Final Objective: 5.8666
mstar: 0.42314
astar: 227.491
bstar: 286.4273
cstar: 0.48007
```

The second output shows that MultiStart completed 100 runs and 47 of them converged with a positive local solver exit flag, and the final objective value was 5.8666. The parameter estimate `astar` is roughly similar to the first run estimate with the exception of The parameters `mstar`, `bstar` and `cstar` which show

a significant decrease, thus displays a jumping effect in the optimization iterations. Overall, the second run outperformed the first run due to more local solver runs converging and a lower ultimate objective value.

4.4 Refined Iteration Results

To study the variation in the results of our optimization, the following adjustments were taken.

1. Objective function: by not factoring in the plant estimation variations and only focusing on the livestock sum of squares. Setting the plant variation expression $\frac{(P(t)_{obs}-P(t)_{est})^2}{P(t)_{obs}^2} = 0$, the objective function can be changed to only reflect the variation in the livestock data as

$$e = \sum_{t=1}^n \left[\frac{(H(t)_{obs}-H(t)_{est})^2}{H(t)_{obs}^2} \right],$$

where $H(t)_{obs}$ and $H(t)_{est}$ are the livestock observed and estimated population weights.

2. Adjustment of starting points: the selection of optimal starting points that are in a reasonable neighbourhood of the the previous year's livestock totals.
3. Adjustment of livestock death rates due to drought conditions in 2017-2019.
4. Boundaries : narrowing of parameter boundaries in order reduce iteration times and increase local solver convergences.
5. Weighting factors: the removal of w_i in the 2019 weighting summations equations.

4.4.1 First run

Initial Objective: 0.93729

MultiStart completed some of the runs from the start points. 49 out of 100 local solver runs converged with a positive local solver exit flag.

Final Objective: 0.83915
mstar: 0.51655
astar: 0.39034
bstar: 278.7595
cstar: 1.6264e-07

This run is based on the first 3 adjustments with the optimization process yielding significant outcomes. The initial objective value, set at 0.93729, served as the starting point for the optimization. From the multiple runs initiated by MultiStart, 49 out of 100 local solver runs successfully converged to optimal solutions. These converged runs were accompanied by positive local solver exit flags, indicating successful convergence. The final objective value achieved after the optimization process was 0.83915, representing a substantial improvement from the initial objective value.

4.4.2 Second run

Initial Objective: 0.84709

MultiStart completed some of the runs from the start points. 498 out of 500 local solver

runs converged with a positive local solver exit flag.

```
Final Objective: 0.83216
mstar: 0.065361
astar: 499.2778
bstar: 1.3051
cstar: 0.0010726
```

The initial objective value, set at 0.84709, served as the starting point for the optimization. From the multiple runs initiated by MultiStart, 498 out of 500 local solver runs successfully converged to optimal solutions. The final objective value achieved after the optimization process was 0.83216, representing an insignificant improvement from the initial objective value.

4.4.3 Third run

```
Initial Objective: 0.20624
```

MultiStart completed some of the runs from the start points. 99 out of 100 local solver runs converged with a positive local solver exit flag.

```
Final Objective: 0.12908
mstar: 0.51537
astar: 0.00093154
bstar: 268.4473
cstar: 0.00013806
```

In this code run the initial objective value, set at 0.20624, represented the starting point for the optimization. From the multiple runs initiated by MultiStart, 99 out of 100 local solver runs successfully converged to optimal solutions. These runs exhibited positive local solver exit flags, indicating successful convergence. The final objective value achieved after the optimization process was 0.12908, demonstrating a significant improvement from the initial objective value with a changing in the weighting factors. This reduction in the objective function signifies an enhanced fit of the model to the observed data.

4.4.4 Fourth runs

```
Initial Objective: 0.06239
```

MultiStart completed the runs from all start points. All 10 local solver runs converged with a positive local solver exit flag.

```
Final Objective: 0.0034832
mstar: 0.32462
astar: 0.12357
bstar: 1915.0084
cstar: 0.0087768
```

```
Initial Objective: 0.06239
```

MultiStart completed some of the runs from the start points. 97 out of 100 local

solver runs converged with a positive local solver exit flag.

```
Final Objective: 0.0034829
      mstar: 0.89334
      astar: 0.090594
      bstar: 1488.0572
      cstar: 0.00022837
```

In this code run the initial objective value, set at 0.06239, represented the starting point for the optimization. From the multiple runs initiated by MultiStart, with a 10 of the solvers all converging in the first output and 97 out of 100 local solver runs in the second output successfully converged to optimal solutions. These runs exhibited positive local solver exit flags, indicating successful convergence. The final objective value achieved after the optimization process was 0.0034829, demonstrating a significant improvement from the initial objective value with a changing in the weighting factors. This reduction in the objective function signifies an enhanced fit of the model to the observed data.



5. Forecasting Plant-Herbivore Dynamics Using Machine Learning Techniques

ETS (Error-Trend-Seasonality) and neural networks have recently been combined as a potential method for predicting in a variety of areas. The integration of forecasting methods can take different forms, for this thesis it involve a sequential approach, where the outputs of forecasting model is used as an input or a conditioning factor for the dynamic model. This approach offers a number of benefits which include increased accuracy, improved robustness against uncertainties, and enhanced decision-making capabilities. By leveraging the strengths of multiple methods, integrated forecasting can provide a more complete and reliable picture of future trends This chapter aims to give readers a thorough understanding of the potential of using Ordinary differential equations (ODEs), ETS (Error-Trend-Seasonality) and neural networks together for forecasting of plant-herbivore dynamics.

We explore the application of machine learning techniques, specifically soil moisture forecasting and harvest and death rate forecasting, in order to be able to forecast the plant-livestock dynamics. In the methodology data on the soil moisture for 11 regions, harvest and death rates in Botswana are incorporated within the dynamic model, thus to obtain forecasts from the model it is essential that to come up with reliable forecasts for the variables stated. We compare the outputs of two popular methods, Exponential Smoothing (Error-Trend-Seasonality) Modeling and Neural Network Auto-regression modeling (NNAR), and assess their forecasting accuracy.

5.1 Soil Moisture Forecasting

5.1.1 Exponential Smoothing (Error-Trend-Seasonality) Modeling

In this subsection, we fit Error-Trend-Seasonality (ETS) models to each of the of the regions soil moisture data sets and discuss their respective outputs. Below is a table summarising the outputs of our ETS model.

KEY	σ^2	$\log - lik$	AIC	AICc	BIC	MSE	AMSE	MAE
A	0.155	171	-337	-336	-325	0.00109	0.00109	0.301
B	0.103	237	-468	-468	-456	0.00079	0.00079	0.250
C	0.150	185	-365	-365	-353	0.00103	0.00103	0.298
D	0.176	163	-315	-314	-291	0.00125	0.00126	0.320
E	0.136	215	-423	-423	-411	0.00088	0.00091	0.269
F	0.150	187	-364	-364	-345	0.00107	0.00107	0.310
G	0.155	192	-378	-378	-366	0.00099	0.00100	0.298
H	0.145	178	-350	-350	-339	0.00109	0.00110	0.289
I	0.123	210	-414	-414	-402	0.00093	0.00094	0.264
J	0.109	255	-497	-497	-474	0.00072	0.00073	0.260
K	0.149	142	-274	-274	-254	0.00137	0.00135	0.297

Table 5.1: Results of ETS Analysis

The ETS (Error-Trend-Seasonality) output shows the results of a time series analysis conducted on eleven different models for each regions soil moisture. The models are labeled A through K, and each model is evaluated based on several measures of fit.

Model Selection Criteria

The data have been modeled using the ETS model in R, with the output presented in Table 5.1. The table includes various parameters such as the key, model, σ^2 , log-likelihood, AIC (Akaike Information Criterion), AICc (corrected Akaike Information Criterion), BIC (Bayesian Information Criterion), MSE (Mean Squared Error), AMSE (Adjusted Mean Squared Error), and MAE (Mean Absolute Error). The key column identifies each individual time series being modeled. The model column indicates that all time series have been modeled using the ETS model with a single exponential smoothing component. The σ^2 column represents the variance of the error term in the model.

The log-likelihood value is the log-likelihood of the data given the parameters of the model, and AIC and BIC are measures of the goodness of fit of the model. AICc is the corrected AIC, which is appropriate when the sample size is small. Lower AIC, AICc, and BIC values indicate a better fit. The MSE represents the average of the squared errors, and the AMSE is the adjusted version of the MSE that takes into account the number of parameters in the model. The MAE is the mean absolute error, which is the average of the absolute differences between the predicted and observed values.

Based on these measures, model K appears to be the best fit for the data. It has the lowest AIC, AICc, and BIC values, indicating that it has the best balance between model complexity and fit to the regions data. Additionally, it has the lowest MSE and AMSE values, indicating that it has the best fit for the regions soil moisture data overall. A second modeling technique is explored in the following subsection.

5.1.2 Neural Network Auto-regression Modelling (NNAR)

Key	Model	σ^2
A	NNETAR(value1)	0.000250
B	NNETAR(value2)	0.000408
C	NNETAR(value3)	0.0000893
D	NNETAR(value4)	0.000101
E	NNETAR(value5)	0.000473
F	NNETAR(value6)	0.000521
G	NNETAR(value7)	0.00000429
H	NNETAR(value8)	0.000562
I	NNETAR(value9)	0.000180
J	NNETAR(value10)	0.000395
K	NNETAR(value11)	0.000379

Table 5.2: NNETAR Model σ^2 Values

The output above shows the results of applying the NNETAR(value) model to 11 different time series data represented by the keys A to K. The σ^2 column represents the estimated variance of the residuals from the model, which is a measure of the goodness of fit of the model to the data.

The results indicate that the model performed differently on each of the 11 time series, as evidenced

by the varying σ^2 values. For instance, keys G and J have the smallest σ^2 values, indicating that the NNAR(value) model fit the data well and can be used for accurate forecasting. Conversely, keys A, B, E, F, H, I, and K have larger σ^2 values, indicating that the data is erratic and the model may need to be modified to incorporate log-based transformations to fit the data better.

5.1.3 Comparison of Model Outputs

In comparing these outputs, we can see that the two models differ in their approach to forecasting. The NNAR model uses a neural network to learn patterns in the data and make predictions based on those patterns. The ETS model, on the other hand, uses a statistical method that relies on the previous observation and a set of smoothing parameters to make predictions. Below is a table summarizing the outputs of a sample of the first 4 regions with each of these approaches.

Model	A- NNAR(12,6)	B - NNAR(8,4)	C- NNAR(16,8)	D-NNAR(16,8)
σ^2	0.0002491	0.0004072	8.906e-05	0.0001004
Weights	85	41	145	145

Table 5.3: NNAR Output Summaries for Regions A,B,C & D

The first table outputs are the result of fitting a neural network autoregression model (NNAR) to the soil moisture data for region A, B, C and D, but with different numbers of hidden units and weights. The models were fitted using linear output units, and the estimated variance of the residuals (σ^2) for the four models were 0.0002491, 0.0004072, 8.906e-05, and 0.0001004, respectively. The first and fourth models have the same architecture of a 16-8-1 network, while the second and third models have 8-4-1 and 16-8-1 architectures, respectively.

The second set of outputs are from exponential smoothing state space models (ETS) fitted on the regional soil moisture data, but with different values of smoothing parameter (alpha) and initial states ($l[0]$). The estimated variance of the residuals (σ^2) for the four models were 0.1554, 0.1028, 0.1498, and 0.1763, respectively. The first three models are of the form M,N,N, which means the model uses multiplicative errors (M) and no trend or seasonality (N,N), while the fourth model is of the form M,Ad,N, which means it also includes an additive trend component (Ad).

Model	A-ETS(M,N,N)	B-ETS(M,N,N)	C-ETS(M,N,N)	D-ETS(M,Ad,N)
Alpha	0.05757763	0.05512469	0.04646635	0.04556309
σ^2	0.1554	0.1028	0.1498	0.1763
AIC	-336.5255	-468.2840	-364.7488	-314.6345
AICc	-336.4630	-468.2215	-364.6863	-314.4140
BIC	-324.6425	-456.4009	-352.8658	-290.8684

Table 5.4: ETS Output Summaries for Regions A,B,C & D

Looking at region A's soil moisture models, both NNAR and ETS we can formulate comparisons.

Series: value
Model: NNAR(12,6)
Average of 20 networks, each of which is

