



School of Earth Sciences and Engineering
Department of Chemical, Material and Metallurgical Engineering

Modelling and Optimization of Ultra-Fine Grinding of Copper Sulphide Ore.

by

Name: **Moyo Nkosilamandla**

Student ID number: **22017793**

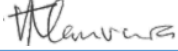
mn22017793@studentmail.biust.ac.bw

The thesis presented in fulfilment for the degree of Master of Engineering in
Chemical Engineering in the School of Earth Sciences and Engineering at the
Botswana International University of Science and Technology (BIUST).

Main Supervisor: Dr. Tirivaviri Mamvura

Department of Chemical, Materials and Metallurgical Engineering, School of Earth Sciences and
Engineering, BIUST

E-mail Address: mamvurat@biust.ac.bw

Signature:  Date: 05-09-2025

Co- Supervisor: Prof. Gwiranai Danha

Department of Chemical, Materials and Metallurgical Engineering, School of Earth Sciences and
Engineering, BIUST

E-mail Address: danhag@biust.ac.bw

Signature:  Date: 05-09-2025

September/2025

Co Supervisor: Dr. Prasad Raghupatruni

Department of Chemical, Materials and Metallurgical Engineering, School of Earth Sciences and Engineering, BIUST

E-mail Address: raghupatrunip@biust.ac.bw

Signature:  Date: 05-09-2025

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Candidate (please write in caps or type): NKOSILAMANDLA MOYO

Student ID number: 22017793

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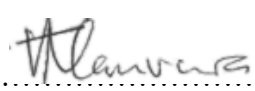
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Primary Supervisor: Dr. Tirivaviri Mamvura

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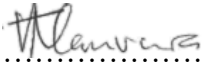
CERTIFICATION

This dissertation titled “**Modelling and optimisation of ultra-fine grinding of copper sulphide ore**” was prepared by **Nkosilamandla Moyo** and submitted as a fulfilment of the requirement for a degree of Master of Engineering in Chemical Engineering at Botswana International University of Science and Technology.

Approved by:

Primary Supervisor:

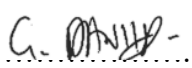
Dr. Tirivaviri Mamvura

Signed: 

Date: 05/09/2025

Co-Supervisor:

Prof. Gwiranai Danha

Signed: 

Date: 05/09/2025

Co-Supervisor:

Dr. Prasad Raghupatruni

Signed: 

Date: 05/09/2025

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DEDICATION

This work is dedicated to my family, my love and to aspiring young academics. I leave this quote for aspiring engineers, by Marie Curie, “Nothing is to be feared, only to be understood.”

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List of Abbreviations.

BIUST- Botswana International University of Science and Technology.

TNMC- Tati Nickel Mining Company.

BCL- Bamangwato Concessions Limited.

XRD- X-ray Diffraction.

XRF- X-ray Fluorescence spectrometry.

SEM- Scanning Electron Microscopy.

EDS- Energy Dispersive Spectroscopy.

ICP-OES- Inductively Coupled Plasma Optical Emission Spectroscopy.

KCM- Khoemacau Copper Mine.

BMR- Botswana Metal Refinery.

HDP- Hydrometallurgy Demonstration Plant.

UFG- Ultra-Fine Grinding.

OPAL- Oxidative Pressure Acid Leaching.

HPAL- High Pressure Acid Leaching.

ANN- Artificial Neural Networks.

ANFIS- Artificial Neuro-Fuzzy Inference System.

ML- Machine Learning.

AI- Artificial Intelligence.

List of Publications

1. Moyo, N., Marakalala, B., Gaogane, T., Danha, G., Mamvura, T. Assessing the viability of magnetic separator tailings as a secondary source of nickel, cobalt and copper: a case study for Tati Nickel Mining Company. *J. Eng. Appl. Sci.* **72**, 75 (2025). <https://doi.org/10.1186/s44147-025-00663-3>
2. Moyo, N., Mamvura, T., Danha, G., Raghupatruni, P. Modelling and optimisation of ultra-fine copper sulphide grinding. A hybrid statistical and machine learning approach. *Particuology* (2025). <https://doi.org/10.1016/j.partic.2025.08.010>

ABSTRACT

The global demand of base metals has been steadily on the rise since the fourth industrial revolution, and in the past 20 years, the rise has sharply increased, driven by the paradigm energy transition from traditional fossils utilization to green sustainable technologies. Base metals are key in these sectors, particularly in structural developments, smart materials for the health sector and as key elements in the renewable energy storage facilities, energy transmission and in the automotive industry.

Copper is an essential base metal, particularly in driving energy transition and electrification, in electronics and technology advancements, in urbanization and infrastructure growth etc. For instance, the utilization of copper in electric vehicles is reported as 2-4 times that of internal combustion engine cars therefore its demand has been steadily on the rise. The mining sector remains crucial to the development of Southern African economies, Botswana having its economy being driven by the diamond industry, however, observing the dangers of relying on diamonds, the government has since set its sights towards diversification of its industrial sector. This starts with the prominent mining sector, by promoting and resuscitating once operational base metals mines, particularly reviving the copper industry, which currently is the second mined mineral in the country.

Copper reserves in Botswana are predominantly sulphide ores therefore they have been beneficiated through the pyrometallurgy route. However, with the depletion of rich ore bodies, and the imposed environmental laws set to reduce climate effects resulting from anthropogenic emissions, the traditional route has not been favorable anymore. This led to the country exporting its resources solely in concentrate form, losing some valuable revenue in that transaction, therefore the need to extract, separate and purify base metals from these concentrates is more beneficial for the nation. Hydrometallurgy technologies developed in the late 20th century to the beginning of the 21st century, e.g. the Activox, CESL technologies, have demonstrated effective extraction, countering the traditional drawbacks of sulphide mineral leaching. This work looks therefore into adoption of the Activox technology for the efficient beneficiation of copper in Botswana.

The Activox technology is parent to two daughter processes, the ultra-fine grinding process and the oxidative pressure acid leaching. The scope of this work dwelt on the primary process,

the ultra-fine grinding process, an energy intensive process, for the mechanical activation of the mineral surfaces for effective extraction of copper from its mineral matrix. Due to equipment constraints faced, the work was conducted using a 5litre laboratory ball mill, with the aim of modelling and optimizing the process, i.e. the product particle size and energy consumption. This was achieved through hybrid modelling, i.e. machine learning modelling surrogated with statistical modelling. The StatEase Design Expert software was utilized for statistical modelling while the Artificial Neural Network and Artificial Neuro-Fuzzy Inference System packages in Matlab 2021a were used.

The UFG process is highly complex, as evidenced by the increasing research conducted on it aimed at assessing the impact of various factors on its efficiency, i.e. the grinding extent and energy consumption, e.g. rotational speed, powder filling, media type, media size, media filling, solids/liquids ratio and use of viscosity modifiers etc. In this study, the effects of grinding media size, milling speed, milling time and media filling ration were tested experimentally on the efficiency of the process on both counts. The outright realization herein was that media size greatly impacts the process, and this aligned with reports from literature with the generalisation that fine media best suits fine milling, and the effect of this was also noticed in energy consumption as lengthy milling periods were needed with coarse media for effective grinding compared to fine media. Further analysis of the observations from experiments using fine media showed that the base factor of importance was the media filling ratio, from which mill speed and milling time impacts relied on. This was a positive realisation as it highlighted the potential of improved production rate at maximum filling of the media, subject to optimisation of this filling rate using the attainable region as thus recommended in this work.

Modelling of the processed ore led to effective optimisation, firstly after realisation of 20 μ m being the optimum P₈₀ grind size, compared to the technology's 10 μ m from characterization. As such, the hybrid modelling exercise was effectively done, with regression analysis-validation results when using a different sulphide ore obtained at of 97.34% for P₈₀ predictions using the ANFIS technique while that of SE at 85.71% using the ANN technique. To ascertain the validity of this optimisation, a cost benefit analysis was conducted, and it proved that with optimizing the process from 10 μ m to 20 μ m, a 24.45% energy saving could be realized. This translates to a massive saving especially in a process that is energy intensive as such this could lead to the feasible adoption of the technology, leading to reviving the economy, creating more

employment opportunities. To comprehensively conclude on this optimisation, a comparison of the leaching at 10 μ m and 20 μ m is required to ascertain the extraction efficiency as validation of the entire work.

1. INTRODUCTION

1.1 Introduction to the problem

The importance of mining to most African countries including Botswana cannot be overemphasized as it is the major foreign exchange earner for the economies. Mineral deposits are assets, and so are part of a country's natural capital. Mining sector is the key that converts dormant mineral wealth into infrastructure such as schools, homes, ports, and other forms of capital that directly contribute to economic development, (Kosanke, 2019). For some time now, Botswana had been trying to diversify its economy so as not be only reliant on the mining sector and on diamonds to be specific. Despite Botswana's efforts to diversify its economic activities, mining still remains the backbone of the economy, contributing majorly to the government's revenue and generating majority of the export earnings (Nkundla et al., 2005).

For a greater amount of time, diamonds have been at the forefront, contributing over 90% of the mining sector's contribution to the nation's GDP (*Botswana*, 2001). However, of late, base mineralisation is increasingly becoming significant in the economic outlook for Botswana (Soto-Urbe et al., 2023). Sulphide mineral deposits in the country have been to date concentrated around the Northern part of the country, owing to the Greenstone Belt and the Kalahari Copper Belt (KCB). The Greenstone Belt, Fig. 1-1, gave rise to daughter Tati and Selebi-Phikwe belts of nickel-copper (Ni-Cu) deposits, (2-9% Ni and 0.05-4% Cu), and (1-3% Ni and 0.1-4% Cu), respectively (Hepworth & Marengwa, 1976), while exploration to the KCB Fig. 1-2, denoted it as copper-silver (Cu-Ag) deposit. Currently in operation are the copper mines, MMG Khoemacau Copper Mine (after expansion to 130 ktpa Cu and 5 Mozpa Ag) located 77 km away from Maun and Sandfire Motheo Copper Project (49.6 metric tons of resource at 0.96% Cu and 13.6 g/t Ag) in the Western part of the country (Keeditse, 2022), North-East of Ghanzi town, with hopes of resuscitation of Phoenix pit by Nickel Resources. Operation of MMG KCM and Motheo copper project have aided in the nation's GDP due to their concentrate export.

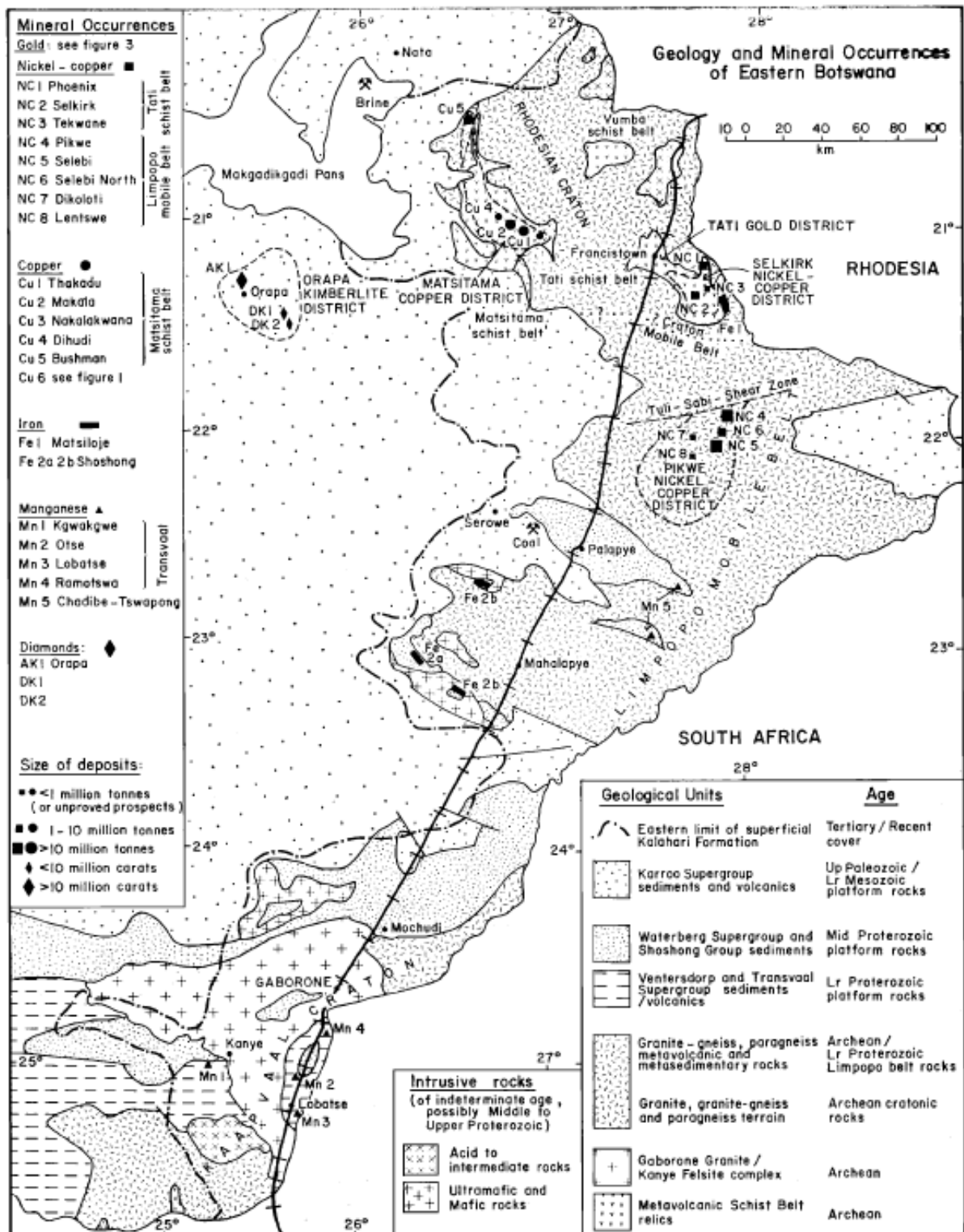


Figure 1-1: Geology and mineral occurrences of Eastern Botswana i.e. Greenstone belt, (Hepworth & Marengwa, 1976).