```
a 12-6-1 network with 85 weights
options were - linear output units
sigma^2 estimated as 0.0002491
```

We observe that the model has 12 input nodes, 6 hidden nodes, and 1 output node, with linear output units as shown in the output above. The reported value is the forecast for the next period using this model. The model parameters include 85 weights, which were estimated based on an average of 20 networks. The estimated variance of the errors (σ^2) is 0.0002491.

```
Series: value
Model: ETS(M,N,N)
Smoothing parameters:
alpha = 0.05757763
Initial states:
1[0]
0.08946941
sigma^2: 0.1554
AIC      AICc      BIC
-336.5255 -336.4630 -324.6425
```

The comparative model from the second table is the result of fitting an exponential smoothing state space model (ETS) with the MNN parameterization. The model has a smoothing parameter alpha of 0.05757763, an initial state value of 0.08946941, and an estimated variance of the errors (σ^2) of 0.1554. The reported values include the AIC, AICc, and BIC information criteria.

Comparing the two sets of outputs, it is clear that the neural network models generally have lower variance estimates than the ETS models, which suggests better goodness-of-fit to the data. This suggests that the NNAR model may be better able to capture the underlying patterns in the data and make accurate predictions, while the ETS model may be more prone to error due to the relatively large variance. We can now look into how each of these modelling techniques forecasts the soil moisture data from the 11 regions.

5.1.4 Comparisons of Model Forecasts

To compare the variability of forecasts of the ETS and NNAR models, plots of the 10 week forecasts and their corresponding prediction intervals have been obtained below. The prediction intervals indicate the uncertainty in the forecasts and their width reflects the variability of the model.

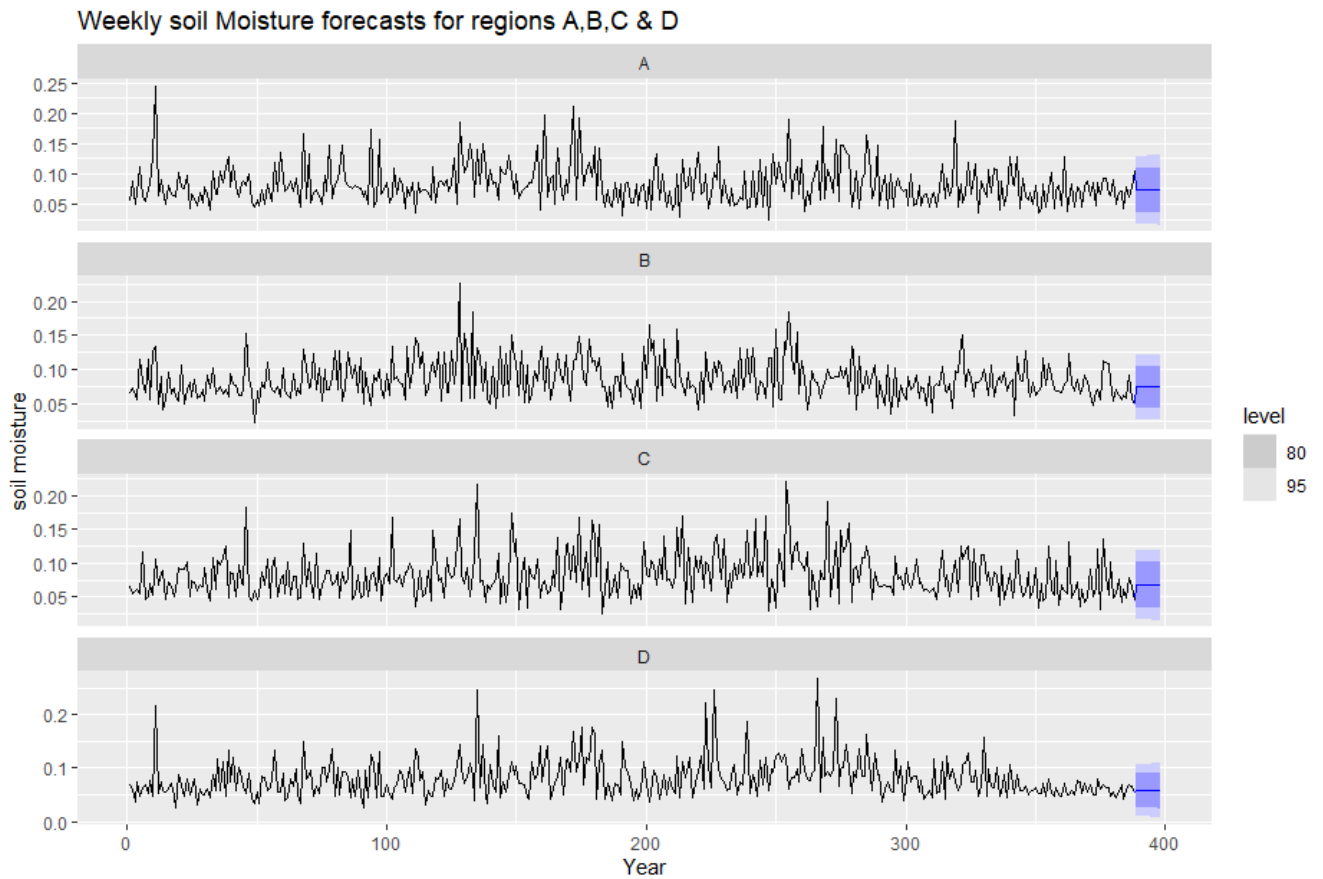


Figure 5.1: ETS 10 Week Forecasts for Regions A,B ,C & D

Visually we observe that the ETS forecasts have a constant forecast value characterized by a significantly larger forecast confidence interval. This is due to the higher values of σ^2 observed in the model outputs. These forecasts are quite rudimentary compared to the ones shown by the NNAR models below.

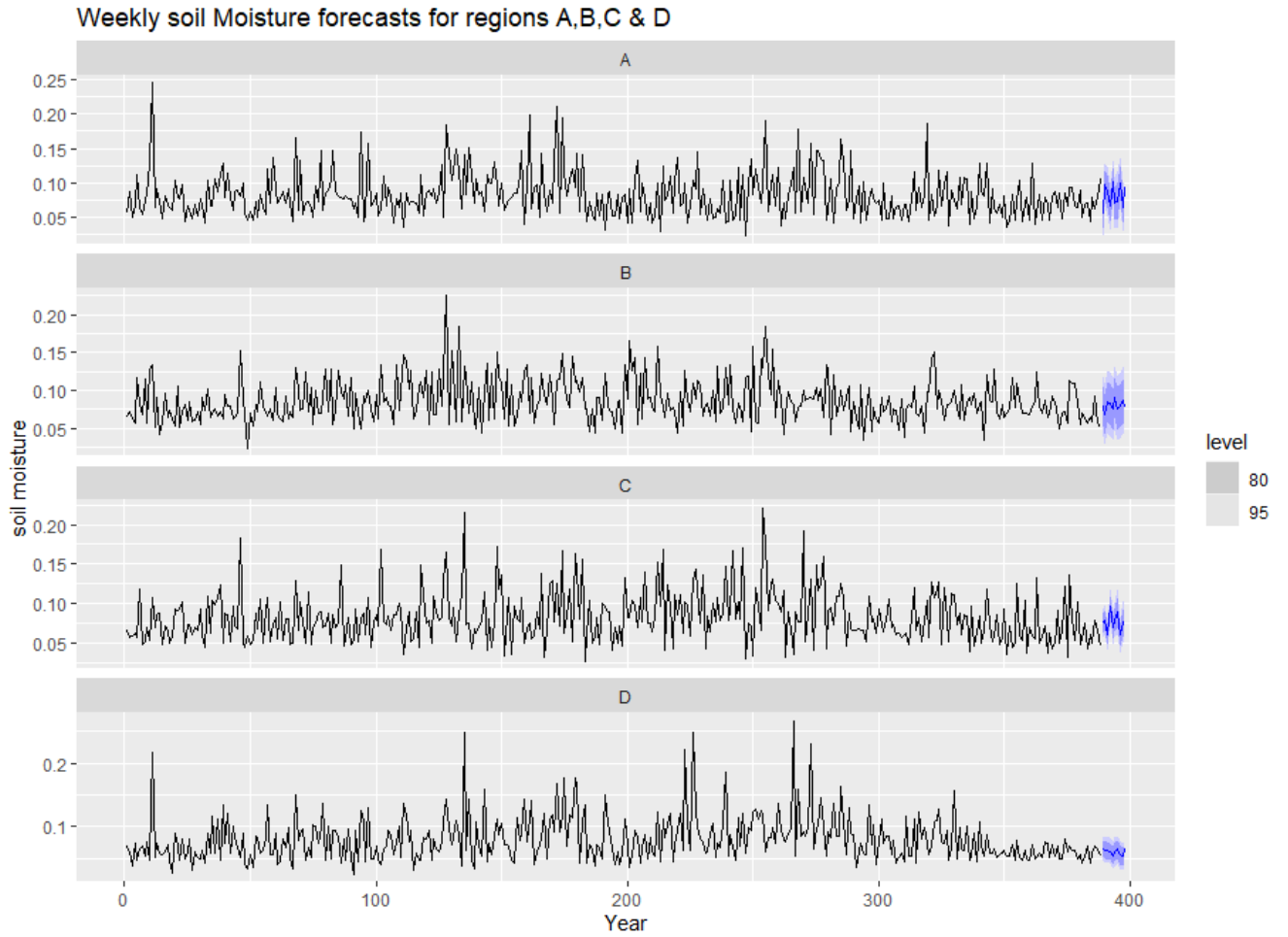


Figure 5.2: NNAR 10 Week Forecasts for Regions A,B ,C & D

The NNAR forecasts above have small confidence intervals as compared to the ETS forecasts, this is due to the difference in variance of the models for each region. This approach yields estimates (forecasts) that mimic the pattern of the soil moisture data for each region. In terms of visual comparisons, the NNAR models seem to out-perform the ETS in both forecasting and modeling the data. To test this claim an accuracy test is be validate these results was conducted.

5.1.5 Forecasting Accuracy

This section includes forecast accuracy results for NNAR and ETS models for two levels of training and testing data sets. Table 5.6 and 5.5 present the accuracy measures obtained from the ETS (Error-Trend-Seasonality) and NNAR forecasting models for various spatial points. The accuracy measures serve as quantitative indicators to assess the performance and reliability of the forecasting models.

The tables the following accuracy measures for each spatial point: $ME = \frac{1}{n} \sum_{i=1}^n |y_i - \hat{y}_i|$ (Mean Error), $RMSE = \sqrt{\frac{1}{n} \sum_{i=1}^n (y_i - \hat{y}_i)^2}$ (Root Mean Squared Error), $MAE = \frac{1}{n} \sum_{i=1}^n |y_i - \hat{y}_i|$ (Mean Absolute Error) , $MPE = \frac{1}{n} \sum_{i=1}^n \left(\frac{y_i - \hat{y}_i}{y_i} \right) \times 100\%$ (Mean Percentage Error), and $MAPE = \frac{100}{n} \sum_{i=1}^n \left| \frac{y_i - \hat{y}_i}{y_i} \right|$ (Mean Absolute Percentage Error). These measures provide insights into the forecast errors and the overall accuracy of the models. The spatial points, labeled A to K, represent specific locations or regions for which the forecasting models were applied. Each spatial point is associated with its respective accuracy measures.

The ME indicates the average difference between the forecast values and the actual values, where positive values represent overestimation and negative values denote underestimation. The RMSE represents the standard deviation of the forecast errors, providing a measure of the model's accuracy. The MAE measures the average absolute forecast error, giving an indication of the magnitude of the errors. The MPE expresses the average percentage difference between the forecast values and the actual values. A positive MPE indicates an overall overestimation, while a negative MPE suggests an overall underestimation. The MAPE represents the average percentage error in relation to the actual values, providing a measure of the relative accuracy of the models.

Spatial Point	ME	RMSE	MAE	MPE	MAPE
A	0.0053	0.0379	0.0294	-14.5537	39.9288
B	-0.0131	0.0213	0.0174	-23.0742	27.4946
C	0.0083	0.0479	0.0356	-18.553	46.2562
D	0.0216	0.0466	0.0291	11.6971	23.8531
E	0.0139	0.0452	0.0355	-1.8442	36.7475
F	0.0242	0.0462	0.0354	14.2053	30.3568
G	0.0078	0.0240	0.0190	2.4621	21.2762
H	0.0040	0.0348	0.0273	-9.3397	33.1465
I	0.0108	0.0355	0.0267	0.6273	26.0392
J	-0.0002	0.0363	0.0283	-13.9308	33.5954
K	-0.0110	0.0229	0.0186	-17.0899	23.5822

Table 5.5: Accuracy Metrics for NNAR Forecasting Models

Spatial Point	ME	RMSE	MAE	MPE	MAPE
A	0.0181	0.0429	0.0333	-16.0140	40.1976
B	-0.0099	0.0238	0.0200	-19.7214	31.4952
C	0.0049	0.04833	0.0357	-24.5420	49.0535
D	0.0047	0.0414	0.0268	-6.5751	25.5100
E	-0.4001	0.0403	0.0330	-17.6467	38.7501
F	0.0074	0.0338	0.0277	-3.2577	27.4979
G	-0.0300	0.0226	0.0177	-7.2596	21.66115
H	0.0109	0.0353	0.0252	0.1363	27.8202
I	0.038	0.0504	0.0386	31.3984	32.5180
J	0.0009	0.0353	0.0282	-13.0201	33.7898
K	0.0022	0.0190	0.0154	-1.8476	17.8175

Table 5.6: Accuracy Measures for the ETS Forecasting Models

Table 5.5 displays the accuracy metrics for NNAR forecasting above. Table 5.6 displays the accuracy metrics for ETS forecasting above. By examining the accuracy measures for each spatial point, it is possible to evaluate the performance of the ETS and NNAR forecasting models and compare their effectiveness across different locations. These results are discussed in detail in Chapter 6.

5.2 Harvest and Death Rate Forecasting

Series	Harvest Rates (%)	Transformed Harvest Rates(%)
Model	NNAR(1,1)	NNAR(1,1)
Transformation	-	square root
Average of Networks	20	20
Network Architecture	1-1-1	1-1-1
Number of Weights	4	4
Output Units	Linear	Linear
σ^2	0.8067	0.02499

Table 5.7: Harvest Rate NNAR Model Details

The model is an ensemble of 20 individual neural networks. Each network is trained separately on the data and produces its own set of predictions. The final forecast is obtained by taking the average of these individual predictions. A 1-1-1 network means that it has one input neuron, one hidden neuron, and one output neuron. The network contains four weights, which are parameters that are learned during the training process. These weights are adjusted to minimize the difference between the predicted harvest rates and the actual observed values.

The neural network is a linear function of the weighted inputs. The model outputs σ^2 as 0.8067, but this number is reduced when a square root transformation is applied it reduces to 0.02499 therefore the transformation over a significant improvement in estimated variance.

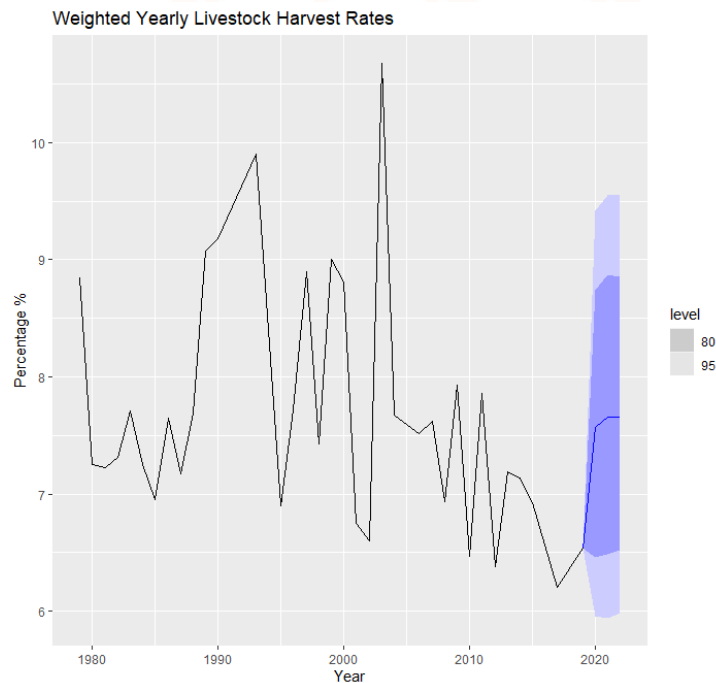


Figure 5.3: Harvest Rate Forecast

Figure 5.3 shows the 3 years forecasts for the harvest rates. There seems to be a decreasing trend and a non-constant variance. There highest harvest rates can be seen in the period between the 1990 and

2005, with the largest being in 2003. The confidence intervals for the forecasts are wide which can be attributed to the volatility in the data.

The Death model is a 1-1-1 network, it has one input neuron, one hidden neuron, and one output neuron which is similar to the harvest forecasts. Similarly the network contains four weights, these weights are adjusted to minimize the difference between the predicted harvest rates and the actual observed values. The neural network is a linear function of the weighted inputs. The model outputs σ^2 as 6.233, but this number is reduced when a square root transformation is applied it reduces to 0.09744 therefore the transformation over a significant improvement in estimated variance.

Series	Death Rates (%)	Transformed Death Rates(%)
Model	NNAR(1,1)	NNAR(1,1)
Transformation	-	square root
Average of Networks	20	20
Network Architecture	1-1-1	1-1-1
Number of Weights	4	4
Output Units	Linear	Linear
σ^2	6.233	0.09744

Table 5.8: Death Rate Model Details

Figure 5.4 shows the 3 years forecasts for the death rates. There seems to be a decreasing trend and a non-constant variance in the time series data. A large spike in the death rate during the early 90's is seen and could be linked to out break of disease or famine. The forecast confidence intervals are narrower than that of the death rate as the data is less erratic in nature.

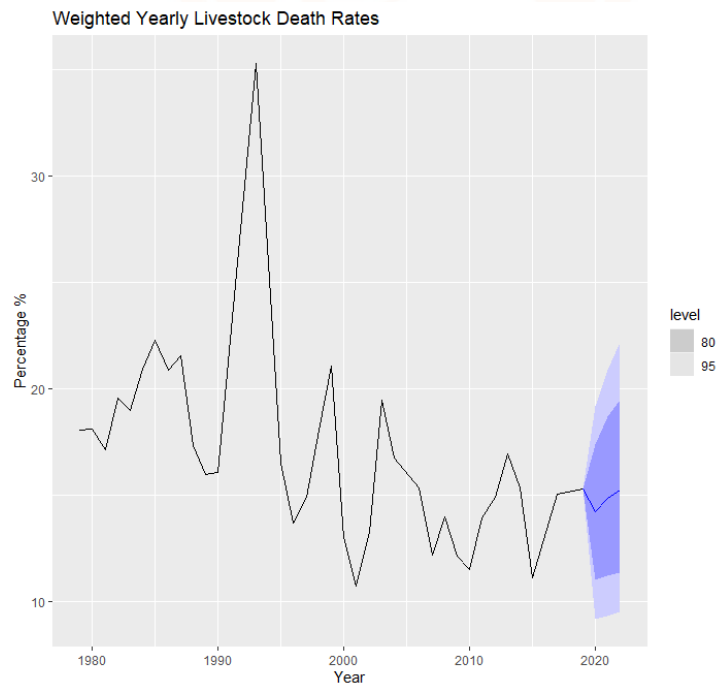


Figure 5.4: Death Rate Forecast

6. Discussion

6.1 Parameter Estimation Breakdown

In Chapter 4, we presented a comparative analysis of two classes (initial and adjusted iterations) of MultiStart optimization results. The objective of the optimization was to minimize an initial objective value through multiple runs of a local solver algorithm. The focus of the analysis was on the convergence rates, final objective values, and the optimized variable values. Table 6.1 below shows a summary of the initial iteration results.

Iteration	N° of Solvers	Initial Objective (e^2)	Final Objective (e^2)	Convergence (%)
1	10	1563.4521	81.3626	60
2	100	1563.4521	5.8666	47
3	500	1563.4521	5.7372	64.5

Table 6.1: Initial Iteration Results

The first set of results from the initial iterations class comprised of 10 local solver runs, out of which 6 converged successfully with a positive local solver exit flag. The final objective function error (81.3626) achieved after optimization was significantly lower than the initial objective (1563.4521), indicating a successful optimization process. The optimized variable values of m, a, b and c were 0.79109, 258.9894, 190.1992, and 0.62262, respectively.

The second set of results included 100 local solver runs, with 47 converging successfully. The final objective function value (5.8666) achieved after optimization was even lower than the previous set, demonstrating further improvement. The optimized variable values of m, a, b and c were 0.42314, 227.491, 286.4273, and 0.48007, respectively. When the local solver *fmincon* is initialized 500 times, we see an increase in convergence rate from the second run but a minimal change in the final objective function. This set of results indicates that increasing the number of local solver runs can enhance the optimization process and yield more favorable outcomes.

The total percentage error for each of the final objective function is 5.20%, 0.38 % and 0.37 % of the initial error. The percentage-based analysis reveals a clear trend of continuous improvement in error reduction rates, signifying that the optimization process was progressively refining the objective function with each iteration. Under the the hypothesis of plant density simulations amplifying the true error in estimation in our initial iteration results, we tested this as shown in Table 6.2.

Iteration	N° of Solvers	Initial Objective (e^2)	Final Objective (e^2)	Convergence (%)
1	100	0.93729	0.83915	49
2	500	0.84709	0.83216	99.6

Table 6.2: Adjusted Iteration Results

After adjustments in the objective function, a significant decrease in the initial objective functions from Table 6.1 and Table 6.2 can be noted. This confirms that the plant simulations amplified the error in estimation therefore removing it was justified. Further adjustments in the form of testing multiple population start points, narrowing of parameter intervals and increases in local solvers were made. The initial objective value was reduced to 0.84709 from 1563.4521. The numerous runs initiated by MultiStart show that 498 out of 500 local solver runs successfully converged to optimal solutions. The convergence of such a high number of runs with positive local solver exit flags indicates that narrowing of the estimation intervals was significant. The high number of convergence also reveal how robust and reliable Multistart with *fmincon* and non-linear least squares are in the optimization process.

Iteration	N° of Solvers	Initial Objective (e^2)	Final Objective (e^2)	Convergence (%)
3	100	0.20624	0.12908	99
3	500	0.20624	0.12850	99.6

Table 6.3: Refined Iteration Results

To investigate whether the weighting parameter w_i was increasing the error of estimation, we tested whether its presence had a significant impact on the initial and final the objective functions values. The final objective value achieved after the optimization process was significantly improved to 0.12908 achieving the threshold set at 1.0 (Table ??). This improvement can be attributed to the change in weighting parameter w_i for the year 2019. It is important to note that the error function is normalized and thus the thresholds were assigned with this in mind. The 3 thresholds used in the estimation are 1.0, 0.1 and 0.01, all of which are applied in the order stated throughout the optimization process.

Iteration	N° of Solvers	Initial Objective (e^2)	Final Objective (e^2)	Convergence (%)
4	10	0.06239	0.0034832	100
4	100	0.06239	0.0034829	97

Table 6.4: Final Iteration Results

The last iterations yielded the best results compared to previous iterations as shown in Table 6.4. The objective function had values that were lower than the adjusted threshold (0.01). Increasing the number of solvers did not produce much of a difference between the runs thus we can state the results of optimization as ; the minimum plant growth rate (m) = 0.32462, consumption/feeding rate of herbivores (a) = 0.12357, half saturation constant of herbivore feeding on the plant biomass (b) = 1915.0084, and Conversion rate of plant biomass to herbivore offspring (c) = 0.0087768 from the first run. Figure 6.1 shows graphs of livestock population weights and livestock population using the optimized parameters for the period 2015-2019 along with their corresponding observed data values.

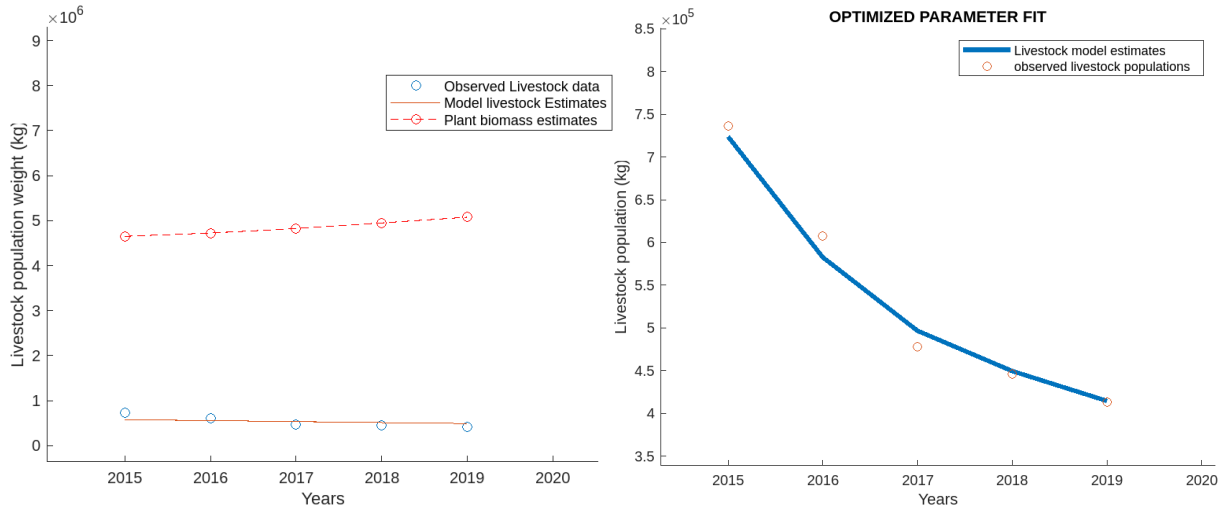


Figure 6.1: Optimal Parameter Plot

Model runs based on the final optimized parameters values have produced results that have well captured the observations. Refining the death rates actually yielded better fits for our model. The final objective value falling within the set thresholds (1.0, 0.1 & 0.01) and the high convergence rate achieved by the MultiStart optimization method all demonstrate the effectiveness and reliability of the methods. The optimized parameter values offer valuable insights into the parameter configurations associated with optimal solutions despite limited observational data points. Overall, while the improvement in the objective value are significant, it is still does not account for the models relationship with plant biomass (vegetative coverage); this means that the parameters do not offer an accurate representation of the plant and livestock interaction.

6.2 Forecasting Results Summaries

6.2.1 Soil Moisture Forecasting

The forecast accuracy results from the NNAR models and the ETS models provide insights into the performance of these two forecasting approaches for the four spatial points. Comparing the mean error (ME), we observe that the NNAR models generally produce smaller errors 0.0053, 0.0083 and 0.0216 for three out of the four spatial points A, C, and D compared to 0.0181, 0.049 and 0.0047 from the ETS models. However, for spatial point B, the ETS model performs slightly better with a lower ME value. In terms of root mean square error (RMSE), the NNAR models outperform the ETS models for spatial points A and D, as they have lower RMSE values. Conversely, the ETS models achieve lower RMSE values for spatial points B and C. When considering mean absolute error (MAE), both modeling approaches show similar performance, with no clear dominance observed for any of the spatial points.

Analyzing the mean percentage error (MPE), the NNAR model performs significantly better for spatial point D with a positive MPE value, indicating a closer match between the predicted and actual values. In contrast, the ETS model achieves better results for spatial points B and C, exhibiting lower MPE values. Lastly, examining the mean absolute percentage error (MAPE), the NNAR models demonstrate superior accuracy for spatial point D, providing a smaller MAPE value. However, for the remaining spatial points (A, B, and C), the ETS models yield lower MAPE values, indicating better accuracy.

The MAPE values provide an understanding of the average percentage deviation of the forecasts from the actual values, while the MPE values indicate the direction and magnitude of bias in the forecasts.

Looking at the forecasting results from Tables 5.5 and 5.6 four spatial points, A, B, C and D we can note the following;

- Spatial Point A: The NNAR model has a MAPE of 39.9288%, indicating that, on average, the forecasts have a deviation of approximately 39.9% while the ETS model has average deviation of approximately 40.2% from the actual values. This shows that the NNAR model has a slightly better average deviation. The negative MPE (-14.6%) suggests that, on average, the forecasts tend to underestimate the actual values.
- Spatial Point B: The NNAR model has a MAPE of 27.5% which is lower as compared to the 31.5% for the ETS model, Thus the NNAR model has a lower average deviation from the true values. The negative MPE of -23.1% for the NNAR and -19.7% for the ETS model suggest that, on average, both models tend to underestimate the actual values, with the ETS having the slightly larger underestimation compared to the ETS models.
- Spatial Point C: The NNAR model has a MAPE of 46.3% which is slightly less than 49.1% exhibited by the ETS model. This indicates that the NNAR model's average deviation from the actual values is less than that of the ETS model. The NNAR model has a negative MPE (-18.6%) which is preferable over -24.5% for the ETS model. This suggests that, on average, the forecasts for the NNAR tend to underestimate the actual values less ETS model.
- Spatial Point D: The NNAR model has an average deviation of approximately 23.9% from the actual values which is slightly less than 25.5% for the ETS model. The positive MPE 11.8% for the NNAR model suggests that, on average, the forecasts tend to overestimate the actual values while the ETS model has a negative MPE of -6.6% which suggests that, on average the forecast from this model tend to underestimate the actual values. Overall, the NNAR model performs better in terms of RMSE, MAE, and MPE for spatial points A, B, and D, indicating lower errors and better accuracy compared to the ETS model. However, for spatial point C, the ETS model outperforms the NNAR model in terms of RMSE, MAPE, and MPE

In summary the MPE values indicate that the NNAR model tends to underestimate the actual values for spatial points A, B, and C, while overestimating for spatial point D. The ETS model exhibits a negligible bias for spatial point A, underestimation for spatial points B and C, and slight underestimation for spatial point D. This can be explained by the pattern prediction nature of the NNAR model which is Incorporated into the forecasts, unlike the ETS forecasts which are linear which wide confidence intervals. This is shown in Figures 5.1 and 5.2, thus NNAR models are more prone to overestimate or underestimate when compared to ETS models.