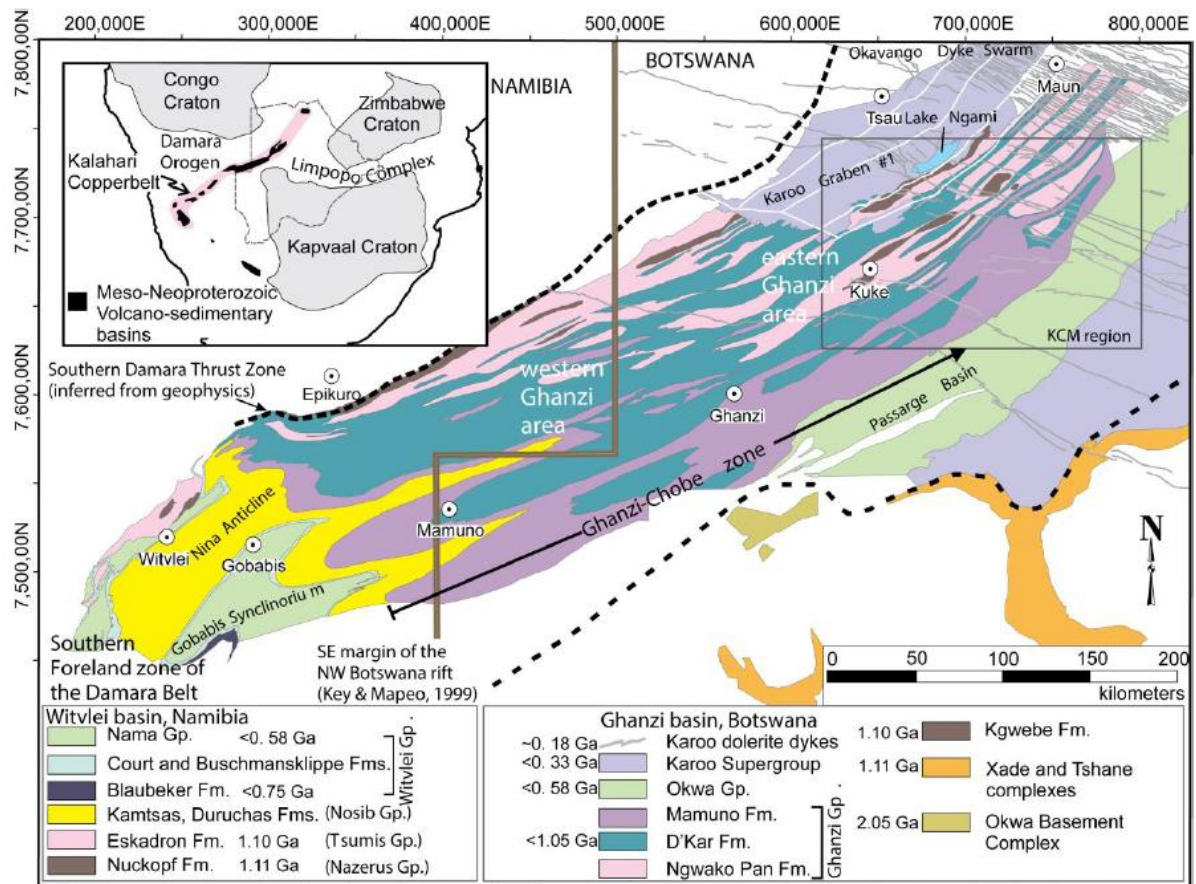


Figure 1-2: Geological setting of the Kalahari Copper Belt, (Keeditse, 2022).

1.1.1 Background study

Botswana has been praised for its effective mineral resource management by making sure it benefits both the present and future generations through investing into education, health care and asset acquisition, unfortunately the impact of the mining sector to the country's fiscal stability is slowly degenerating and its long-term impact is not guaranteed no more. Many of these challenges emanate from metal price market volatility, fiscal challenges of capturing resource rents, failure to plan properly and the negative effect of the corruption activities from government officials as well the macroeconomic impacts of monetary index rate fluctuations, i.e. the Dutch disease (Nkundla et al., 2005).

Also, the economic growth of Botswana has been backed by the diamond industry for decades and this has led to unwarranted dependency and vulnerability on the prosperity of the precious mineral, which also comes at the mercy of the market controllers i.e. the De Beers group company. There is therefore a dire need to diversify from the diamond industry and as the nation invests in other industrial sectors, base metal beneficiation holds an immense potential

in revenue generation for the nation.

Base metals are vital elements in the electrical power industry, particularly copper, nickel and cobalt which play a pivotal role in renewable energy, energy storage technology for electrical vehicles production amongst many other applications. Although the base mineral market is faced with volatility, the secondary products market, however, is consistent with noticeable growth patterns, this alludes to the need for resource base economies to consider diversification of their operations from mineral processing to extraction and product development from these base minerals.

Traditional beneficiation of base minerals in Botswana and the Southern region of the continent has majorly been attributed to pyrometallurgy, owing to the high-grade ore reserves present. However, through extensive exploitation of these mineral resources over time, the ore grades have depreciated, considerably bringing about challenges with the recovery processes as low grade ores beneficiation would directly constitute high production costs. Therefore, a greater need for low-cost processing methods has arisen (Watling et al., 2009).

Activox technology, a novel patent by Western Minerals Technology Pty Ltd of Australia, (Palmer & Johnson, 2005a) adopted under Botswana Mineral Refinery major was demonstrated in the early 21st century for the efficient recovery of low-grade base metals of sulphide nature at Tati Nickel Mining Company (TNMC), particularly for the recovery of nickel, cobalt and copper. This technology basically involves ultra-fine grinding (UFG)- of sulphide concentrates up to P₈₀ of 10 microns and oxidative pressure acid leaching (OPAL) with temperature in the range 90-120°C, pressure in the range 8-10 bars, whilst bubbling with pure oxygen, which governs the performance of the technology, i.e.

- ❖ High recoveries of base metals, e.g. nickel, cobalt, copper recoveries exceeded 95% at the Tati Hydrometallurgy Demonstration Plant (HDP).
- High production rate per unit size of equipment as oxidative kinetics aid in relatively fast reaction rates.

Like all other processes, this technology has some of its drawbacks and it is mostly on energy utilisation, and materials selection amongst other issues. The issue on high energy utilisation means high production costs which could not be supplemented by the plummeting base metal market prices, (Nel & van der Berg, 2009), and eventually causing the technology not to be fully

implemented commercially in Botswana (Faris et al., 2022). However, there is a need to optimise the technology so that it can be fully implemented in Botswana as previously demonstrated.

1.1.2 Problem Statement

The intensiveness of the UFG and the OPAL operating conditions negatively impact on the techno-economic feasibility of the technology. Milling contributes in general to above 50% of the operational costs, and with ultrafine grinding, the higher the energy drawn by the mill, the higher the operating costs (Kumar et al., 2023) hence this makes UFG an energy intensive process. OPAL operating conditions also have huge implications on the economics of the process, though with higher recovery efficiencies, it needs backing of volatile market prices i.e., higher metal prices.

To ensure better understanding of these processes, modelling can be roped in to assist as it conserves the costs while reducing the number of experiments to be done. Modelling is a technique trusted for over decades to fully understand a process' mechanisms, project the attainable regions with various operating conditions and optimise the process with certain confinements or process constraints. To successfully optimise the Activox process requires modelling of the two main processes, UFG and OPAL.

Morrell (Morrell, 2004) developed a model to quantify energy consumption for fine milling, as the traditional comminution models could only accurately be used for milling up to product sizes of 75 microns. There existed a gap to properly understand and precisely quantify energy consumption during fine milling. Morrell then proposed a model; however, it can only be accurately implemented for milling up to 45 microns, leaving again a gap to quantify for ultra-fine milling (Morrell, 2004). The need to develop such a model is because as milling goes to the ultra-fine region, more energy per unit mass is consumed thus making UFG even a more energy intensive process.

Mathematical modelling is a tool that can assist as the basis to address this challenge, supported with Artificial Intelligence (AI), incorporating Machine learning as a tool to simply and effectively learn from the process and gather insights on it (Schweidtmann et al., 2021). The developed model's target should be to effectively predict energy consumption during UFG and the product P_{80} to avoid overgrinding thus reducing unnecessary power consumption.

OPAL by its nature, though aiding improved process kinetics leading to higher recoveries at

shorter time periods, it is relatively expensive as compared to prevalent ambient conditions (Mbaya et al., 2013). Its high product yields per unit volume of the equipment aids its advantages, however, metal market fluctuations negatively affect the process therefore there is need to optimise the process so that as much benefit is obtained from it with the smallest of expenses incurred. The feed particle size distribution (PSD), oxygen bubbling rate, pressure, temperature, stirring rate and time contribute to the efficient recovery of base metal elements overall and the extent to which each parameter affects efficiency can only be understood through process modelling hence leading to optimisation.

Therefore, this work investigates modelling of the ultra-fine grinding process (UFG) for a copper sulphide ore, to optimise energy consumption and product quality i.e. the P_{80} of the product particle size distribution.

1.1.3 Research Questions.

This research work seeks to address and answer the following questions:

1. What is the impact of UFG on overall efficiency of Activox technology?
2. Which parameters directly affect the UFG of copper sulphide ores and to what extent is media diameter size a contributing factor on power consumption?
3. How effective is the UFG of copper sulphide ores process using traditional ball mills and what mechanism dominates the process?
4. Which predictive modelling tool best simulates the UFG of copper sulphide ores process?

1.2 Aim and Objectives

1.2.1 Aim

The aim of this project is to model the ultra-fine grinding process of a copper sulphide ore, to optimise its energy consumption and product quality.

1.2.2 Specific Objectives

1. To sample and characterise the local sulphide concentrate on determining its

mineralogy.

2. To evaluate the effects of mill speed, time, media size and media filling on process efficiency and power consumption.
3. To model, optimise, validate and simulate ultra-fine grinding (UFG) using the developed model from hybridised statistical and machine learning tools- artificial neural networks (ANN) and adaptive neuro-fuzzy inference system (ANFIS).
4. To conduct an economic analysis to ascertain the milling efficiency of ultra-fine grinding.

1.3 Hypothesis

UFG mechanically activates the surface of mineral particles, fully separating the mineral surface from gangue material thereby increasing the exposure of the mineral for ease of access by the lixiviant hence improved dissolution kinetics. Leaching kinetics of sulphide minerals follow mostly the shrinking core model therefore with mechanically activated surface, the transport phenomena during leaching is greatly enhanced, reducing the effects of sulphide encapsulation, prone to hydrometallurgy of sulphide minerals. Predictive modelling allows for effective optimisation of the process i.e. attaining fine grinding target regions at minimum energy expenditure. Precise utilisation of statistical and machine learning modelling tools eases the complexities of analytical modelling while not compromising on the predictive accuracy. Exercising all this work with the conventional ball mills addresses the potential use of the mills which have been otherwise deemed not fit for such applications.

1.4 Dissertation outline.

The dissertation outline is as follows:

- ❖ Chapter 1: Introduction.
- ❖ Chapter 2: Literature review.
- ❖ Chapter 3: Methodology.
- ❖ Chapter 4: Sulphide concentrate characterisation.

- ❖ Chapter 5: Ultra-fine grinding analysis, modelling, optimisation and validation.
- ❖ Chapter 6: Economic evaluation, conclusion and recommendations.

2. Literature review.

2.1 Introduction.

With the depletion of rich ore bodies, the necessitated move to beneficiation of low-grade ores and secondary mineral sources had to be backed up by complimentary mineral processing techniques for maximum efficient recoveries at minimum costs. The complexity of these ores, according to their mineralogy and fine grain dissemination in the gangue material is one important factor that must be addressed to enhance efficiencies of downstream process recovery systems. Hydrometallurgy is the most economic route to extracting minerals from these typical sources and the efficiency of the process is based on how fast the leaching agent breaks through the boundary layers into the mineral surface where dissolution takes place, and then diffuse back into the bulk solution, Fig 2.1.

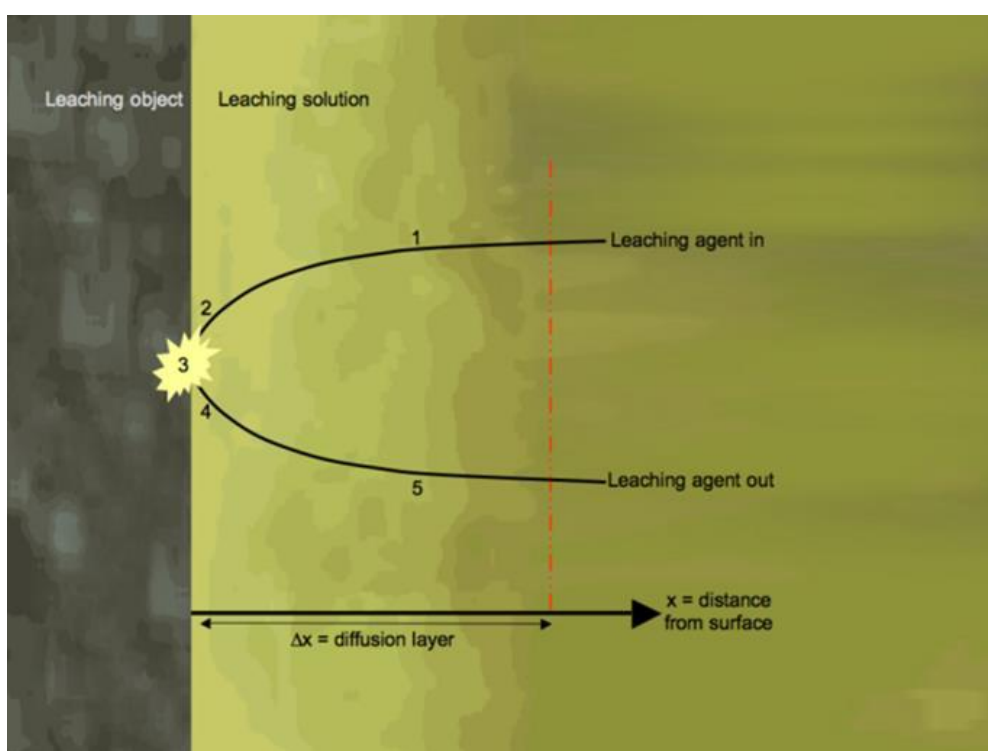


Figure 2-1: Transport phenomena involved in leaching.

Generally, processing sulphide mineral sources through hydrometallurgy has impeding challenges owing to the growth of the passivation layer because of elemental sulphur i.e. engulfing of mineral particles by a surface layer during oxidative leaching, Fig 2.2. This barrier hinders ease of diffusion, in and out of the mineral particle, by the leaching agent. As such, extraction of the valuable mineral is halted, hence the reason why pyrometallurgy was favored over hydrometallurgy when processing sulphide ores..