In general the NNAR models and the ETS models exhibit different forecast accuracy for each spatial point. The NNAR models perform differently in comparison to the ETS models, with some points showing higher accuracy and others showing lower accuracy. Each of the 11 models show varying estimates and do not outline a single preferable model among two forecasting techniques. This can be explained the following

- Model Complexity: NNAR models are based on neural networks, which are capable of capturing complex relationships in the data. On the other hand, ETS models are exponential smoothing models that assume a simpler pattern in the data. The complexity of the underlying data patterns may favor one model over the other in different spatial points.
- Data Patterns: each spatial point may have unique patterns and characteristics that are better captured by one model compared to the other. The NNAR models may excel in capturing nonlinear or intricate patterns, while the ETS models may be more suitable for capturing simpler, linear trends.

- **Training Data:** the performance of the models heavily relies on the quality and representativeness of the training data. If the training data for certain spatial points lacks diversity or contains outliers, it may affect the accuracy of the models differently.
- **Model Hyper parameters :** the performance of the models can also be influenced by the chosen hyper-parameters during model training. The selection of hyper parameters such as the number of hidden layers, neurons, or learning rate for the NNAR models, as well as the smoothing parameters for the ETS models, can impact the accuracy.
- **Randomness :** Neural networks are subject to random initialization and can produce different results with each run. It's possible that the NNAR models' performance varies due to this randomness.

Comparing the models, we see that the NNAR model generally performs better in terms of RMSE, MAE, and MPE for high volatility soil moisture spatial points than ETS models. NNAR models indicate lower σ^2 estimates across the board and better accuracy compared to the ETS model when forecasting fewer estimates as NNAR are pattern identifying models. An interesting area to branch into would be a hybrid of both these two methods. Another benefits of NNAR models is that they can handle missing values unlike their counterparts which require external imputation methods. By training on available data, neural networks captured non-linearity and non-stationarity that may exist in the soil moisture from the different spatial points

6.2.2 Harvest and Death Rates

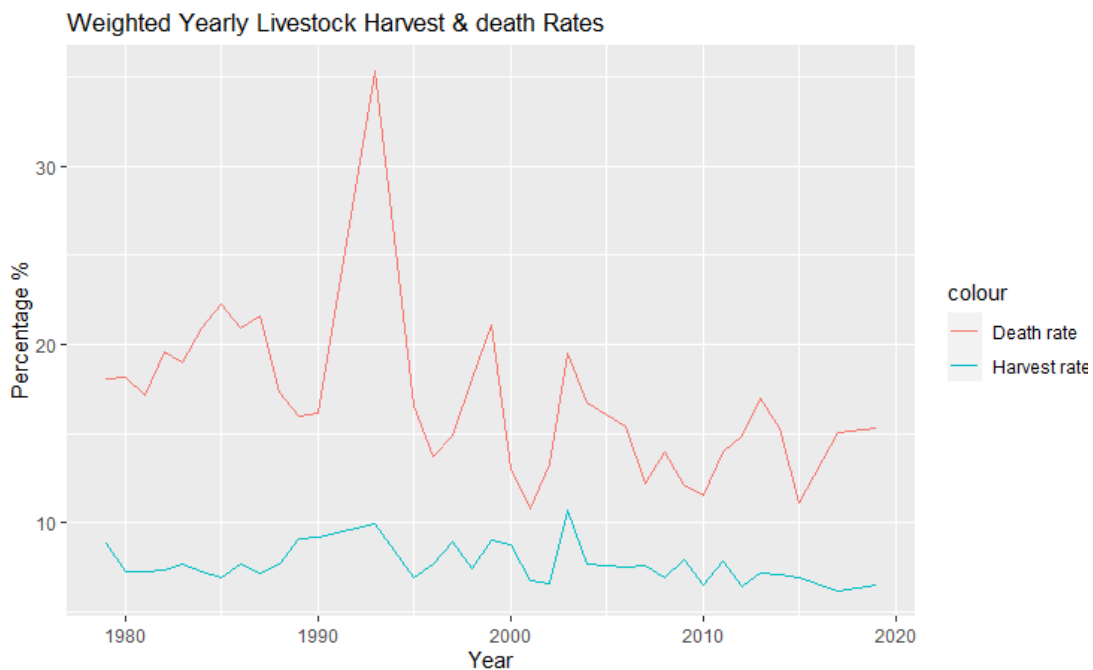


Figure 6.2: Weighted Livestock Harvest and Death Rates : Statistics Botswana

From Figure 6.2, we can see that livestock death rates have always been higher than harvest rates. This is a cause for concern since it suggests that farmers are losing more cattle to disease and other causes of death than they sell or feed off. However, these high death rates can be explain by the criteria in which Statistics Botswana classifies a livestock death, this is because cattle lost to thieves, gone missing, culled

and killed by car accidents are all included in these rates. This classification amplifies the rates with the inclusion of human intervention rather than natural death rates.

The NNAR model for death and harvest rate are relatively simple with each having only 4 weights, one lagged input, one neuron in the hidden layer with a linear model structure. This can be attributed to the relative size of the data which has only 41 observations. The forecast confidence intervals for the harvest rates are narrower than that of the death rate as the data is less erratic in nature as shown in Figure above 6.2 above. Square root transformation on both time series data sets had significant improvements in σ^2 estimates, with initial estimates of 6.233 and 0.8067 which later reduced to 0.09744 and 0.02499 for death and harvest rates respectively.

6.2.3 Forecasting of Livestock Population

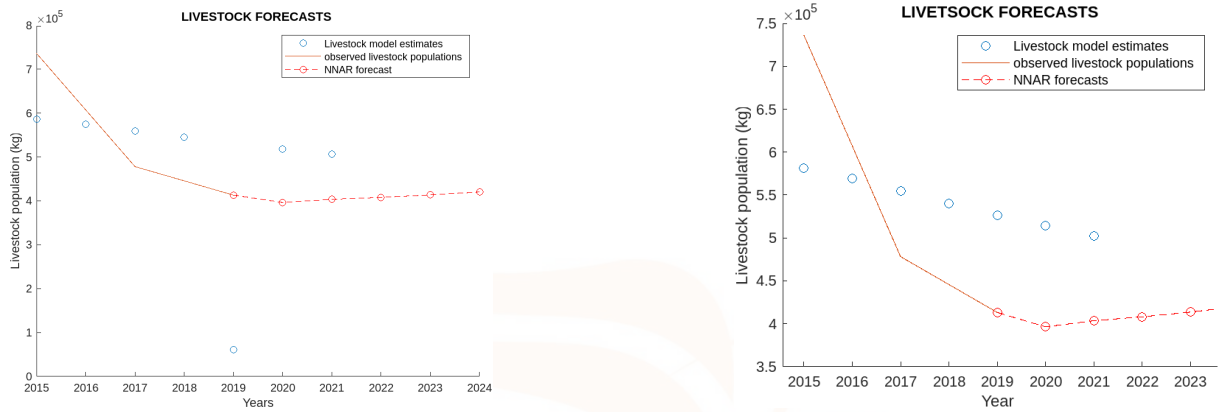


Figure 6.3: NNAR and Initial Parameter from [25] P-H Model Forecasts

After optimizing the parameters and forecasting the soil moisture, death and birth rates. WE now move onto utilizing these forecasts to come up with an integrated model forecasts. Figure 6.3 below shows the model behaviour for the initial parameter estimates taken from literature. We can clearly see the lack of fit for these estimates, with the model over estimating the values of the livestock population. It can be noted that even with the change in the weighting factors in the figure to the right, the model does not perform better and still over estimates the true population figures.

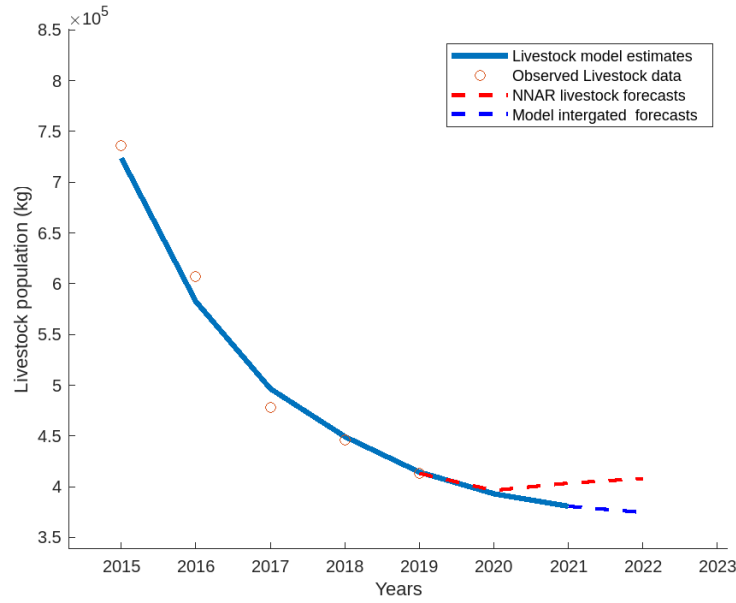


Figure 6.4: NNAR and Optimized Parameter P-H Model Forecasts

Figure 6.4 displays the optimized model fit from 2015-2019 (solid blue line) along with the NNAR forecasts for the livestock data from 2019-2022 (red dash line) and the model forecasts that are divided into two, those formulated using observed soil moisture (2019-2021, green dash line) and those that integrated NNAR soil moisture forecasts (2021-2022, blue dash line). The two forecasting methods used above differ on the basis that the NNAR livestock forecasts are generated solely from the observed livestock data while the model forecasts are trajectories based on parameter such as soil moisture, death and harvest rate forecasts which are integrated within the model. Between the years of 2019-2020 the NNAR livestock forecasts and the model-parameter estimates forecasts were predicting similar population declines which confirms the continuity of the historical livestock data trends.

This agreement is expected as NNAR has low uncertainty at shorter lead time and the Plant-Herbivore model used observed soil moisture data. However, the NNAR forecasts project a recovery in livestock populations from 2020 - 2022, while the model forecasts indicate a further decline. This discrepancy reflects the fundamental difference between the two methodologies. NNAR forecasts are myopic, relying mainly on historical data, and may not consider external factors like soil moisture or other ecological variables. The NNAR models generate forecasts that are unaffected by climatic and environmental variations thus do not take into account the dynamic nature livestock populations. As a result, the NNAR models are prone to high uncertainty with longer lead time. On the other hand, the Plant-Herbivore model is constrained by observed SMAP soil moisture until 2021 and by soil moisture forecast from NNAR between 2021 to 2022. As noted earlier for the NNAR livestock forecast, soil moisture forecasts from NNAR is also expected to have low uncertainty at shorter lead time. Therefore, the integration of SMAP observation and NNAR soil moisture into the Plant-Herbivore model lead to consistent simulation of livestock population. In addition, the Plant-Herbivore model given by Eqn (3.1) is an initial value problem whose forecasts depend on the initial values. Therefore, smooth evolution of livestock population that retains historical trend shows physical consistence inherent in the model.

7. Conclusions

7.1 Conclusions

The Plant-Herbivore models proposed by [3; 25] served as a good reference in studying the relationship between traditional livestock production and vegetative conditions. The models were found to effectively model the complex interplay of parameters in Plant-Herbivore interactions. These models were extended in order to incorporate the existing information for Botswana's climate and livestock factors thus addressing the absence of real data for parameter estimation. Estimate aggregation through weighting and time conversion are incorporated to reduce the error brought about by the assumption of homogeneity within the data. This approach effectively bridges the variations in base data, often represented by the Plant-Herbivore model, and the observed data. This is in line with the first two specific objectives in our research.

The utilization of the Runge-Kutta method with Multistart and local solver *fmincon* enabled efficient exploration of parameter spaces, improved the accuracy of parameter estimation, with objective functions achieving set thresholds in each of the runs. The combination of these optimized parameter estimation techniques, with the normalized nonlinear least squares, presented very robust results which provide a promising avenues for dynamic modelling in agro-environmental control systems. The incorporation of nonlinear least squares is explored to further enhance the optimization process, allowing for the identification of parameters that best fit the observed livestock data, even in the presence of non-linearity. The combination of these optimized parameter estimation techniques with the normalized nonlinear least squares, achieved a final cumulative error of 0.0038 from initial error of 1563.4521, falling within the set thresholds (1.0, 0.1 & 0.01). These techniques offered a systematic approach to estimate parameters of the extended Plant-Herbivore models accurately tackling the issue of estimation of parameters for dynamic model systems that exhibit non-convex parameter relationships despite the low number of observation points.

Neural Network forecasting models were integrated into the parameter estimation process by incorporating them as a component of the optimization framework. Soil moisture is a dynamic variable within the model that can exhibit complex temporal and spatial patterns. Neural Networks effectively captured these dynamics by utilizing their inherent ability to model sequential data. The ARIMA models yielded significantly higher variance estimates (0.103 - 0.155) than NNAR models (0.000004 - 0.000562), NNAR models had ME (Mean Error) values ranging from -0.0012 to 0.0140, indicating that the model's forecasts are generally close to the actual values with small deviations. The MAPE values for the ARIMA model range from 30.9511 to 43.1502, which is higher than the NNAR model's MAPE values (30.1192 - 39.4099). The NNAR models were found to generally perform better for smaller numbers of predictions than ARIMA models. NNAR forecasts were found to have narrower confidence intervals than ETS for soil moisture forecasts, the NNAR models were found to better capture the evolving nature of soil moisture and make more informed predictions. NNAR forecasting was used to obtain soil moisture, death and harvest rates forecasts, which were then integrated within the extended Plant-Herbivore model to estimate 2021 - 2022 livestock production trajectories This enables accurate and adaptable forecasting addressing the lack of forecasting capabilities for Plant-Herbivore models.

The project primarily focuses on the dynamics of Plant-Herbivore interactions and their relationship with traditional livestock production. However, there might be other external factors, such as land use changes, human interventions, or disease outbreaks, that can significantly influence the vegetative conditions and livestock dynamics. These factors are not explicitly considered in the models. The accuracy and reliability of the results heavily depend on the availability and quality of data, the low number of livestock observation points used within the optimization, may limit of the generality of the findings. The increase in NNAR uncertainty with increasing lead time might affect the accuracy of forecast from the Plant-Herbivore model. Therefore, future work should take this limitation into account and explore other robust data-based forecast such as Long Short-term Memory Network (LSTM) used in the field of Deep learning for integration with the Plant-Herbivore dynamic model.

7.2 Limitations

1. **Data Availability:** The accuracy and reliability of the results heavily depend on the availability and quality of data. The thesis acknowledges the low number of livestock observation points used within the optimization, which may limit the generalizability of the findings. This issue further limits optimization in terms of there being no data on plant biomass in Botswana thus the objective function could not include the variation between plant biomass estimates and observed plant biomass. The scarcity of long-term and comprehensive datasets for both climate variables and livestock factors could introduce uncertainties and affect the accuracy of predictions.
2. **Assumptions and Generalization:** The thesis makes certain assumptions about the homogeneity of data and system behavior, which may not hold true in all cases. These assumptions made for the weights could limit the applicability of the findings to specific contexts or regions. Generalizing the results beyond the specific conditions of Botswana's climate and livestock factors should be done cautiously.
3. **External Factors:** The thesis primarily focuses on the dynamics of plant-herbivore interactions and their relationship with traditional livestock production. However, there might be other external factors, such as land use changes, human interventions, or disease outbreaks, that can significantly influence the vegetative conditions and livestock dynamics. These factors are not explicitly considered in the models, which may limit the comprehensiveness of the analysis.

7.3 Recommendations

- Harvesting before cattle die is a critical recommendation to ensure the welfare and economic viability of livestock operations. We have found that the harvest rate is lower than the death rate during our study and we expect a further population decline. Thus, setting clear culling criteria based on the animal's health, productivity, and economic viability could help farmers have economic viability.
- Rotational grazing is a recommended practice for livestock management that involves dividing a pasture into smaller sections or paddocks and systematically rotating livestock between these areas this prevent reduction in plant biomass for select areas. This is taken from the decreased plant biomass thus evening out the vegetation grazing would help in allowing pasture recovery. Regular rotational grazing prevents overgrazing in specific areas and promotes more uniform grazing pressure across the pasture.

- Traditional farmers can consider implementing supplementary feeding practices to their livestock for period of drought as plant biomass decreases in these times. In situations where the available feed resources are inadequate to meet the nutritional requirements of livestock, supplementation can be considered. This involves providing additional nutrients, such as protein, minerals, and vitamins, to complement the existing diet. Supplementation can be done through locally available alternatives such as legume forages or molasses.

7.4 Future Work

Based on the current research findings and the limitations identified, there are several areas that can be explored for future work. These potential future directions aim to further enhance the understanding of the dynamics between traditional livestock production and vegetative conditions, as well as address the limitations of the current study. The following sections outline some suggestions for future research

7.4.1 Conducting a Survey

One major limitation in the project is the unavailability of livestock data and plant biomass data we intended to further our study by conducting our own data collection through a survey. We have already made advances with the implementation of a pilot survey as detailed below

7.4.2 Pilot Survey

In order to form a justifiable basis of analysis for the Plant-Herbivore models discussed in our research projects. Data collection and sampling is a mandatory task that has to be undertaken. A pilot study was carried out in late November 2022 to test out field equipment such as the mobile network adapters and cellular tablets, get a feel for the process involved in sample collection. A three day field trip to 6 (10 × 10 m) sites around the greater Palapye (Tamasane, Kgagodi, Mogapi) region to identify vegetative cover. Four (4) soil samples were obtained for validation with soil moisture satellite image data.

7.4.3 Cross Sectional Sampling of Farms

A cross-sectional pilot survey was chosen as a base study design for our research project, with a digital questionnaire serving as the data collection medium. Variables such as, geographic location, type of livestock and count have been included in the questions asked to farmers. The questionnaire was administered by a researcher who is equipped to explain the questions in the Setswana if necessary. The survey was conducted in farms surrounding Palapye. 4 Farms were visited in total.

7.4.4 Definition of the Scope and Objectives of the Survey

Before conducting the livestock survey, the following scopes and objectives were established. These survey objectives were formulated with consideration for the projects scope and objectives. Below is how we formulated the scope of the survey :

The survey is to be targeted to only subsistence livestock farmers. It can cover various aspects related to livestock farming such as animal health, nutrition, breed, livestock count, and area of grazing. The survey can target different types of livestock such as cattle, sheep and goats. It can be conducted on farms of different sizes, locations, and production systems granted that the farmers is farming for domestic purposes.

Objectives:

1. To assess the current status of livestock farming in the region, including the number and types of animals being raised, their health status, and the farming practices being used.
2. To identify the major challenges and opportunities facing livestock farmers, such as access to markets, disease and availability of supplements.
3. To evaluate the effectiveness of current policies and programs related to livestock farming and identify areas for improvement.
4. To provide baseline data for monitoring changes in livestock production and management practices over time.
5. To generate information that can be used to develop strategies to improve livestock productivity and profitability, and to enhance the sustainability of livestock farming.

7.4.5 Vegetation Survey and Soil Sampling

Vegetation survey was conducted following the method by [41]. Three quadrants (size: 10×10 m each) were made at the hailstorm occurrence site and unaffected site (control). From each quadrant, the vegetation (grasses, herbs, shrubs, and trees) was identified using morphology. Soil samples from dominant plant species and open area were taken at a depth of 30 cm from each quadrant. Soil samples were air-dried at room temperature for 24 hours and sieved with a 2.0 mm sieve and stored until analysis.

7.5 Utilization of New Methods

In the future, there is a promising scope for further advancements in forecasting methodologies, particularly in the areas of hybrid approaches, continuous learning neural network algorithms, and forecast generation. We intend on using multiple optimization techniques such as Gradient-Based Optimization, Markov Chain Monte Carlo (MCMC) methods and Bayesian estimation to compare how well they work.

An interesting area of further research is the formulation of model forecast confidence interval. Due to the variety of estimation techniques used in the model outputs the formulation of confidence intervals for the model forecasts is not straight forward as in the NNAR techniques.

Another future avenues is the building of Neural Networks algorithms rather than using already made one that offer limited control over the learning algorithms. The development of custom neural network algorithms that embrace continuous learning capabilities is a promising direction for future research. These algorithms enable neural networks to adapt and update their knowledge incrementally, continually learning from new data without requiring extensive retraining. This approach holds tremendous potential for real-time forecasting.

8. Appendix

8.1 Matlab Codes

8.2 Euler Method Solution Curves