To mitigate this challenge, several techniques have been employed to counteract the negative impact of the passivation layer i.e. use of oxidants e.g. strong oxidants like KMnO_4 , H_2O_2 etc., ferric ion and bubbling pure oxygen or air; use of chemical additives like surfactants or sulphur complexing agents which prevent passivation through reducing adhesion of sulphur to the mineral particles and by keeping the sulphur in dissolution via formation of complexes, respectively; adjusting leaching conditions e.g. controlling pH e.g. in acidic form, operating at elevated temperatures or maintaining high redox potential, to destabilize the passivation layer; through mechanical means e.g. agitation or mechanical activation, to dislodge and increase surface area of mineral particles; biological means; organic solvents or electrochemical techniques; all these to destabilize the elemental sulphur layer. The selection of the technique to employ is then dependent on the sample mineralogy, leaching agent of choice and operating conditions to be used, in which it is rather a common practice to combine these strategies for optimum leaching efficiency e.g. Activox technology.

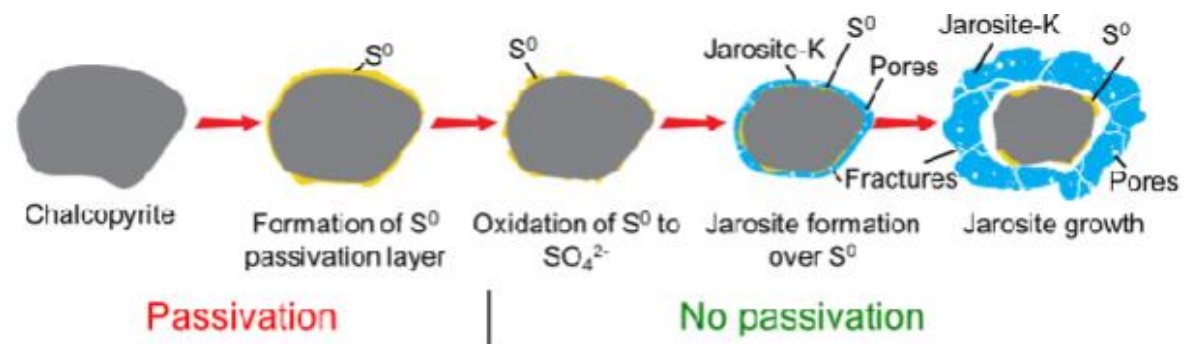


Figure 2-2: Effect of passivation layer growth during leaching.

In the Activox technology, ultra-fine grinding technique is employed to activate the mineral surfaces by exposing them for ease of access by the leaching agent thereby improving the kinetics of the involved reactions i.e. oxidative pressure acid leaching (OPAL). The extent to which grinding is conducted, whether fine or ultra-fine will be dependent on the mineral grain sizes disseminated in the ore as well as the economics for the entire process i.e. fine grinding generally depicts size reduction to product sizes less than 45 microns while ultra-fine defines the grinding to less than 20 microns. Because this is energy intensive, to conserve expenditure, it is typically applied to concentrate forms rather than raw ore.

2.2 Ultra-fine grinding.

As observed in Fig. 2.3, mechanical activation entails further grinding of the ore to create fine

and ultrafine powders by using compression, attrition, abrasion, and impact force (Kumar et al., 2023).

As previously mentioned, low recoveries from dissolution difficulties caused by elemental sulphur formation, high operating costs from reagent use, and environmental effects have all been associated with the serious difficulties of hydrometallurgy sulphide mineral resources (Dreisinger, 2006; McDonald & Muir, 2007a; Zheng et al., 2023a). UFG (mechanical activation) offers improved metal recoveries with no environmental impact and is a straightforward, affordable method with a more universal approach. Higher leaching rates and improved leaching selectivity are the outcomes of using UFG before leaching because it increases the surface area of the particles and creates nano crystal mineral particles with higher reactivity and solubility.

The growing need to efficiently and economically exploit, beneficiate, and extract metals from low-grade ores and secondary sources, such as tailings and electronic waste, in order to improve the liberation of mineral values from gangue material and produce feedstock amenable to subsequent extraction processes, is one of the reasons why UFG technology has become more and more adoptable over time (Harbort et al., 1996).

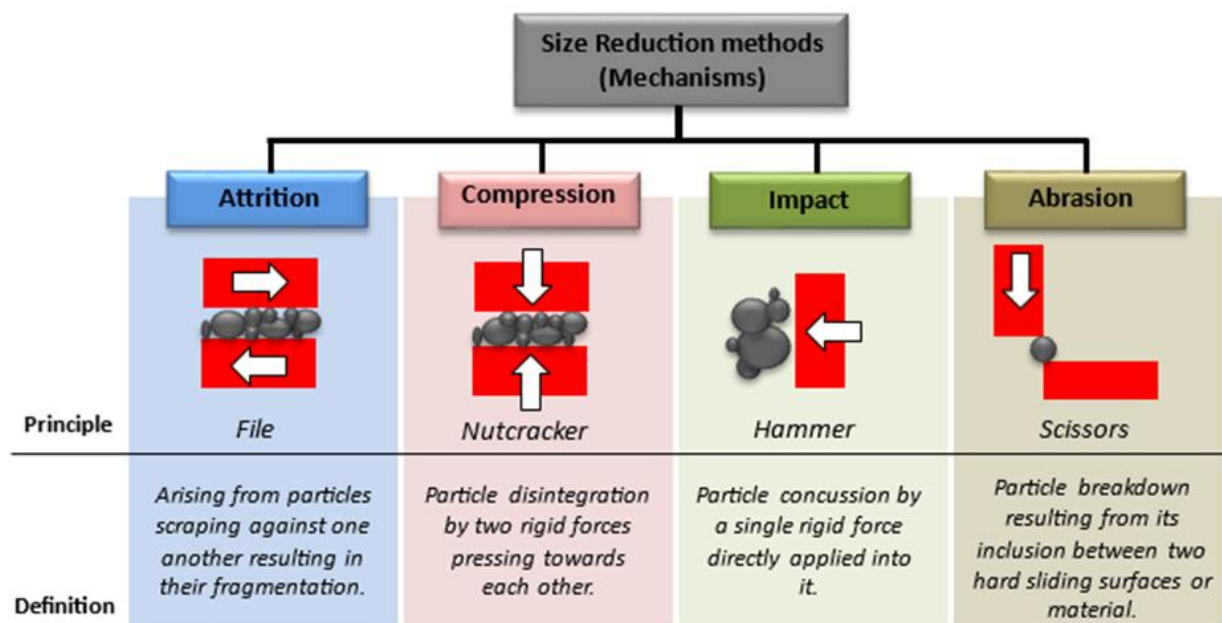


Figure 2-3: Different particle grinding mechanisms

According to Fig. 2.4 below, which shows the overall trend of specific energy consumption by a milling circuit product size range, UFG is generally an energy-intensive process. This is also

consistent with (Partyka & Yan, 2007; Patino et al., 2022). Thus, innovations to optimise design parameters, operating conditions, and system energy efficiency, among many others, have been in the forefront of UFG research to increase the efficiency of the grinding process (Allen & Ntsele, 2012; Jayasundara et al., 2010a; Ohenoja & Illikainen, 2015; Öksüzoğlu & Uçurum, 2016; Patino et al., 2022; Tanaka, 1995).

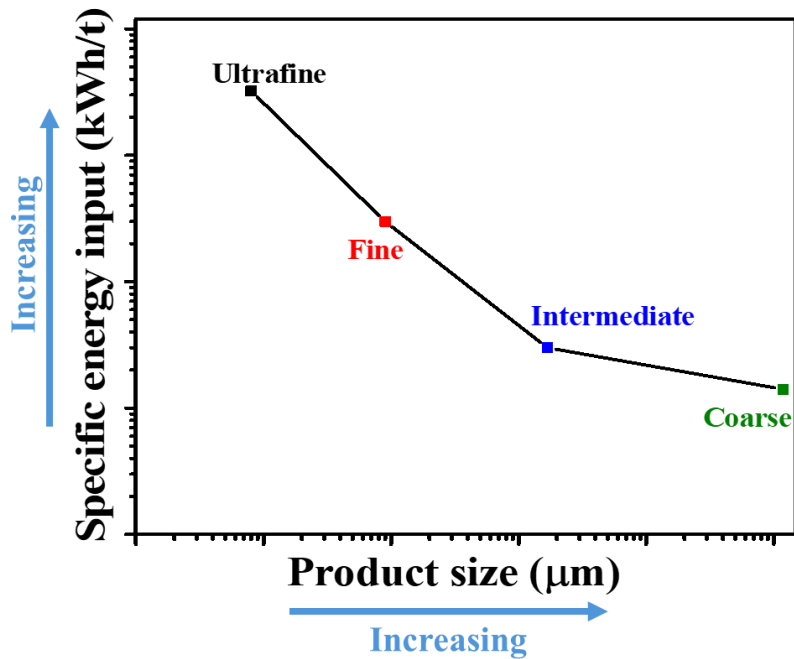


Figure 2-4: Energy input vs extent of grinding trend.

2.2.1 Principle of operation.

The main force in UFG is attrition, which depends on the mill's interstitial filling and needs intimate contact between grinding media to be effective (Choi et al., 2007a; Patino et al., 2022). Through impact force, abrasion between medium and particles, and frictional forces, attrition affects UFG through particle-to-particle interaction inside the grinding mill. The amount of internal space in the mill that is occupied by the grinding media is known as interstitial filling, and it has the following effects on UFG (Tembo et al., 2019a):

1. Stress intensity: As the stress intensity rises, better particle breaking is encouraged by higher filling.
2. Grinding efficiency: very low filling has a detrimental effect on grinding efficiency, whereas excessive filling increases the rate at which media wears down.
3. Resident time: resident time influences the grinding process's overall kinetics, which in

turn influences filling rate and, ultimately, throughput per unit of equipment size.

Generally speaking, attrition is a function of the media-particle contact surface area, which rises as the size of the grinding media decreases, leading to effective particle size reduction (Hlabangana, Danha, & Muzenda, 2018; Tanaka, 1995). The next step is to have a better understanding of the elements influencing this process' efficiency.

2.2.2 Effect of milling parameters on efficiency and energy consumption.

The following factors typically influence the production of ultra-fine powders:

1. Grinding media size (Tanaka, 1995).

The efficiency of UFG is examined in relation to the surface area of the media's contact sites, the frequency of collisions between them, the stress that the striking balls apply, and the likelihood of grinding. In general, all these factors are enhanced as grinding media size decreases, improving grinding efficiency. However, as media size decreases, media separation from products becomes more difficult, requiring extra energy input to facilitate separation, as such a balance is necessitated.

2. Mill/ impeller rotational speed (Choi et al., 2007a).

Increasing speed enhances the kinetic energy transferred to the impeller blades to increase the frequency of collisions between media and particles, to the media to increase the impact force of the media on the particles, and to prevent dead grinding zones within the grinding chamber. This significantly improves milling efficiency, however, it comes at the cost of excessive energy consumption. Also, the increase in mill speed causes centrifugation of mill contents especially in ball mills as such a balance is required which will optimise both the milling efficiency and power consumption (Corrans & Angove, 1991).

3. Ball filling ratio (Kumar et al., 2023)

The media filling ratio has a significant impact on attrition, which is the main mechanism in UFG. According to the operating principle, this is regarded as the interstitial filling ratio, and a higher ratio often corresponds to a finer product size. Also, the increase in ball filling ratio directly affects the required rotation of the mill/impeller to effect proper mixing, thus an

increased energy requirement, therefore, a balance must be struck to overcome the disadvantages of excessive media filling.

4. Powder filling ratio (Choi et al., 2007a).

Filling the feed material is essential to assessing the process's efficiency. For the feed powder particles to break effectively, they must be enclosed within the media collision zones; thus, they must only occupy the vacuum spaces within the media i.e. interstitial filling. Down filling this space leads to energy wastage while overfilling leads to reduced milling efficiency. Also, increased filling rates increase the viscosity of the mill contents, and this negatively impacts energy consumption as more impeller energy is required to break this viscous lattice. A balance between these two parameters, as well as total throughput, is a crucial element in fostering efficient UFG. The feed material also plays a significant role in lowering the media's wear rate by preventing media-to-media collisions.

5. Grinding time (Öksüzoğlu & Uçurum, 2016).

Time is an important parameter across all grinding processes, correlating to efficiency, together with the process's economic feasibility. Longer residence durations lead to enhanced size reduction efficiency, however, lowers the overall production rate, which impacts subsequent operations to acquire the final product for commercial uses, consequently, lowering the income earned. Increase milling periods though may enhance the fineness of the grind (Klimpel, 1999), however leads to excessive energy consumptions as power is rated per time. Also, increased milling time consequently leads to overgrinding, that is energy waste, as such, it is one of the key parameters to be optimized in UFG.

In addition to these important factors, there are other factors that also contribute to the process's efficiency. These include the impact of chemical dispersants, the size distribution of the media, the size distribution of the feed particles (Ortiz-Landazuri & Colay, 2019), the density of the feed slurry for wet milling (He et al., 2004) and how its rheological behaviour affects the level of interparticle interactions (Muster & Prestidge, 1995). For each application instance, the ideal values of these parameters are crucial to the effective use of the UFG process. The characterisation of the material (concentration) to be processed determines the extent of the grinding.

2.3 Characterisation of sulphide concentrate.

X-Ray Powder Diffraction (XRD) was used to identify the crystalline phases in the concentrate material, X-Ray Fluorescence (XRF) and Inductively Coupled Plasma-Optical Emission Spectrometry (ICP-OES) were used for bulk chemical analysis, and Scanning Electron Microscopy-Energy Dispersive X-ray Spectroscopy (SEM/EDS) was used to analyse the morphology and surface of the concentrate particles. These techniques were necessary to solidify a potential beneficiation approach.

2.3.1 X-Ray Powder Diffraction.

Crystalline material's phases, mineral content, and structural characteristics can all be determined using this analytical non-destructive method. The three primary components of the apparatus—the x-ray tube, sample holder, and x-ray detector—allow for the observation of the distinctive diffraction pattern depending on the mineralogy of the concentrate. The constructive interference of the scattered rays creates the diffraction pattern, which is specific to the material's crystalline structure and consists of a sequence of peaks with varying intensities and angles (2θ). Each peak corresponds to the different plane spacing (d-values) and the wavelength of the x-rays.



Figure 2-5: XRD analytic equipment housed at materials testing laboratory, BIUST.

It is a primary analytical instrument used to estimate the quantity and purity of a target mineral, as well as its physical and chemical characteristics, in a sample. XRD is used for process monitoring, quality control, and optimisation in addition to ore characterisation (Ayedzi et al., 2024).