```
%%Estimating values of PLANT HERBIVORE MODEL
given a system of linear first order ODEs
clear;
clc;
clf;
%%Initial Conditions
r = 0.25;
a = 0.75;
K =10000;
b = 1100;
c = 0.25;
h= 40;
d1 = 0.1;
d2 = 0.3;
u = 0.025;
t=2/0.6; %Change the value of t until answer is determined
deltax = 3;
y1(1)=5000;
y2(1)=1000;
n=100/deltax +1;

%%Get Functions and Other Parameters
X(1)=0;
func1=@(y1,y2) r*y1*(1-y1/K)-(a/(b+y1))*y1*y2;
func2=@(y1,y2) y2*(c*(a/(b+y1))*y1*((y2/(h+y2))))-(d2*K*(1+d1 -d2)*(b+y1)
+ y1*(b+K)*(d2-d1)*(d2-1))/(K*(1+d1-d2)*(b+y1) + y1*(b+K)*(d2-d1))- u);
%second equation

for i =1:n-1
x(i+1)=i*deltax;
y1(i+1)=y1(i)+deltax*feval(func1,y1(i),y2(i));
y2(i+1)=y2(i)+deltax*feval(func2,y1(i),y2(i));
end
disp(x);
disp(y1);
disp(y2);

%%Graphing of ODEs
```

```
plot(x,y1,x,y2);
legend('P','H');
title("Plot of Plant and herbivour trajectories Using Euler Method");
ylabel('Population');
xlabel('Time');
```

8.3 Runge-Kutta Method Solution Curves

```
%This function gives the predator prey model with known konstant parameters
function pdottdot= lvPH(t,y)
    global r a K b c h u d1 d2
    r = 0.25;
    a = 0.7;
    K =10000;
    b = 1100;
    c = 0.25;
    h= 40;
    d1 = 0.1;
    d2 = 0.3;
    u = 0.025;
    pdottdot = [r*y(1)*(1-y(1)/K)-(a/(b+y(1)))*y(1)*y(2);
                y(2)*(c*(a/(b+y(1)))*y(1)*((y(2)/(h+y(2))))-
                (d2*K*(1+d1 -d2)*(b+y(1)) +
                y(1)*(b+K)*(d2-d1)*(d2-1))/(K*(1+d1-d2)*(b+y(1))
                + y(1)*(b+K)*(d2-d1))- u]

%This program creates the data points for the curves of prey and predators
%and then plots them in one figure
clear;
options=odeset('RelTol',1e-5,'AbsTol',[1e-5 1e-5]);
%Order is necessary in the next line,that is, y1 and y2, prey & predator
respectively.
[t, y]=ode45(@lvPH,[0 100],[5000 500],options);
plot(t,y(:,1),'-r',t,y(:,2),'r-');
legend('P','H')
%title('Plot of Plant and herbivour trajectories Using Runge Kunta');
```

TABLE 1.1: LIVESTOCK POPULATION TRENDS (Traditional & Commercial): ('000)

YEAR	CATTLE	GOATS	SHEEP	DONKEYS	HORSES	CHICKEN	PIGS	OSTRICH
1979	2,840	616	152	127	18	740	6	-
1980	2,911	638	149	130	22	833	6	-
1981	2,967	621	140	127	24	1,046	5	-
1982	2,979	636	140	138	24	1,146	5	-
1983	2,818	783	165	142	23	961	5	-
1984	2,685	889	167	139	23	714	7	-
1985	2,459	1,138	200	146	23	1,020	9	-
1986	2,332	1,332	229	142	24	1,179	11	-
1987	2,264	1,470	240	147	24	1,283	11	-
1988	2,408	1,691	259	150	28	1,810	13	-
1989	2,543	1,897	286	151	32	2,013	15	-
1990	2,696	2,092	317	158	34	2,126	16	-
1993	1,821	1,838	250	231	31	1,077	4	-
1995	2,530	2,624	337	303	35	3,157	1	-
1996	2,249	2,205	349	336	5	1,355	3	-
1997	2,212	2,615	409	404	7	1,191	2	-
1998	2,345	2,199	393	400	8	776	4	-
1999	2,581	1,916	369	373	5	874	4	-
2000	2,099	1,576	256	319	48	1,015	1	-
2001	2,468	1,887	306	409	5	928	5	-
2002	3,060	1,683	273	405	46	866	2	1
2003	2,028	1,355	220	493	41	650	4	3
2004	2,155	1,550	244	403	48	1,046	5	1
2006	2,071	1,630	229	371	41	1,639	8	1
2007	1,788	1,640	253	306	30	858	11	-
2008	2,220	1,879	303	366	37	1,170	10	2
2009	2,413	1,867	293	360	42	1,739	10	-
2010	2,635	1,938	279	339	41	1,761	3	2
2011	2,554	1,770	296	351	40	1,499	5	-
2012	2,247	1,650	294	290	36	1,130	24	1
2013	2,084	1,533	274	310	37	1,081	3	-
2014	1,597	1,606	227	227	27	1,017	11	-
2015	1,744	1,205	242	178	32	760	4	-
2017	1,100	1,200	235	140	15	710	1	-
2019	935	1,229	243	125	20	521	11	-

Figures for 1991 -1992, 1994 not available, 2001, 2014, 2017 and 2019 are traditional sector only, 2005 are commercial sector only

TABLE 1.2: CATTLE PRODUCTION INDICATOR TRENDS: BIRTHS, DEATHS AND OFFTAKE RATES (%)

YEAR	BIRTH RATES		DEATH RATES		OFF-TAKE RATES	
	TRADITIONAL SECTOR	COMMERCIAL SECTOR	TRADITIONAL SECTOR	COMMERCIAL SECTOR	TRADITIONAL SECTOR	COMMERCIAL SECTOR
1979	58.4	63.4	10.9	3.6	8.8	17.7
1980	58.4	60.3	13.8	3.9	6.8	13.4
1981	57.5	54.1	13.6	4.0	7.2	12.1
1982	59.4	61.0	17.0	5.9	7.1	13.7
1983	52.8	56.2	17.8	8.9	8.0	24.7
1984	52.5	61.5	19.3	10.0	7.2	23.0
1985	54.3	56.0	23.6	6.2	10.3	22.4
1986	55.7	62.1	23.2	8.8	8.7	14.5
1987	58.5	49.4	22.7	6.3	7.2	14.9
1988	65.5	67.7	15.4	5.1	6.8	13.4
1989	61.9	65.0	12.9	5.0	7.6	16.0
1990	61.6	69.8	12.7	4.2	7.6	17.9
1993	57.0	56.1	32.0	6.7	10.2	58.4
1995	61.0	59.6	14.3	8.2	7.4	3.9
1996	58.4	64.0	4.9	3.5	7.9	23.4
1997	54.6	51.1	4.3	3.6	11.8	1.7
1998	59.9	51.5	6.1	4.5	6.3	13.9
1999	54.0	55.9	7.9	3.6	9.6	14.1
2000	51.3	33.2	6.0	2.6	10.6	1.6
2001	55.5	-	4.9	-	7.7	-
2002	55.9	45.8	5.4	4.8	6.8	15.0
2003	66.6	49.1	16.4	4.7	12.2	15.4
2004	54.4	49.4	12.7	4.8	8.1	12.3
2006	56.9	46.5	9.8	4.3	7.7	10.7
2007	61.3	41.4	8.5	4.5	8.7	18.2
2008	47.3	42.3	6.0	4.2	7.5	12.9
2009	57.8	44.2	5.8	3.4	8.8	10.9
2010	54.6	40.9	6.0	3.2	7.3	12.5
2011	54.4	38.9	6.6	1.6	6.0	13.5
2012	53.7	48.6	9.9	3.3	6.9	11.9
2013	52.1	40.0	12.9	8.0	7.7	10.0
2014	48.8	-	12.4	-	8.1	-
2015	58.0	52.0	7.0	3.0	7.0	12.0
2017	47.3	-	5.9	-	5.5	-
2019	56.5	-	10.9	-	7.0	-

Figures for 1991 -1992, 1994 not available, 2001, 2014, 2017 and 2019 are traditional sector only, 2005 are commercial sector only

TABLE 1.3: GOATS PRODUCTION INDICATOR TRENDS: BIRTHS, DEATHS AND OFFTAKE RATES (%)

YEAR	BIRTH RATES		DEATH RATES		OFF-TAKE RATES	
	TRADITIONAL SECTOR	COMMERCIAL SECTOR	TRADITIONAL SECTOR	COMMERCIAL SECTOR	TRADITIONAL SECTOR	COMMERCIAL SECTOR
1979	42.1	36.7	45.9	17.5	9.3	15.8
1980	45.0	40.7	34.7	20.7	7.1	8.9
1981	44.0	40.1	32.1	24.7	7.0	13.2
1982	42.1	42.2	27.6	16.2	8.2	14.6
1983	44.6	38.5	22.5	23.1	6.2	12.0
1984	41.9	39.0	25.4	22.4	6.8	18.7
1985	40.6	37.3	19.7	19.2	5.7	12.0
1986	40.9	42.2	18.2	16.8	6.3	11.5
1987	40.3	37.8	20.9	18.0	7.2	14.9
1988	42.9	44.9	19.7	13.3	9.0	15.3
1989	41.4	47.2	19.5	11.6	10.9	23.1
1990	40.2	44.2	19.7	17.4	11.1	18.2
1993	49.6	42.5	38.4	22.8	9.8	12.4
1995	43.7	42.4	18.6	23.3	6.5	10.2
1996	43.3	41.6	22.7	27.9	7.7	17.4
1997	42.7	40.5	23.6	21.1	6.8	10.0
1998	46.4	34.7	30.6	21.3	9.0	12.8
1999	45.5	39.0	37.4	23.1	8.8	14.5
2000	45.3	42.1	22.4	17.2	6.9	7.1
2001	43.8	-	17.7	-	5.7	-
2002	42.6	38.4	26.8	27.7	6.5	10.7
2003	46.7	37.1	23.9	20.8	8.6	13.8
2004	48.2	37.9	21.9	20.7	7.2	9.7
2006	42.5	42.6	22.1	22.3	7.5	12.1
2007	42.3	36.6	16.3	16.4	6.6	6.8
2008	44.9	39.9	23.5	23.1	6.5	10.4
2009	43.5	37.6	19.8	13.2	6.9	9.5
2010	41.7	36.7	18.7	15.0	5.6	9.8
2011	41.5	39.5	24.1	21.1	10.9	5.8
2012	41.4	39.9	21.8	23.5	6.0	8.7
2013	42.3	40.0	22.8	25.0	7.0	10.0
2014	41.9	-	18.3	-	6.8	-
2015	44.0	39.0	17.0	16.0	7.0	10.0
2017	39.1	-	23.3	-	7.3	-
2019	39.2	-	18.9	-	6.6	-

Figures for 1991 -1992, 1994 not available, 2001, 2014, 2017 and 2019 are traditional sector only, 2005 are commercial sector only

TABLE 1.4: SHEEP PRODUCTION INDICATOR TRENDS: BIRTHS, DEATHS AND OFFTAKE RATES (%)

YEAR	BIRTH RATES		DEATH RATES		OFF-TAKE RATES	
	TRADITIONAL SECTOR	COMMERCIAL SECTOR	TRADITIONAL SECTOR	COMMERCIAL SECTOR	TRADITIONAL SECTOR	COMMERCIAL SECTOR
1979	33.7	33.8	38.9	15.4	7.9	13.8
1980	45.4	39.3	32.1	16.6	16.7	5.5
1981	36.0	36.4	25.6	19.3	8.8	11.8
1982	39.8	29.6	38.1	17.5	7.8	10.0
1983	36.2	34.1	22.2	19.3	9.9	18.1
1984	35.8	38.3	23.9	26.0	10.5	26.4
1985	33.1	36.6	20.6	11.0	11.1	17.2
1986	33.9	39.9	13.1	21.5	4.8	9.2
1987	33.9	37.5	14.9	27.2	6.8	7.8
1988	34.8	34.3	20.0	13.1	7.5	19.3
1989	38.8	40.3	19.8	11.8	10.1	16.8
1990	42.6	38.5	21.3	18.1	11.3	14.3
1993	40.9	38.7	36.5	17.9	8.5	8.9
1995	35.0	37.2	17.2	18.4	6.2	22.0
1996	32.2	48.0	13.8	24.9	6.7	15.0
1997	32.8	32.9	16.6	14.9	6.6	11.4
1998	37.9	35.4	19.5	17.3	5.4	12.4
1999	41.9	31.1	28.3	12.4	5.9	5.0
2000	35.5	33.8	13.1	16.7	5.9	1.7
2001	33.4	-	14.9	-	5.6	-
2002	33.5	34.5	18.2	21.2	5.0	15.1
2003	37.2	31.4	20.3	11.9	9.4	13.4
2004	39.7	30.7	19.6	15.5	6.9	13.3
2006	32.9	35.1	17.6	15.2	5.9	12.8
2007	34.2	32.0	11.6	36.2	6.6	6.8
2008	35.3	30.6	13.4	14.6	5.5	7.5
2009	35.4	31.9	15.8	12.5	7.3	5.8
2010	32.7	31.8	13.6	6.5	4.7	8.6
2011	32.8	33.4	16.3	17.9	5.7	10.0
2012	33.0	35.0	14.5	16.2	4.6	10.7
2013	36.0	35.0	15.0	14.0	4.4	9.0
2014	35.1	-	15.1	-	2.7	-
2015	37.0	34.0	12.0	12.0	6.0	9.0
2017	33.6	-	16.1	-	3.9	-
2019	32.8	-	14.1	-	4.5	-

Figures for 1991 -1992, 1994 not available, 2001, 2014, 2017 and 2019 are traditional sector only, 2005 are commercial sector only

TABLE 1.5: TRADITIONAL SECTOR : AREA PLANTED TRENDS ('000 HECTARES) BY CROP

YEAR	SORGHUM	MAIZE	MILLET	PULSES	SUNFLOWER	GROUNDNUTS
1979	67	50	13	26	5	4