2.3.2 X-Ray Fluorescence.

This is a simple, inexpensive, non-destructive, and quantitative analytical method for figuring out a material's bulk elemental makeup (Mohanraj et al., 2022). When the sample is exposed to a high-energy x-ray beam, the atoms inside it are excited, which leads to the ejection of low-energy inner shell electrons, creating vacancies that are subsequently replaced by electrons of higher energy levels. The strength of the x-ray peak generated will thereafter be proportional to the element's content in the sample. This transition causes the formation of typical x-rays of energy levels characteristic to each element. For this reason, Fig. is a common tool.



Figure 2-6: XRF analytic equipment

2.3.3 Inductively coupled plasma- optical emission spectroscopy.

This analytical method is employed to analyse the concentration of the metal elements in solution. Before analysis, the concentrate is completely dissolved by employing strong acids and microwave digestion of the sample at high temperatures. It is an effective and adaptable method that offers multi-elemental analysis for trace elements, impurities, and valuable metals all at once. Its sensitivity in many elements, even at very low concentrations, makes it profitable when working with low-grade ores and in anthropogenic mining (Gabasiane et al., 2021). Figure 2.7 illustrates the economical and effective ICP-OES Analyser SPECTRO GENESIS for use in academic labs.



Figure 2-7: ICP-OES analytic equipment.

2.3.4 Scanning Electron Microscopy- Energy Dispersive X-ray Spectroscopy.

Although it is semi-quantitative, this potent surface analytical method is utilised to ascertain the concentrate's surface characteristics, such as shape, microstructural characteristics, and elemental composition estimation (Olubambi et al., 2008). Like the XRF, the device consists of an electron gun that shoots a concentrated electron beam across the sample surface in the direction of the sample holder. This causes electrons to interact with the atoms in the sample, producing backscattered electrons, secondary electrons, and distinctive x-rays.

The EDS is then used for detection with an elemental contrast, meaning that high atomic elements appear brighter compared to lighter elements and gangue, which appear darker i.e. bright, shiny segments signal presence of high atomic elements, high electron density elements and conductive materials, while dark, grey parts are for low atomic, low density and non-conductive elements (Mohanraj et al., 2022). These signals are collected by the detector, which creates detailed imaging at high resolution for surface morphology. The EDS can map the elemental composition of the entire sample by characterising and performing target-specific

analysis. Sample preparation must be done carefully since it plays a crucial role in establishing the degree of accuracy and imaging. A Zeiss SEM/EDS analytical tool is depicted in Fig 2.8.

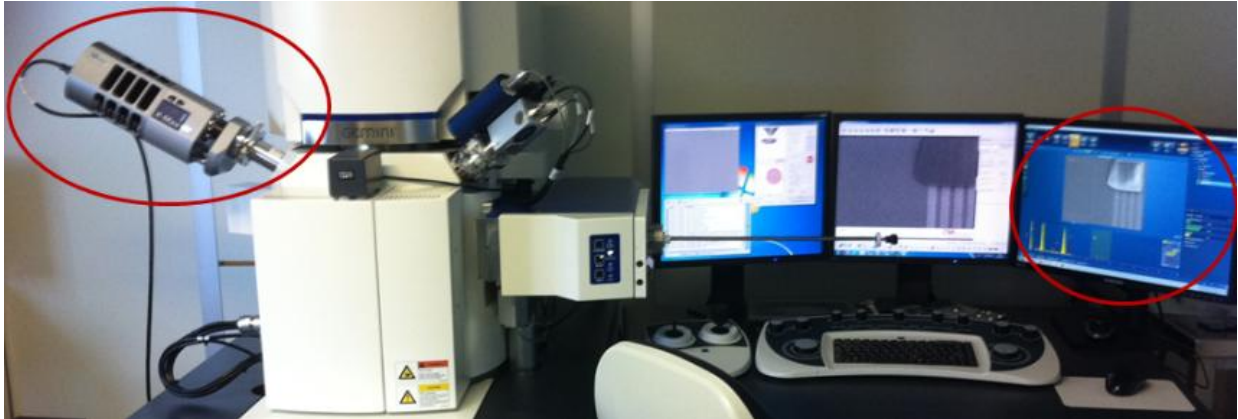


Figure 2-8: SEM/EDS analytic equipment.

2.4 Selection of equipment used.

The choice of equipment is crucial when designing the milling circuit and is regarded as the parent of aspects that contribute to the effective operation of the UFG process. The following factors are typically taken into consideration: ore hardness, equipment availability, maintenance and operating expenses, feed particle size distribution (PSD)- F_{80} , projected product PSD- P_{80} , and so on (Stanujkic et al., 2019). High output rates of ultrafine powders in the shortest amount of time with the lowest maintenance costs are the key attributes for the chosen mill, i.e., to improve the design characteristic for the milling circuit.

Because of the specific energy usage, Fig. 2.9 illustrates the overall cost trend of employing tumbling mills as opposed to stirred media mills.

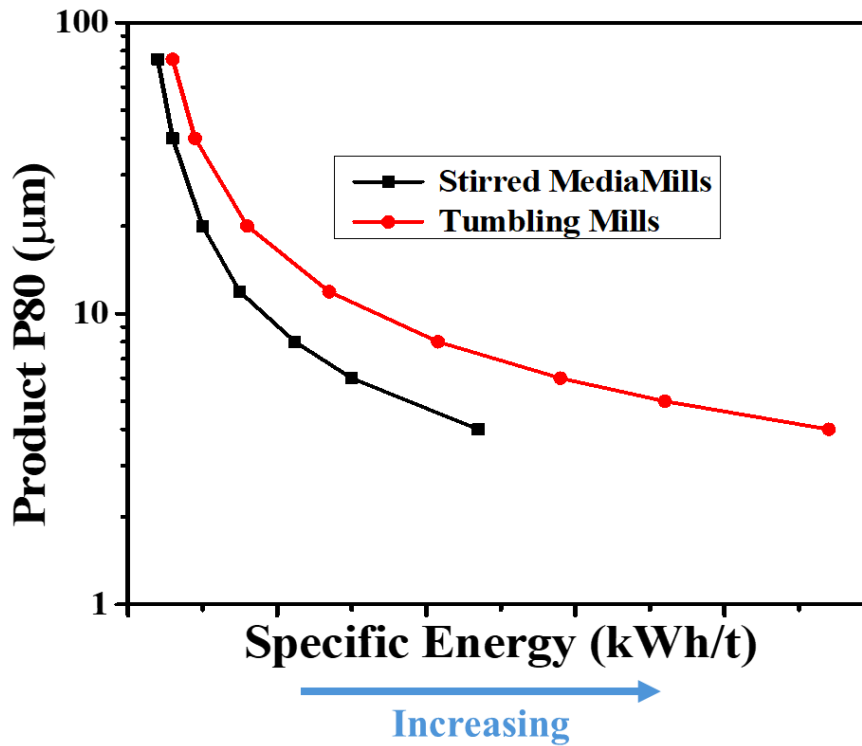


Figure 2-9: Performance comparison between tumbling mills and stirred media mills.

Due to their great energy efficiency and large unit yields, stirred mills are preferred (Bernhardt et al., 1999; Kapur et al., 1996). These mills have lower investment costs, more energy efficiency, a more accurate P₈₀, and a shorter retention time than traditional (tumbling) mills. A stationary chamber with a high-speed stirrer mounted on a drive shaft is part of their design. To create an efficient grinding action while churning the feed slurry-media mixture inside the chamber at high speeds, the grinding media is loaded into the chamber at high loading ratios (Kwade et al., 1996). Planetary mills, centrifugal mills, jet mills, and other primary mills can also be used for ultra-fine grinding. The two types of stirred mills that are most used are horizontal and vertical.

The primary disadvantage of tumbling mills is that they require longer grinding times, which raises energy consumption at the price of output (Sterling et al., 2023). Since they can handle huge tonnages at once and are currently widely used in industry operations, efforts to optimise their performance for UFG applications have been reported with success (Little et al., 2017; Partyka & Yan, 2007). Although Partyka et al.'s study suggests using finer grinding media in these tumbling mills for fine milling, there is still a dearth of thorough research to definitively identify the ideal range of parameters that tumbling mills can use for UFG. Developing a laboratory-scale tumbling mill modelling outline for UFG applications is the aim of this work.

The various UFG mill technologies utilised, along with the corresponding performance and operating parameters, are displayed in Table 1.

Table 2-1: Different mill technologies applied for UFG.

Mill type	Mill Volume (L)	Ore type	Feed F_{80} (μm)	Product size- P_{80} (μm)	Specific Energy consumption (kWh/t)	Reference
Lab-scale (Hosokawa Alpine 90 AHM hydro mill)	1.1.2	Titanium dioxide (TiO_2)	1	$D_{100}=0.3$	100	(Jayasundara et al., 2010a)
Lab scale stirred mill	22.3	Kokaksu Bauxite ore	780	$D_{50}=5$	300	(Hacifazlıoğlu, 2018)
Batch type lab scale pin-type vertical stirred mill	0.58	Au/Ag ores from Akoluk (Ordu in Turkey)	60	$D_{80}=3.37$		(Celep et al., 2011)
Netsch, GmbH with disk-type stirrer V-mill: lab scale	5.3	PGM bearing chromite ore	22.09	$P_{80}=18.73$	4.29	(Altun et al., 2013)
Netsch, GmbH with disk-type stirrer V-mill: lab scale	5.3	UG2 ore (PGM bearing chromite ore)	29.42	$P_{80}=19.61$	13.33	(Altun et al., 2013)
Stirred media detritor (SMD) lab scale	Diameter = 219mm	UG2 ore (PGM)	380	$P_{80}=41$	22.1	(Little et al., 2017)
Ball mill- lab scale	Diameter = 300mm	UG2 ore (PGM)	380	$P_{80}=33$	26.0	(Little et al., 2017)

Vertical stirred ball mill	0.6	Potassium feldspar	75	3.3		(Bilir & Ipek, 2015)
NETZSCH stirred mill- lab scale	5.3	Chromite ore	800	$D_{80}= 247.7$	26.01	(Ohenoja & Illikainen, 2015)
Bradken-Metprotech V-mill, pilot scale	200	Pentlandite concentrate	60	$P_{80}=10$	45	(Palmer & Johnson, 2005b)
Netsch IsaMill	45	Pentlandite concentrate	60	$P_{80}=10$	45	(Palmer & Johnson, 2005b)

The table clearly shows that stirred mills are superior to traditional ball mills for fine and ultra-fine grinding. They are commended for producing ultra- and nano-fine items in the shortest amount of time and with the least amount of specific energy. Yes, that is true; nevertheless, the table also shows that ball mills can perform fine and ultrafine grinding, albeit at a cost in energy. This implies that there is potential for their use in sectors that now employ them in their milling circuits, provided that significant operating parameter optimisation is done. The performance variations in Table 2-1 confirm that operational parameters, which were covered in the previous section, have an impact on each mill type's performance.

It is interesting to notice that (Hacifazlıoğlu, 2018) suggests that there is a limit to fine milling where the most efficient use of ball mills occurs; beyond that, the stirred mill becomes the most efficient mill type (Fig 2.10). This then implies that ball mills can be utilised for bigger mineral particle sizes, based on the analytical results, especially the QEMSCAN or SEM/EDX on sizing of mineral particles, to guide the extent of milling. This discovery is consistent with (Little et al., 2017) findings indicating both stirred mill and ball mill types show comparable patterns in energy consumption during fine milling; however, at UFG, the stirred mill is 15% more efficient than ball mills.

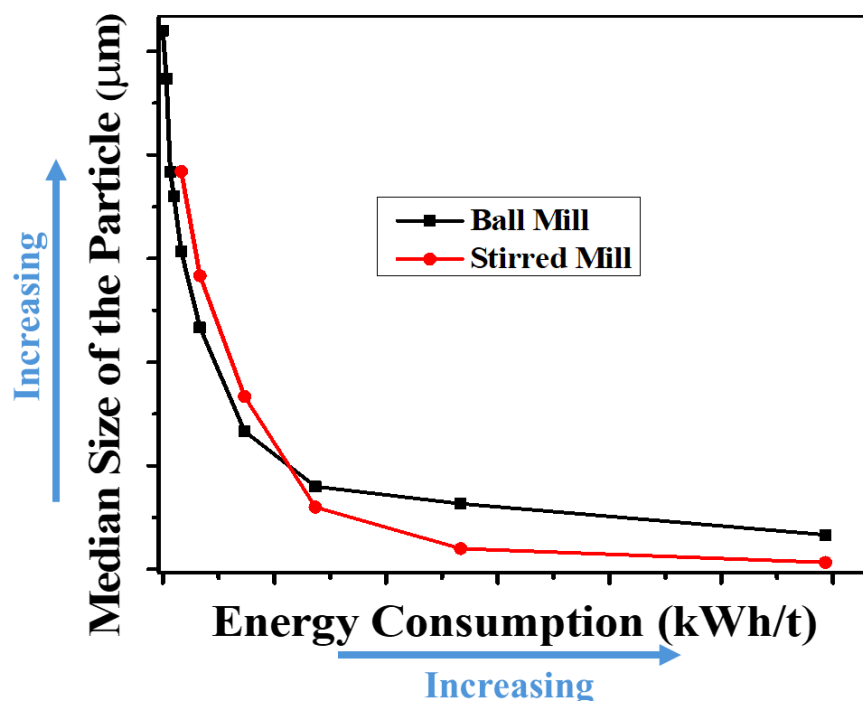


Figure 2-10: Fine milling energy consumption behaviour of stirred mill versus ball mill.

As a result, UFG of base metal sulphide mineral resources is conceivable when energy costs

per area, in this case, Botswana, as well as other capital and operating costs are considered. The Activox technology (Palmer & Johnson, 2005b), uses mechanical activation as a pre-treatment for effective metal recovery (Y. Li et al., 2019), to enhance the efficiency for oxidative pressure acid leaching.

2.5 Modelling techniques for UFG.

As industrial operations progressed, numerous gaps emerged that needed to be filled to expand processes, optimise economic outcomes, and provide staff with the necessary training to operate these processes efficiently (Saloojee & Crundwell, 2016). Among many other things, process engineers had to deal with operational issues that required process updates, which ultimately resulted in plant modifications. They also had to improve process control to guarantee record standards for product quality. These made it necessary to model and simulate processes to gain a thorough understanding of them, from the overall performance of the process to the sub-processes and systems that are essential to its overall efficiency (Chin, 2016).

If maximal recovery of base metals is to be accomplished, no general process design parameters of the Activox technology can be used when working with low grade ores, whose mineralogy is complicated and constantly changing. Modelling is the method used to understand process behaviour due to changing mineralogy and the way these parameters interact with one another. This helps to avoid increased economic losses due to recovery inefficiency, which necessitates ongoing analytical work and expensive experimental work.