1980	143	75	17	23	5	2
1981	136	81	20	26	1	3
1982	91	52	16	20	1	2
1983	125	57	16	21	-	1
1984	108	44	17	16	-	2
1985	119	48	15	16	3	1
1986	153	42	17	16	1	1
1987	195	43	14	15	1	2
1988	246	51	13	17	1	1
1989	237	76	20	28	1	1
1990	181	69	12	19	1	1
1993	138	84	15	27	2	1
1995	164	85	11	26	4	1
1996	191	134	11	35	-	2
1997	154	117	11	77	1	5
1998	65	88	9	32	-	11
1999	114	123	8	38	-	3
2000	106	123	5	35	1	4
2001	23	48	3	15	1	1
2002	66	95	3	30	-	2
2003	25	38	3	11	-	-
2004	48	63	7	15	-	-
2006	58	78	7	23	-	1
2007	30	40	6	18	1	3
2008	63	88	11	29	1	3
2009	67	121	9	31	1	3
2010	61	110	10	26	1	4
2011	70	151	13	29	14	2
2012	52	141	10	31	1	2
2013	62	124	12	36	1	2
2014	65	130	8	37	1	2
2015	23	62	5	23	-	2
2017	24	62	3	27	-	1
2019	17	39	3	19	1	1

Figures for 1991 -1992, 1994 not available, 2001, 2014, 2017 and 2019 are traditional sector only, 2005 are commercial sector only

TABLE 1.6: TRADITIONAL SECTOR : AREA HARVESTED TRENDS ('000 HECTARES) BY CROP

YEAR	SORGHUM	MAIZE	MILLET	PULSES	SUNFLOWER	GROUNDNUTS
1979	41	19	8	14	4	2
1980	126	41	14	12	5	1
1981	118	54	13	15	1	2
1982	36	12	3	3	1	1
1983	34	16	6	3	-	-
1984	46	1	8	3	-	-
1985	78	7	10	3	-	1
1986	90	8	10	5	-	-
1987	103	3	5	2	-	-
1988	219	30	9	10	-	-
1989	184	45	11	16	-	1
1990	132	34	9	12	-	1
1993	67	22	11	9	1	1
1995	104	18	7	10	3	-
1996	164	100	9	25	-	1
1997	65	78	8	34	1	4
1998	26	24	5	13	-	9
1999	45	34	5	11	-	1
2000	53	78	3	22	1	2
2001	13	25	2	8	1	1
2002	31	35	-	11	-	-
2003	5	8	1	3	-	-
2004	39	43	6	8	-	-
2006	46	60	6	16	0	1
2007	7	8	2	3	0	0
2008	35	49	7	13	0	1
2009	47	82	8	18	1	2
2010	44	65	9	15	0	3
2011	49	101	11	20	14	1
2012	24	56	7	12	0	1
2013	34	44	8	16	-	1
2014	50	104	7	25	1	1
2015	8	13	3	6	-	1
2017	17	51	3	16	-	1
2019	5	7	2	6	-	-

Figures for 1991 -1992, 1994 not available, 2001, 2014, 2017 and 2019 are traditional sector only, 2005 are commercial sector only

TABLE 1.7: TOTAL PRODUCTION TRENDS (METRIC TONNES) BY CROP (Traditional & Commercial)

YEAR	SORGHUM	MAIZE	MILLET	PULSES	SUNFLOWER	GROUNDNUTS
1979	5,000	3,000	1,000	1,000	900	200
1980	29,000	12,000	3,000	2,000	1,400	1,400
1981	28,000	22,000	2,000	3,000	1,200	2,000
1982	4,000	13,000	500	500	7,000	300
1983	5,000	9,000	500	300	200	800
1984	6,000	500	700	400	400	600
1985	15,000	1,500	1,800	400	500	800
1986	16,000	3,600	1,300	600	200	200
1987	18,000	3,300	400	100	100	100
1988	94,000	7,300	3,700	2,300	200	200
1989	53,000	20,000	2,000	2,500	200	500
1990	38,200	12,000	1,700	2,000	240	540
1993	10,797	2,976	1,624	530	113	126
1995	113,547	18,953	19,390	10,181	962	951
1996	59,200	24,629	2,508	3,914	39	551
1997	13,450	22,647	1,357	4,467	444	1,040
1998	3,743	2,344	507	1,198	1,267	121
1999	6,658	3,796	860	1,348	2,829	217
2000	12,671	19,962	439	4,102	163	545
2001	1,583	4,976	472	1,280	150	147
2002	15,805	16,447	54	1,907	2,250	137
2003	23,501	1,633	91	460	960	15
2004	6,095	4,291	958	1,093	479	119
2006	12,369	14,896	3,201	4,918	133	558
2007	11,774	2,158	532	805	2,558	76
2008	23,632	8,969	1,636	2,305	8,140	161
2009	29,379	19,247	2,757	4,317	1,765	1,538
2010	31,326	16,575	3,368	3,617	3,843	1,828
2011	32,591	29,235	2,511	4,700	15,837	833
2012	24,021	7,677	1,959	2,285	6,000	200
2013	10,231	3,844	1,391	3,655	2,021	112
2014	14,310	28,550	3,398	6,826	361	553
2015	35,508	3,792	555	7,187	1,388	151
2017	5,975	13,911	1,099	2,348	78	145
2019	826	987	313	583	73	6

Figures for 1991 -1992, 1994 not available, 2001, 2014, 2017 and 2019 are traditional sector only, 2005 are commercial sector only

TABLE 1.8: TRADITIONAL SECTOR: YIELD PER HECTARE PLANTED TRENDS (KG/HA) BY CROP

YEAR	SORGHUM	MAIZE	MILLET	PULSES	SUNFLOWER	GROUNDNUTS
1979	74	58	77	37	150	50
1980	197	150	176	83	200	280
1981	199	253	100	115	400	400
1982	43	203	31	25	350	150
1983	40	148	29	14	200	800
1984	53	10	41	24	200	150
1985	120	31	113	27	167	400
1986	99	82	74	33	211	129
1987	86	70	26	20	175	33
1988	338	141	225	128	200	282
1989	199	224	97	83	200	232
1990	185	147	135	98	300	257
1993	78	36	108	20	51	90
1995	685	220	1,708	394	197	856
1996	310	180	227	109	65	240
1997	67	189	129	58	147	193
1998	55	25	55	38	32	10
1999	47	30	109	36	69	73
2000	119	162	82	116	178	146
2001	68	104	141	87	156	100
2002	89	75	15	63	86	64
2003	37	42	36	41	24	23
2004	213	134	329	134	97	250
2006	211	191	444	218	313	442
2007	42	45	61	34	7	25
2008	82	95	132	69	158	49
2009	118	108	267	99	191	447
2010	101	96	301	111	180	429
2011	93	193	195	133	437	334
2012	144	53	153	63	165	91
2013	71	22	109	7	169	71
2014	220	220	400	182	263	337
2015	83	23	103	35	209	71
2017	251	225	353	86	216	98
2019	48	25	91	31	103	10

Figures for 1991 -1992, 1994 not available, 2001, 2014, 2017 and 2019 are traditional sector only, 2005 are commercial sector only

TABLE 2.1: SUMMARY OF TRADITIONAL SECTORS LIVESTOCK STATISTICS

INDICATORS		2014 TRADITIONAL	2015 TRADITIONAL	2017 TRADITIONAL	2019 TRADITIONAL
CATTLE:-	TOTAL HOLDINGS	65,853	37,753	33,819	29,355
	TOTAL CATTLE	1,596,605	1,360,467	1,100,375	934,732
	TOTAL COWS	710,704	599,420	494,229	420,873
	TOTAL BIRTHS	347,128	347,297	233,736	237,974
	TOTAL DEATHS	197,765	97,099	64,447	102,255
	TOTAL SALES	122,426	81,716	51,734	58,388
	TOTAL HOME SLAUGHTER	24,745	20,427	18,991	13,939
	TOTAL PURCHASES	17,725	12,263	10,145	6,660
	OFFTAKE	129,446	89,879	60,580	65,667
	OFFTAKE RATE	8.1	6.6	5.5	7.0
	BIRTH RATE	48.8	57.9	47.3	56.5
	DEATH RATE	12.4	7.1	5.9	10.9
GOATS:-	TOTAL HOLDINGS	76,913	43,853	58,332	42,179
	TOTAL GOATS	1,605,642	1,143,251	1,199,661	1,228,744
	TOTAL BIRTHS	672,844	498,855	469,240	482,226
	TOTAL DEATHS	293,144	191,487	279,215	231,798
	TOTAL SALES	71,656	53,185	44,043	50,972
	TOTAL HOME SLAUGHTER	56,441	44,364	48,299	47,969
	TOTAL PURCHASES	18,452	16,598	4,843	17,650
	OFFTAKE	109,645	80,951	87,499	81,291
	OFFTAKE RATE	6.8	7.1	7.3	6.6
	BIRTH RATE	41.9	43.6	39.1	39.2
	DEATH RATE	18.3	16.7	23.3	18.9
SHEEP:-	TOTAL HOLDINGS	18,690	12,362	12,992	11,905
	TOTAL SHEEP	227,247	214,234	234,621	242,911
	TOTAL BIRTHS	79,759	78,540	78,831	79,669
	TOTAL DEATHS	34,223	24,990	37,672	34,369
	TOTAL SALES	8,369	10,152	7,949	13,230
	TOTAL HOME SLAUGHTER	7,405	7,230	7,026	8,919
	TOTAL PURCHASES	9,663	4,007	5,821	3,454
	OFFTAKE	6,111	13,375	9,154	18,695
	OFFTAKE RATE	2.7	6.2	3.9	7.7
	BIRTH RATE	35.1	36.7	33.6	32.8
	DEATH RATE	15.1	11.7	16.1	14.1
OTHER LIVESTOCK:-	TOTAL DONKEYS HOLDINGS	39,470	27,679	24,630	21,824
	TOTAL DONKEYS	227,200	174,392	140,215	125,374
	TOTAL MULES HOLDINGS	-	-	-	192
	TOTAL MULES	-	-	-	426
	TOTAL HORSE HOLDINGS	7,116	6,893	4,219	4,637
	TOTAL HORSES	26,827	26,608	15,402	19,604
	TOTAL PIG HOLDINGS	1,742	433	222	484
	TOTAL PIGS	11,104	3,286	1,437	10,988
	TOTAL OSTRICH HOLDINGS	-	-	-	-
	TOTAL OSTRICHES	-	-	-	-
	TOTAL CHICKEN HOLDINGS	71,027	37,305	42,648	34,464
	TOTAL CHICKENS	1,017,444	569,935	709,829	520,893

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