In essence, process modelling uses knowledge information and measurable data transcribed to extract facts based on process dynamics. Models used in mineral processing often fall into one of three categories: mechanistic, data-driven, or integrated hybrid models, which combine the two approaches stated above into a single model (Long et al., 2020). Data-driven models include statistical models e.g. response surface methodology (RSM) , and machine learning (ML) models (McCoy & Auret, 2019), whereas mechanistic models are usually mathematical models, kinetic models, population balance models (Meloy & Williams, 1992), etc. Since UFG and PL have been mentioned as the article's focus, a detailed discussion of the modelling tools' applications specifically to these two processes follows.

2.5.1 Mechanistic models.

In the following manner, models are typically categorized and organized into classes: grey shade pellets in the spectra from 0 to 1 (Sun et al., 2020);

1. White models- first principles models defined by a set of differential/algebraic equations.
2. Off-white models- groups those of first principles, however, with certain unknown parameters e.g. state-space models.
3. Slate-grey models- hybrid models.
4. Black models- data driven machine learning models e.g. ANN and ANFIS.

Mechanistic models fall within the category of white models, which are characterised as reasonably trustworthy models that generally depict the internal matrix or phenomena of a process (Long et al., 2020). By describing them precisely and showing how changing any one of the parameters will undoubtedly impact on the process, they make use of the physics of the process. Differential or algebraic equations based on thermodynamics, conservation laws, and process kinetics are used for this. From these, several phenomena that describe changes in both internal and external systems over time can be deduced. These include the Rosin-Rammler-Benre models (Guo et al., 2022), which describe the size distribution of particles when working with solids, and Population Balance (Adeniji, 2018; Bilgili & Scarlett, 2005; Connell & Professional, 2002). Computational Fluid Dynamics (CFD) models (McBride et al., 2012; Saldaña, Gálvez, et al., 2022) describe the flow of fluids during a process and how it affects particle-to-particle or phase-to-phase interactions, which in turn account for the mass and heat transfer efficiencies.

2.5.1.1 Ultra-Fine Grinding mechanistic modelling.

Several variables, including rotation/grinding speed, slurry concentration, media size, media type, media filling, powder filling, media mass, interstitial filling, feed size, grinding time, dispersant dosage, and bed porosity, affect the process of size reduction to such fine particle sizes (Celep & Yazici, 2013a; Choi et al., 2007b; He et al., 2006). All these factors have an impact on the process and are interconnected, therefore the first step in studying modelling techniques that are relevant to it is to ascertain how they affect it (Patino et al., 2022), with the goal of clearly

outlining the step-by-step process.

2.5.1.1.1 Mathematical Modelling.

These descriptive classes below serve as a broad framework for mathematical modelling, which necessitates a thorough examination of the procedure (Departent & Engineering, 2022; Mikleš & Fikar, 2007);

1. Type of the process- deterministic or stochastic process.
2. Type of information employed- mechanistic, empirical or combined.
3. Distribution of system variables in space- lumped or distributed parameter system.
4. Type of equations employed- linear or non-linear equations.
5. Response of variables with time- continuous or discrete mannerism.
6. Time distribution of variables- whether dynamic or steady state.

After the entire process has been properly and thoroughly described, the model must be constructed using the following steps (90), as seen in Figure 2.11.

A positive model that is certain to produce results that are satisfactory for real-world applications is guided by proper due diligence at every stage. When calibrating the model, optimisation is a crucial tool to consider. This is accomplished by performing a sensitivity analysis on the parameters and by optimising the model's decision variables so that it can forecast results that are correct or acceptable and comparable to those obtained on the field (Celep et al., 2011).

As illustrated in Fig. 2.12, ultra-fine grinding's descriptive features make it a complex stochastic process. It is time-consuming and it requires professional expertise to properly define the process matrix using analytical equations. To overcome this time-consuming challenge, the intricacies of the grinding process are primarily defined through modelling using data from industry or experimental work.

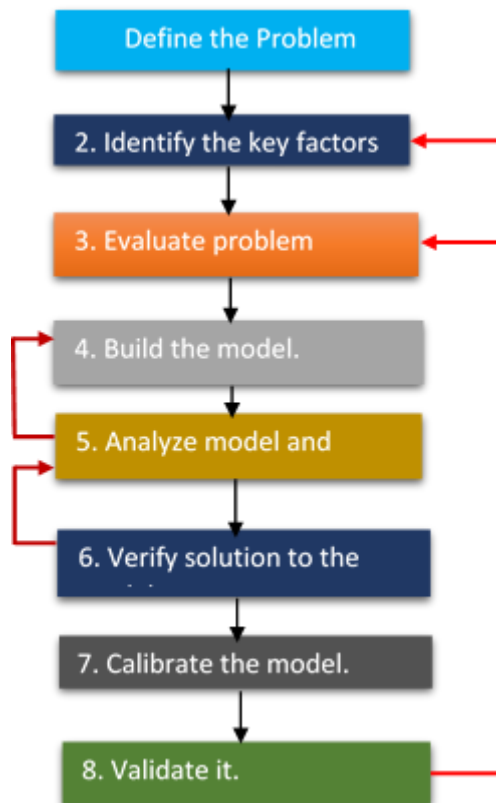


Figure 2-11: Stepwise guide to model building.

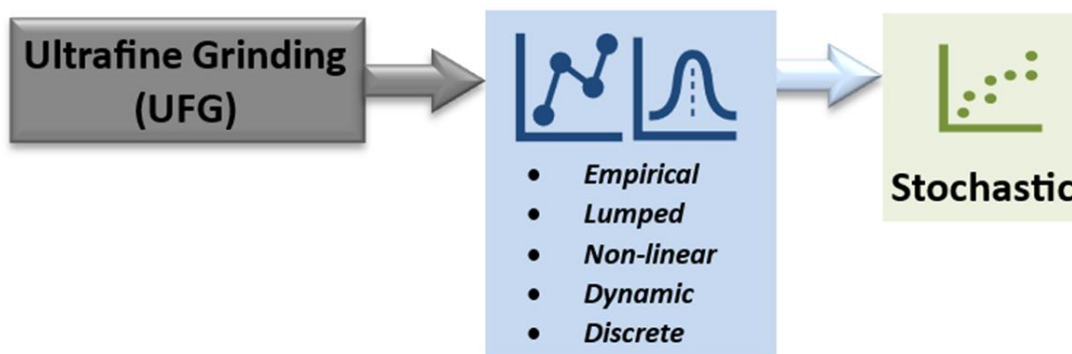


Figure 2-12: Description of UFG process characteristics.

In mechanical activation assisted hydrometallurgy, the main problem with the UFG, especially when using a ball mill, is its excessive energy consumption. As a result, a lot of research has focused on optimising its consumption by adjusting the independent variables mentioned above (Celep & Yazici, 2013b; Sterling et al., 2023; Zhao et al., 2022). Determining the relative influence of the independent factors on the dependent variable and evaluating the problem can be

accomplished using industrial or practical data. Reaching a particle size distribution with P_{80} in the 5–20 μm range at the lowest feasible energy cost is the aim. To be used in steps 7 and 8, the evaluated data must be divided into calibration and validation data sets.

A basic method is used to build the model before moving on to a more thorough one. The basic approach entails specifying the model's components, after which the comprehensive approach will comprise (Mikleš & Fikar, 2007):

- Stating the process's presumptions.
- Establishing the constitutive equations, the boundary volumes, the balancing equations, and the characterising variables.
- Establishing the beginning and boundary conditions precisely.
- Lastly, a description of the parameters and the units of measurement.

It is essential to analyse the model after it has been constructed before attempting to solve it. To do this, begin with the fundamental analysis tool, degrees of freedom analysis, to ascertain the type of solution that will be obtained. Next, analyse the type of equations involved, such as partial differential equations or algebraic equations, to determine the most effective method of solving the model using both analytical and numerical methods. Next, the solution is verified to see how the model behaves and whether it accurately predicts the product P_{80} in relation to the observed energy consumption. This is known as model testing, which is followed by model calibration and validation utilising

2.5.1.1.2 Population Balance Modelling.

The intricacy of UFG has made it impossible to represent the process primarily through mathematical modelling, as was mentioned in the section above. Instead, additional methods that characterise the process in a more straightforward manner with high predictability have been used. A paradigm shift from mathematical models to population balance models, wherein the controlled variable is the particle size distribution of the feed material, emerged in the early 2000s because it has shown to be a laborious endeavour to precisely describe the governing equations of the milling process (Bilgili & Scarlett, 2005; Ramkrishna & Singh, 2014). The conservation law of mass in a system is the source of this data-driven model, while the specific

energy applied is the process's independent variable, and the product P_{80} is its response variable (Adeniji, 2018).

This method's benefit is that it eliminates the need to precisely and meticulously characterise the processes constitutive, balancing, and boundary equations because the model automatically learns these parameters from the data used, thereby defining the mill's matrix. The specific breakage rate function and the breakage distribution function are the two governing equations that make up the model (Connell & Professional, 2002; Petrakis et al., 2017). The rate at which a particle breaks down from one sieve class to a lower one is described by the specific breakage rate function (also known as the selection function). The cumulative breakage rate, which describes the fraction of daughter particles that fragment from the parent particle and appear in the subsequent sieve class, is described by the breakage distribution function. Since these two essentially constitute the model, it is essential to properly tune its parameters, which can be done by using optimisation techniques like particle swarm optimisation (Patino et al., 2022) in Matlab, among many others.

A general energy-based model equation of vector notation is as shown below:

$$\frac{dm(\bar{E})}{d\bar{E}} = -(I - B)Sm(\bar{E}) \quad (2.1)$$

Solution to it derives the non-linear equation:

$$m(\bar{E}) = m(0) \exp(-[I - B]S\bar{E}) \quad (2.2)$$

Where:

n- Number of size intervals

$m(\bar{E})$ - Matrix (n*1) of product mass fractions after specific energy input.

I - Identity matrix

B- Breakage function matrix (n*n) with elements $b_{i,j}$.

S- Selection function matrix (n*n) with diagonal specific selection function parameters S_i .

$\bar{E}$ - Specific energy input.

$m(0)$ - Feed mass fraction matrix (n*1).

The model will be given an appropriate status for practical considerations if this equation is solved using the Matlab platform and the parameters are properly optimised. The fluid flow in the system is modelled using CFD, one of the other types of mechanistic models that can be used. The fluid's viscosity directly affects the particle size distribution of solids in the liquid medium while maintaining mass conservation.

2.5.2 Response surface methodology (RSM) modelling.

This is a powerful tool employed for modelling and optimisation purposes particularly of multivariable processes i.e. those of various input parameters all significantly affecting the process performance, its outcome (Sarma et al., 2022; Swain & Bhima Rao, 2018). It has been applied in mineral processing to model and optimise the various stages involved, enhancing the overall performance of the plant e.g. comminution stages, concentration processes, extraction stages (Chelladurai et al., 2020; Kalyani et al., 2005; Sepúlveda et al., 2024). It possesses the following advantages:

1. Optimisation efficiency- it saves on experimental resources needed, time and energy through reduction of required number of experiments for precise analysis compared to traditional route e.g. one factor at a time (Shanmugam et al., 2023).
2. Interaction effects- it is effective in deciphering the underlying interaction effects between several factors hence developing models effective models (Chelladurai et al., 2020). This inherent property of RSM is critical in multivariable processes like fine and ultra-fine grinding.
3. Predictive models- developed models are highly precise therefore assist in forecasting of process behavior under certain conditions. This is particularly important in optimisation and safe operation of processes (Arancibia-Bravo et al., 2019).

However, it is prone to a few drawbacks when dealing in particular with highly complex processes, as the interaction effects cannot be precisely represented through the polynomial models developed thus developments have been carried out whereby RSM models are coupled with machine learning models to improve on predictive accuracy of the models and their flexibility (Orero et al., 2023a).

2.5.3 Machine Learning modelling.

Computers may learn without explicit programming thanks to machine learning (ML), a branch of artificial intelligence (AI) (Chiang et al., 2022). To learn, develop, and optimise models that may be used to infer plausible predictions based on methods used, it utilises a lot of data. Machine learning's use in chemical engineering, especially in mineral processing, has expanded over time, as shown by its growing influence in both commercial and research applications (Schweidtmann et al., 2021). Improvements in computing power and the availability of vast data pools, which have enabled learning from them, have contributed to this progress throughout time (Dobbelaere et al., 2021; Lee et al., 2018; Venkatasubramanian, 2019).

Over the years, machine learning approaches have been used in mineral processing, specifically in flotation, leaching, smelting, size reduction, and other processes (McCoy & Auret, 2019; Niu et al., 2023). Artificial neural networks and neuro-fuzzy inference systems have become the most commonly used technologies for better control of unit operations (Banza & Seodigeng, 2023b; Kazemi et al., n.d.).

2.5.3.1 Artificial Neural Networks (ANNs).

More research and development have been conducted on process control and optimisation of sulphide base metal hydrometallurgy on the basis of costs and environmental requirements since the start of the 4th Industrial Revolution (4IR) (McDonald & Muir, 2007b; Padilla et al., 2008; Toro et al., 2021). Utilising its primary performance feature of approximating arbitrary non-linear functions—which are typically challenging and complex to model with conventional learning tools—ANNs have been the most extensively used tool in this regard, having been achieved through process modelling (Brest et al., 2021; Long et al., 2020; Setiawan et al., 2018; Srivastava & Dhaker, 2024). Many other engineering fields, such as computer vision, water and wastewater treatment, and separation processes, have used artificial neural networks (ANNs) for predictive modelling, especially for process parameters that are difficult to measure online (Orero et al., 2023a; Sithole et al., 2024a).

Through the breakdown of process complexity, such as extractive metallurgy, the use of these approaches has resulted in safer, more intelligent, and more efficient manufacturing, hence reducing numerous process penalties (Gomez-Flores et al., 2022). This has been accomplished by incorporating this tool into expert systems to assess the interactions between process factors,

forecast performance, optimise process parameters, and implement process control techniques, such as Model Predictive Control (Jovanović et al., 2015).

The presence of machine learning tools in mineral processing and extractive metallurgy dates back to the early 21st century, mostly within milling processes and mineral concentration (Gomez-Flores et al., 2022; McCoy & Auret, 2019) and leaching processes (Brest et al., 2021), but not much presence in ultra-fine grinding. Particularly with UFG using ball mills, an unconventional method, the utilisation of these optimisation tools will aid in the successful adoption of these technologies aiding efficiency and intelligence in their production patterns (Modigell et al., 2000; Sun et al., 2020), particularly to artisanal and small-scale miners for economic operations.

2.5.3.1.1 Architecture of ANNs.

The network's initial architecture is determined by the type of learning that is used, whether it is supervised or unsupervised. Most models developed for mineral processing applications use supervised learning (Yan et al., 2020). The multi-layer perception design that is thus integrated consists of the input layer, hidden layers, and the output layer, resulting in the following kinds of networks (Kazemi et al., n.d.);

- Feed-forward Neural Networks (FNNs).
- Convolutional Neural Networks (CNNs).
- Recurrent Neural Networks (RNNs).
- Long-Short Term Memory (LSTMs).
- Radial Basis Function Networks (RBFNs).
- Auto-encoders.
- Genetic Adversarial Networks (GANs).

FNNs, which are mostly used for regression and classification, are the network used for the topic at hand. Fig. 2.13 shows a typical general perspective of the network.

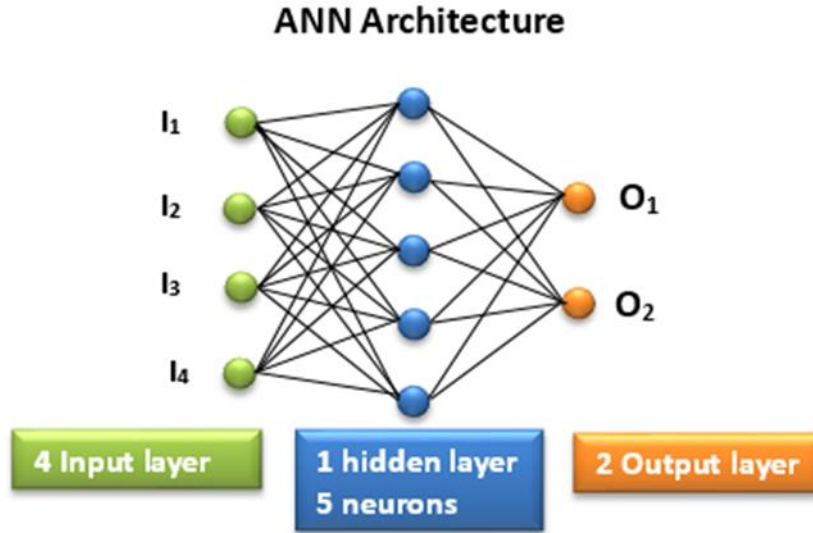


Figure 2-13: Architecture of ANN model.

The model's predictors, which rely on the data set to be learned from, are located in the input layer. These parameters are easily accessed, but the output layer is for the response variables, which are difficult to measure, necessitating the use of a prediction model (Jovanović et al., 2015). The factors influencing each UFG, as described in the previous section, are its inputs, and the process performance indicators, such as the cumulative 80% product particle size passing a sieve size, are its response variables. The neurons that make up the hidden layer include weights and bias parameters whose purpose is to determine the active state of each neuron and the strength of the signal between the input neuron and the hidden layer neuron, respectively.

The algorithm modifies the weight parameters during training to learn explicit features and correlations/patterns from the predictors, while the bias is for baseline neuron activation, which is frequently calculated using the tangent sigmoid function, thus promoting the network's ability to learn a variety of patterns (Kazemi et al., n.d.). Non-linear activation functions built into the training process are used to accomplish these data variations inside the hidden layer. The equation below illustrates how the hidden neuron's output is then equal to the sum of the input weights plus the bias.

$$neuron\ output = \sum_{i=1}^n (w_i * x_i) + b \quad (2.3)$$

Where:

w_i – weight between input neuron i and hidden layer neuron.

x_i – input neuron parameter.

b – hidden layer neuron bias.

The back-propagation algorithm is the main training technique. Iterations are used to calculate the gradient of the loss (error) function with respect to each weight. The gradient descent approach is then used to optimise these weights in order to minimize this error function (Brest et al., 2021). This algorithm has two unique features:

1. It does not converge to the absolute (global) minimum.

The objective function's absolute lowest value among the collection of specified weights and biases is known as the global minimum. It suggests the best model parameters, which produces the most accurate model. However, it causes overtraining, or over-fitting, which produces a rigid model that works well with observed data but badly with unseen data. The model identifies a local minimum that will provide improved generalization and high prediction performance on a larger spectrum of unseen data with less computational cost, hence it is advantageous that it does not converge to this value.

2. It has low convergence rate.

This is a measurement of the speed at which the training process is completed, or the rate at which the local minimum is reached. To prevent under-fitting and over-fitting, respectively, the convergence rate should be proportionate to the training expenses and neither too high nor too low. A low rate of convergence guarantees a stable and effective training procedure.

2.5.3.2 Adaptive Neuro-Fuzzy Inference System (ANFIS)

An extension of Boolean logic for the concept of partial truths, fuzzy logic was first used in mathematics and engineering in the mid-1960s by L.A. Zadeh. Its purpose was to represent approximate thinking that could not be articulated using clear procedures (Jovanović et al., 2015). Its foundation consists of fuzzy sets that are connected by If-Then rules between inputs and outputs. Although they were acclaimed for their ability to reason, they were unable to effectively learn from big data sets. As a result, by combining the two principles, ANN and FIS

emerged to address each other's shortcomings, leading to ANFIS (Jovanović et al., 2015); (Long et al., 2020; Saldaña, Neira, et al., 2022).

The inference system that manages intricate and ill-defined systems was integrated into this modelling tool. Its better dynamic predictive performance can be attributed to its integrated model, which learns from both process physics and observable data. ANFIS and ANNs have some of the same fundamental characteristics, but the fuzzy rules system in ANFIS is integrated.

2.5.3.2.1 Architecture of ANFIS.

Although it is still a component of multi-layer perception and uses the FNNs system, the fuzzy-inference system most frequently uses the Takagi-Sugeno or Mamdani algorithms for training to determine the membership functions (Blej & Azizi, 2016). Like backpropagation, the goal is to reach the local minimum at a slow rate of convergence. Fig. 2.14 depicts the model's basic outline.

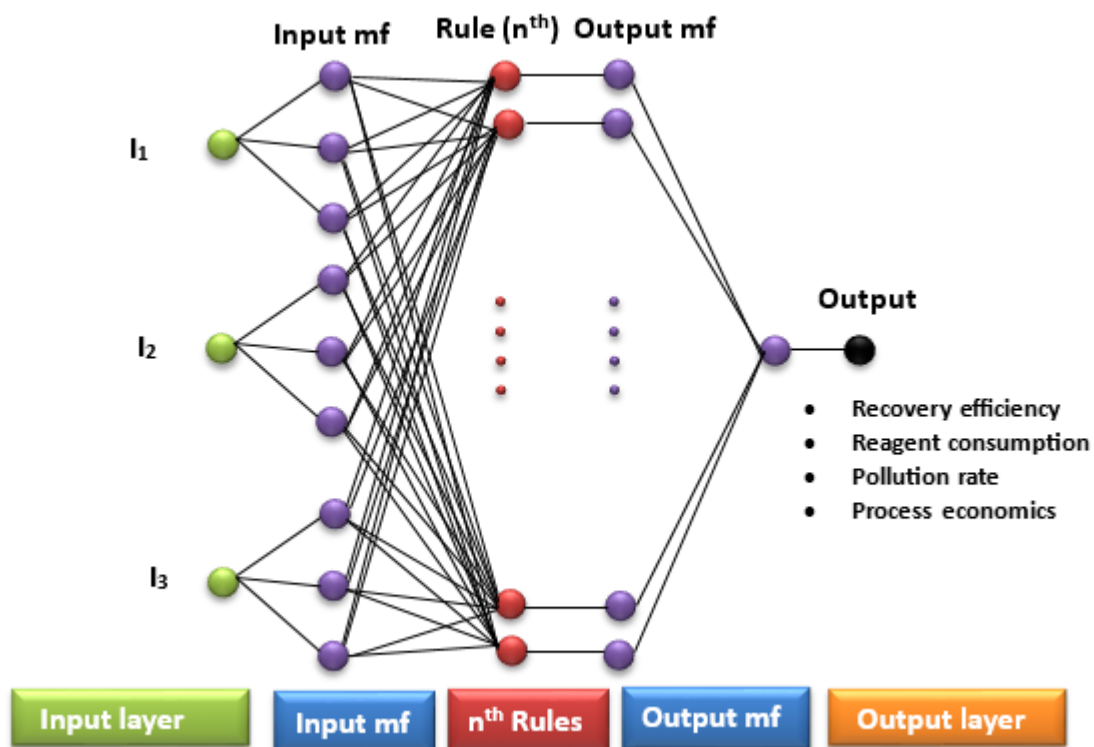


Figure 2-14: Architecture of ANFIS model.

2.5.3.3 Guide to modelling using ML.

ANN and ANFIS are the two machine learning technologies of interest for use in ultra-fine grinding (UFG). Getting an empirical or industrial data set from the process to be modelled is the crucial first step. A big dataset with wide spectra, noise included, and anomalous data sets adequate for training and validation conditions are required for a robust, high-performing model (Gomez-Flores et al., 2022; Jovanović et al., 2015; Kasongo & Mwanat, 2021). To reduce the impact of outlier data points, this data must be normalized.

Typically, 70% of the data set is used for training, with the remaining 30% being split up for testing and validation. Fig. 2.15 provides a broad guideline for creating a model using one of the two tools.

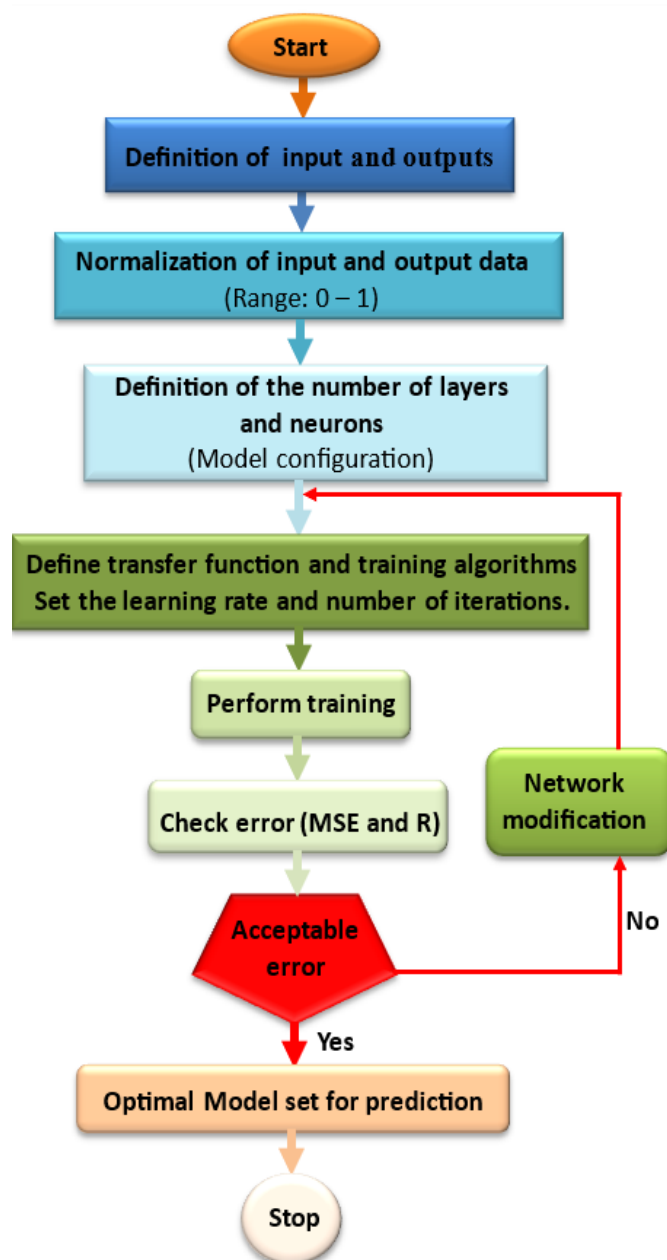


Figure 2-15: Stepwise guide to implementing ML modelling.

2.5.4 Evaluation of model performance.

The following checkpoints serve as the general basis for the criterion used to assess and validate the model's performance (Dawood Salman et al., 2023; H. Li et al., 2017; Su et al., 2023);

1. Correlation coefficient value- R^2 .

It has to do with the dependence variance between the response variable and the predictors, or how well the data fits into a regression model. It falls between 0 and 1, where 1 indicates a good correlation match and 0 the least. Although this parameter is typically employed as a

major indicator, it cannot be used alone to assess the model because good fit values do not always indicate a good model.

2. Mean Squared Error (MSE) value.

A successful model should have the lowest mean squared error (MSE) value, which measures the average squared error between the predicted and actual values.

3. Confidence Interval and p-Value: ANOVA.

The degree to which the model predictions may be implemented is shown by the confidence interval, which is calculated by running hypothesis tests at the 95% or 99% levels. The model's degree of confidence is then ascertained using the inferred p-values.

4. Post-hoc Analysis.

This is an analytical tool to evaluate the statistical significance of models to narrow down the most appropriate model from the multiple developed. It does this through comparing the individual model mean to the experimental mean and identifies the model with the mean closest to the experimental mean as statistically indistinguishable thus best model. Its strength is in its ability to account for abnormalities within each data set therefore ensuring a robust check, however, it has limitations especially in accounting for the model's predictive accuracy hence why it is most applied in-conjunction with the regression test (R^2).

2.6 Optimisation

The drawback faced by ball mills in UFG is the energy demand required, compared to the same performance factor in stirred mills. However, they possess a carrying capacity that outmatches stirred mills therefore there exists a trade-off that can offset the energy expenditure in ball mills, through feed load optimisation. One way to undertake this is through optimising the interstitial filling via the Attainable Region technique.

2.6.1 Attainable region (AR) technique.

It is a model-free powerful optimisation tool developed by Horn in 1964 to govern operational performance of chemical reactors based on various reactor configurations. To date it has extensively grown to be utilised owing to work done by Glasser in 1987 in developing the geometric approach of the AR technique, leading to Khumalo and team applying it to milling processes (Chimwani, 2021; Hlabangana, Danha, Mguni, et al., 2018a). Since then, it has been utilised in optimising of operational parameters in milling circuits as well as in design of flow sheets, as it is a model-free optimisation tool hence its versatility on countless milling scenarios. It generally can be two-dimensional or three-dimensional (2D/3D) graphical representation of boundaries at which optimum performance of the process is achieved i.e. based on the defined objective function (Hlabangana, Danha, Mguni, et al., 2018a).

It possesses the following advantages:

1. It requires definition of process fundamentals and bases on them rather than the equipment type used.
2. It is applicable to wide spectra of objective functions.

However, it has a set of requisites for applicability, of which the major one is:

1. The region generated should be of a convex nature- this ensures optimal system performance as all set of operating conditions are attainable.

2.6.1.1 Optimising interstitial filling (U) using the AR approach.

Particle breakage in milling is a function of U , defined as fraction of media bed void that is occupied by feed powder. It generally governs the probability of particles to be nipped in-between colliding media, at which impact forces are at maximum thus increased probability for breakage (Tembo et al., 2019b). In general, low U values equate to more void in media bed therefore reduced breakage rate and increased media wear, while higher values of U increase the cushioning effect in media rendering no positive effect on breakage rate and it also is a function of feed particle size and material strength, therefore needs optimisation for different milling circuits (Hlabangana, Danha, Mguni, et al., 2018a).

Fine to ultra-fine grinding with ball mills is an energy intensive process therefore to counter this effect, the powder filling rate can be optimised to increase mill throughput, reducing the

specific energy requirements. The breakage mechanism in UFG is dominated by shear forces, which are maximum at low bed void i.e. highest probability of particle to media attrition. This therefore can be done by optimising the interstitial filling (U), as they correlate according to the equation:

$$U = \frac{f_c}{\varepsilon * J}$$

Where: f_c – particle filling rate.

ε - media bed void.

J – media filling rate.

The relationship between U and f_c is direct proportionality, therefore the mandate of optimising mill throughput using U is plausible.

2.7 Cost benefit analysis.

The economics of a process are always the core of any technology adoptability with each industry or even any modifications to the plant. The aim always is to make profit by day end as such the mandate for engineers is to design processes that will minimize plant expenditure while bringing vast returns. In milling processes, the same goes, the particular focus being channeled towards energy consumption as milling is reported to draw about 50% of all mineral processing industries (Wang et al., 2019). (Wang et al., 2019) highlights therefore the 3 main factors to consider, being energy consumption, grinding efficiency and operational costs. What is key is to maximize on throughput, while minimizing on energy consumption.

Traditionally, ball (Bond) mills have been extensively utilised in particle size reduction, however, with the advent of increased processing of low-grade ores, complex fine-grained ores and secondary sources, the need for fine milling arose. The stirred media mills found favor over traditional mills due to low energy consumption due to shorter grind times accompanied with efficient grinding owing to the increased stress intensity imparted to the feed particles by the fine revolving media (Hacıfazlıoğlu & Korkmaz, 2020). This efficient grinding capability thus leads to reduced energy consumption as it generally is a function of time, also the reduced impact forces by media, hence, reduces the maintenance costs as there will be reduced media and mill lining wear (Jayasundara et al., 2010b).

Mill throughput is another factor that is critical in optimization of milling processes, which must be maximized for increased production rates as such while in consideration of energy consumption, mill throughput must be considered too. In general, ball mills have a higher capacity than stirred mills owing to the internally housed stirrer in the latter mill, hence the aim of this study was to consider this advantage of the ball mill. (Hacıfazlıoğlu & Korkmaz, 2020) conducted studies to compare the performance of these two mills in both coarse and fine to ultra-fine grinding. The conclusion was that with coarse grinding, the ball mill performs more economical, while going to the fine-ultra fine region, stirred media mills proved more economical, basing on the specific energy consumption Fig2-10.

3. Methodology.

3.1 Introduction.

Ultra-fine grinding using a ball mill as previously discussed has short falls that carefully need to be apprehended for efficient application of this technology, e.g. to small and medium scale enterprises, which mainly utilize the ball mills, to optimise their operations and benefit from their operations. The modelling and optimisation of the process using a laboratory ball mill involved in essence a hybrid approach involving the response surface method (RSM) and machine learning (ANN/ANFIS) tools, utilising a locally sourced copper concentrate as the raw material. Embarking on this quest necessitated the characterisation of the material to cement the suitability of the concentrate for extraction using the Activox technology, and the analysis conducted were hardness test, bulk density, sample mineralogy i.e. chemical composition, crystallinity, morphology, elemental mapping, grain sizing and particle size distribution. Based on this analysis, an experimental design was conducted to model and optimise the process based on two evaluation parameters, P_{80} - the 80% cumulative undersize passing through the target sieve size and the energy consumption for the process. A model was developed following this hybrid approach and validated, from then, utilising this model, the loading ratio for the ball mill was optimised using the attainable region technique. Finally, a brief cost-benefit analysis had to be conducted to validate the claims for this work and promote its further consideration and finally adoption.

3.2 Materials.

The copper sulphide concentrate used was collected from an active local mine located in the North Western part of Botswana within the Ghantsi and Ngamiland Districts, 77 km southwest of the Maun town, along the Kalahari copper belt, during a 4-days visit to the mine at which sampling was conducted in a random fashion, collecting a kilogram of sample from the discharge point per 2-hours operation i.e. from 8 am to 2 pm, for 3 days for a total of 30 kilograms of sample. The concentrate after filtration was sun-dried to remove dampness, packaged and ferried back to the institution. Upon arrival, the concentrate was thoroughly mixed using a shovel and then to obtain a representative sample, the sample splitter housed in the department of Mining Engineering laboratory was used to obtain a 5 kg sample for characterisation and experimental runs. The concentrate was obtained already in fine powder from a local concentrator, therefore no further pulverization was conducted prior to the analysis

tests.

3.3 Characterisation.

To determine the nature and suitability of the sample, the following tests were conducted using the appropriate equipment as discussed below.

3.3.1 Hardness test

This test was conducted following the Mohs hardness procedure whereby the materials hardness is determined through a simple scratch test by minerals of known hardness. The Mohs hardness test kit was obtained from the department of Mining Engineering laboratory, pencils of known hardness tips. The hardness test was repeated, and the average was determined.

3.3.2 Bulk density test

To conduct this test, the Archimedes' principle was utilised. A 1000ml measuring cylinder was utilised, 300 ml of water was measured into it and a 200 g of sample was submerged, and the displaced fluid was 80 ml. To calculate the density of the concentrate, the following equation was used:

$$\rho_{conc} = \frac{\text{mass of sample}}{\text{volume of sample}} \quad (3.1)$$

Similar procedure was followed for determination of ceramic media density.

The analysis was done in triplicate, and an average was determined.

3.3.3 Method for elemental composition (XRF).

To quantify the chemical composition for the sample, a portable X-ray Fluorescence gun (PXRF) obtained from the department of Environmental Science at BIUST was used to primarily have a snippet of the elemental composition of the sample. The sample was then qualitatively analysed for chemical composition using the XRF spectrometer, located in the department of Chemical Engineering at the University of Johannesburg (UJ) in South Africa, a high-performance energy dispersive spectrometer comprising of a 50W tungsten end window with a silicon drift detector and the test sample was prepared as below:

1. Measure 3 grams of sample and 2 grams of Sasol wax and add into a mortar pestle.
2. Mix the two thoroughly to form a homogenous grind.
3. Transfer the sample mix into the analysis aluminium sample holder.

4. Compress the sample using the pressing machine to 25 kPa to form a sample pellet, to be analysed using the XRF spectrometer.

3.3.4 Method for morphology (SEM-EDS).

The Gemini SEM 500 (Carl Zeiss, German model) from Botswana Institute for Technology Research and Innovation (BITRI), Gaborone, Botswana was used for morphology analysis of the concentrate. Sample preparation was done by BITRI to aid base metals detection during imaging i.e. render their mineral grains to be shiny as compared to the rest of the ore particles. After imaging, the EDS was utilised for elemental mapping of the sample to understand the elemental composition and the association involved between target metals and other inherent minerals in the concentrate, which is pivotal in selecting appropriate extraction route. Finally, grain sizing was conducted to guide the extent to which UFG is conducted.

3.3.5 Method for mineralogy and crystallinity

The sample was compacted onto a glass slide and an XRD (Bruker D8 Model) with Cu-K α of wavelength 1.54056 Å⁰ was used for identification of mineralogy and crystallinity of the obtained concentrate sample. The scan speed was 1.5 degrees/minute, and the scan range was from 5-90 degrees. The XRD used was from the department of Physics at BIUST.

3.3.6 Method for particle size distribution

A Malvern Mastersizer 3000UK- Malvern Hydro EV Model was used to conduct wet analysis for the particles size distribution. The principle of laser beam diffraction is utilised in this regard, where in general, larger particles diffract light at smaller angles and the reverse is true thus the number distribution and volume distribution of the particles can be quantified. The Mastersizer is in the department of Chemical, Materials and Metallurgical Engineering at BIUST. Also, a Mastersizer from UJ was utilised as well.

3.3.7 Method for total copper in the concentrate.

This was quantified using the 4-acid digestion method and analysed using the Inductively Coupled Plasma Optical Emission Spectroscopy (ICP-OES). The reverse aqua-regia technique was used in which the digestion solution was mixed in the ratio HNO₃: HCl of 3:1. 0.3 grams of sample was measured into a round bottom flask and 12 ml of the solution was added; the setup included a condenser to reduce volatiles loss. Digestion was carried out for 3hrs at 110°C, then the solution was filtered and sent for ICP-OES analysis.

3.4 Ultra-fine Grinding.

To achieve desirable liberation to enhance downstream processes, oxidative pressure leaching, particle size reduction is often considered to size ranges below the traditional tumbling mill limits. Ultra-fine grinding was therefore carried out, however as an energy intensive process, multiple factors had to be optimised to achieve an economical liberation point, guided by the grain sizes of the sample.

3.4.1 Experimental procedure.

To obtain a cumulative 80% passing size of $10\mu\text{m}$ at the lowest energy expenditure, the TENCAM 5 litre laboratory ball mill was used to conduct grinding experiments i.e. wet milling and for each run, the energy expenditure was recorded using a Fluke 1738 power logger obtained from the department of Mechanical, Energy and Industrial Engineering. The experiments were conducted by measuring a set mass of sample, of known particle size distribution, produce a slurry as feed to the 45 vol% media charged mill, well mixed and run over a set period. The product particle size distribution was then determined through wet analysis using the Malvern Mastersizer 3000UK particle analyser, while the energy consumed was recorded from the power logger. The experimental setup was as shown below:



Figure 3-1: Experimental setup for the experiments.

With every grinding test, the power logger recorded the energy expenditure through continuous data logging of the voltage and current during the entire duration of each test. To determine the

specific energy consumption in each milling run, the no-load energy consumption was determined prior to feeding the mill contents and perform experiments as per the design. The total mass of mill contents was kept constant by maintaining the media size to be 45 vol% of the mill, and slurry filling at 20 vol% to approximate 2/3 filling of mill. To optimise power consumption, the following parameters were selected and studied:

1. Effect of solids/liquids (S/L) ratio.
2. Effect of mill speed.
3. Effect of time.
4. Effect of media size.

3.4.1.1 Effect of solids/liquids (S/L) ratio.

The total slurry mass was maintained uniform in all runs, at 20 vol% of the mill. The variation of the S/L was studied between the ratios of 1 and 2. The density of the sample, calculated using the volume displacement phenomena (Archimedes principle) in a measuring cylinder, was determined as 2.5 kg/l thus useful in determining the amount of solids sample fed per run.

3.4.1.2 Effect of mill speed.

The rotational speed of the mill was selected as a fraction of the critical speed of the mill varied between the fractions 0.5 and 0.9. To compute the critical speed, the average mill media diameter was determined using vernier calipers, and then computed using the equation:

$$N_c = \frac{42.3}{\sqrt{(D-d)}} \quad (3.2)$$

Where:

N_c - is the critical speed in radians.

D - is the mill diameter in m.

d - is the average mill media diameter in m.

It was noted that the mill speed of rotation was different to that set by the variable speed motor, so to determine the relationship between the two, three tests were conducted at a constant set mill speed and the number of rotations of the mill counted and averaged. The following relationship was obtained, and it was used throughout the experiments:

$$\text{Mill speed} = \frac{(\text{set speed})}{2.913} \quad (3.3)$$

3.4.1.3 *Effect of grinding time.*

To study the effect of grinding time, the milling tests were monitored between periods of 1 to 3 hours. Power consumption is a function of time therefore an optimum time range had to be determined.

3.4.1.4 *Effect of media size.*

To study this effect, two experimental designs were conducted, one following the traditional ball milling operational principles using steel balls of varying size distribution i.e. 5-20 mm, guided by Bond's equation for maximum ball size, and the other mimicking the stirred media concept of using fine ceramic media, size distribution between 1-3 mm. This is due to the dominating breakage mechanism with utilisation of the two media sizes, coarse media for impact forces while finer media for shear forces.

3.4.2 **Design of Experiments.**

When optimising processes through experimental work, various experimental design methods are utilised based on the nature of experimental work and the target goal sought to be addressed. In this study, the integral Box-Behnken Design (BBD) method in RSM was selected as we had limited experimental resources, i.e. the concentrate sample to work with. A 3-level, 3-factor design was developed to study the intrinsic behavior process factors, using actual values as opposed to coded values. The BBD model is praised for its efficiency, simplicity, flexibility, economic nature and appraisal in second order polynomial modelling (Ferreira et al., 2007; Shanmugam et al., 2023). These were generated using the StatEase Design Expert 13 software, Table 3.1 and Table 3.2. The observed results were utilized for modelling the process.

3.4.2.1 *Design #1- following the traditional route.*

Factors: - S/L ratio- [1, 2], speed fraction - [0.5, 0.9] and time- [1, 3].

Responses: -P₈₀ and energy consumption.

N.B. Though it is advised to conduct experiments on triplicates, herein all were conducted in duplicates due to resource limitations.

Table 3-1: DOE parameters for design #1.

Run	S/L	Speed fraction(c)	Time	E (kWh/t)	P ₈₀ (µm)
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			(hr.)		
1	1.5	0.5	1		
2	1	0.9	2		
3	2	0.5	2		
4	1	0.7	1		
5	2	0.7	3		
6	1.5	0.7	2		
7	2	0.9	2		
8	1	0.5	2		
9	1	0.7	3		
10	1.5	0.5	3		
11	1.5	0.7	2		
12	1.5	0.7	2		
13	1.5	0.9	1		
14	1.5	0.7	2		
15	2	0.7	1		
16	1.5	0.7	2		
17	1.5	0.9	3		

3.4.2.2 Design #2- mimicking the stirred mill phenomena.

Based on the shear forces involved with fine media milling, the important factors that are pivotal in inflicting these shear forces for particle breakage are media filling rate, powder filling rate i.e. interstitial filling rate and speed. Based on these, powder filling rate was maintained at a constant i.e. constant slurry concentration, therefore the factors varied were media filling rate (vol %) - [20, 60] speed (rpm)- [250, 450] and time (hr.)- [0.5,1.5], giving a design as shown below.

Table 3-2: DOE parameters for design #2.

Run	Speed (rpm)	Time (hr.)	Media filling (vol%)	E (kWh/t)	P ₈₀ (µm)
1	450	0.5	40		
2	350	0.5	20		

3	350	1.5	60		
4	250	1.5	40		
5	350	1	40		
6	350	0.5	60		
7	450	1.5	40		
8	350	1	40		
9	450	1	20		
10	250	1	60		
11	450	1	60		
12	250	1	20		
13	350	1.5	20		
14	250	0.5	40		
15	350	1	40		

3.4.2.3 Specific energy determination

The total energy expenditure during each run was recorded using the Fluke 1738 power logger and the area under each graph plot of Energy vs time equated to the power consumption. To accurately compute the specific energy expended in each run, no load power was subtracted from the total power consumption i.e. the following equation was used to calculate it:

$$SE = \frac{\int_0^t (N(t) - N(0)) dt}{m_p} \quad (3.4)$$

Where; m_p - mass of solids in slurry, $N(t)$ - power drawn at time t and $N(0)$ - no load power.

3.4.3 Particle analysis.

Representative samples were obtained from each run and collected for analysis. Error analysis was conducted from the duplicate results obtained per run; cumulative undersize PSD graphs were constructed then from hence used to determine the P_{80} size class for each run.

3.5 Modeling.

To improve predictive accuracy and model flexibility, machine learning modelling requires a large dataset (Moraga et al., 2024; Van Duijvenbode et al., 2020), therefore, RSM modelling was conducted to generate a model equation for the response variables to the factors.

3.5.1 RSM modeling.

As a statistical and mathematical tool, RSM is mostly employed to predict the relationship between input factors and their response variables. From its analysis of variance (ANOVA) and p-values analysis (95% confidence interval), the model significance gets determined. Herein, the design model used was adopted from (Ebadnejad et al., 2013a) in their work for modelling ball mills for copper sulphide ore grinding using RSM. From the experimental results obtained, a second order polynomial equation (3.5) is developed, which approximates the true response surface value i.e. experimental value.

$$y = \beta_0 + \sum_{i=1}^k \beta_i x_i + \sum_{i=1}^k \beta_{ii} x_i^2 + \sum_{i=1}^{k-1} \sum_{j=2}^k x_i x_j + \varepsilon \quad (3.5)$$

Where: y - is the response variable to the input factors x_i ($i=1, 2, \dots, k$), the β coefficients are unknown parameters to be determined by the model, together with ε - the random error term.

An ANOVA analysis of the model was conducted, and the most significant model was adopted and used to generate 100 data sets of the input parameters and their respective response variables. The data generated through MATLAB was normalized using equation 3.6, for improved convergence rate and enhanced accuracy due to effective learning and weighting as input ranges will be maintained similar. After normalisation, the datasets were used to develop machine learning models.

$$y_{norm} = \frac{y_i - y_{min}}{y_{max} - y_{min}} \quad (3.6)$$

Where: y_{norm} - normalized value, y_i - value to be normalized, y_{min} - minimum value in the range of y -values and y_{max} - maximum value in the range of y -values.

3.5.2 ANN modeling.

Three layers make up the ANN model's architecture: the input, hidden, and output layers (Multi-Layer Perceptron, or MLP) for feeding, receiving, processing data, and relaying output data according to predetermined activation functions, respectively (Obi et al., 2022). This technique is therefore highly praised for efficiently modelling multivariable complex, non-linear systems. Modelling was done using the neural networks package 'nftool' in MATLAB 2021a. For training, validation, and testing purposes, respectively, 70:15:15 of the normalised data from RSM modeling was split up. The log-sigmoid transfer function was chosen due to

its appraisal for pattern recognition. The Levenberg-Marquardt algorithm was selected due to its acceptable predictability and relative computational ease when compared to the Bayesian approach (Orero et al., 2023b).

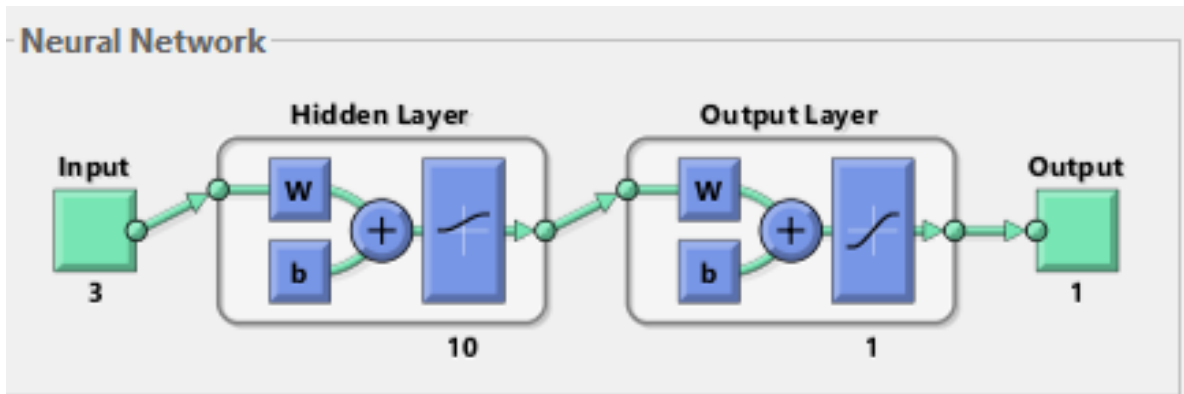


Figure 3-2: ANN layout for 3 inputs to 1 output model, comprising of 1 hidden layer with 10 neurons.

3.5.3 ANFIS Modeling.

ANFIS is a hybrid tool with better capabilities of learning complex non-linear systems. It links input variables to their corresponding outputs by using if-then rules and a learning algorithm. Its five layers comprises of the input, fuzzification, rules, defuzzification, and output layers, which largely pertain to taking in input data, converting it into crisp values, learning and reasoning, analyzing the results, and then decoding it back to provide actual response values. This study used RSM normalized data, which was split 70:30 for training and testing, respectively. Because of its superiority over the Mamdani approach in terms of optimisation and adaptive strategies, particularly for complex systems, the Sugeno Fuzzy Inference System (FIS) was used throughout the modeling process (Banza & Seodigeng, 2023c; Sithole et al., 2024b). Based on the variance of the three factors, membership function (triangular, trapezoidal, generalized belly-shaped, and gaussian), membership function type (linear or constant), and optimization algorithm type (hybrid or optimization), sixteen models were created. To generate the model, MATLAB 21a's ANFIS module was used, and the model architecture was as shown in figure 3.3:

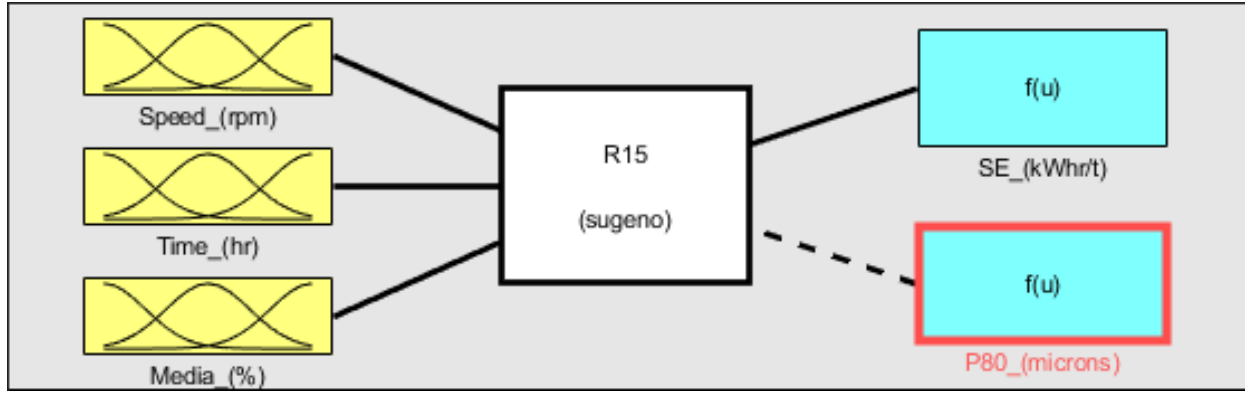


Figure -3-3: ANFIS model layout with Sugeno Fuzzy Inference system (FIS).

3.5.4 Model Analysis.

Within the individual AI models, statistical tools were used to evaluate the performance of the many models that were created. ANFIS employed a hybrid Post-hoc analysis plus the regression coefficient (R^2), whereas ANN used the correlation (regression) coefficient (R^2), (equation 3.7), and mean square error (MSE), (equation 3.8). Regression analysis was performed to determine which model was the most effective in predicting future events, while post-hoc analysis (equations 3.9 and 3.10) was used to look for statistical significance between the model and experimental values in terms of means.

$$R^2 = \left[1 - \frac{\sum_{i=1}^k (y_i - x_i)^2}{\sum_{i=1}^k (y_i - y_m)^2} \right] \quad (3.7)$$

$$MSE = \frac{1}{k} \sum_{i=1}^k (y_i - x_i)^2 \quad (3.8)$$

$$\bar{y}_i = \frac{1}{r} \sum_{k=1}^r \hat{y}_{i,k} \quad (3.9)$$

$$Y = |\bar{y}_{i,model} - \bar{y}_{i,exp}| \quad (3.10)$$

Where y_i - is the predicted value, x_i is the experimental value, y_m is the response mean, k is the total number of experimental runs, $\bar{y}_i$ is the global mean of the experimental data, r is the number of centre points in the experimental design and $\hat{y}_i$ is the experimental responses of the centre points. Y - absolute difference between model prediction global means and experimental global means.

Models with higher R^2 , lower MSE and Y values displayed good performance.

3.5.4.1 Validation.

After the best AI models were identified, they needed to be evaluated for applicability to different locally mined sulphide ores. To perform UFG from its representative sample, a

separate copper-nickel sulphide ore was procured locally from a different place and concentrated in-house (Moyo et al., 2025). A goodness of fit test (equation 3.7) was used to validate the models, namely RSM, RSM-ANN, and RSM-ANFIS. The model with the highest R^2 was chosen.

3.5.4.2 Sensitivity analysis.

Sensitivity analysis for ML models was conducted using the Garson approach after perturbation analysis with RSM. To identify the interactive effects of the input parameters and comprehend the relative influence of each element on the response output, (Sithole et al., 2024b) employed the Garson approach, which makes use of the weight coefficients produced by the model network. Equation 3.11, the sensitivity test, was performed using the accepted model in accordance with this approach.

$$I_j = \frac{\sum_{n=1}^{N_h} \left(\left(\frac{|w_{jn}^{ih}|}{\sum_{k=1}^{N_i} |w_{kn}^{ih}|} \right) * |w_{mn}^{ho}| \right)}{\sum_{k=1}^{N_i} \left\{ \sum_{n=1}^{N_h} \left(\frac{|w_{jn}^{ih}|}{\sum_{k=1}^{N_i} |w_{kn}^{ih}|} \right) * |w_{mn}^{ho}| \right\}} \quad (3.11)$$

Where I_j is the relative importance of factor j on response variable, N_i , N_h – input and hidden neuron numbers respectively, W - connection weights between neurons, subscripts ‘ k ’, ‘ m ’, and ‘ n ’- represent the number of neurons for the input, hidden and output layers; and superscripts ‘ i ’, ‘ h ’ and ‘ o ’ – input, hidden and output layers respectively.

Tables 3.3 and 3.4 give the data used for the sensitivity analysis of P_{80} and SE responses respectively.

Table 3-3: Sensitivity analysis data used for P_{80} response.

Wi			Sum(Wi)	Wo	in1	in2	in3
2.3988	2.1987	1.8107	6.4082	3.7039	1.386492	1.270835	1.046573
3.2405	2.7586	1.8459	7.845	0.42348	0.174925	0.148912	0.099643
0.9755	2.5401	0.55065	4.06625	1.172	0.281165	0.732124	0.158712
1.4503	0.5702	2.9692	4.9897	2.2205	0.645408	0.253749	1.321344
0.9225	1.2299	1.5923	3.7447	6.1797	1.522358	2.029645	2.627697
1.7484	2.8999	3.5706	8.2189	0.35939	0.076453	0.126805	0.156133
4.1797	1.1146	3.8839	9.1782	0.10251	0.046682	0.012449	0.043379

Table 3-4: Sensitivity analysis data for SE response.

Wi	Sum (Wi)	Wo	in1	in2	in3
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2.223	3.5139	1.6795	7.4164	1.1261	0.337538	0.533548	0.255014
3.0319	4.094	0.33932	7.46522	0.58821	0.238894	0.32258	0.026736
2.1838	0.84991	2.0305	5.06421	1.5394	0.663824	0.258353	0.617224
2.8438	0.47243	3.6356	6.95183	0.5273	0.215704	0.035834	0.275762
0.48018	3.7177	0.70148	4.89936	0.41052	0.040235	0.311508	0.058777
0.90704	2.0388	2.3309	5.27674	2.2577	0.388085	0.872319	0.997296
5.034	0.69347	2.7707	8.49817	0.16141	0.095613	0.013171	0.052625

3.6 Optimisation.

Optimisation study is considered a vital process engineering exercise, to identify the best conditions at which the process will operate efficiently and effectively, mostly at minimum cost. To deduce the optimum operating conditions that would align with observations obtained during characterization i.e. the extent of grinding, P_{80} , at the most conserved SE, an optimisation study was conducted using the embedded package in the Design Expert software. Another note about the optimisation problem was to maximize the production rate and that could only be possible at maximum media filling design ratio i.e. 60%.

The package in the software operates in a manner by which an optimization criterion is defined by the user, in the format as of Table 3.5, wherein the goal for each parameter is defined, i.e. in range for input parameters and either at target, minimize or maximize for the response variables. The ranges for each parameter are also specified. The aim is to generate a desirability contour plot, mimicking a heat plot, with different color codes highlighting the different desirability extent the optimization criterion. The optimum conditions are the one defined by the desirability of 1 (maximum) or anything close to it, and it is within the red zone of the desirability plot.

Table 3-5: Optimization criterion levels settings.

Name	Goal	Lower limit	Upper limit	Lower weight	Upper weight	Importance
A-speed	In range	250	350	1	1	3

B-time	Minimize	0.5	1.5	1	1	3
C-media	Maximize	20	60	1	1	3
P ₈₀	Is target- 20	10	34	1	1	3
E	Minimize	8.20896	29.7761	1	1	3

3.7 Cost- benefit analysis.

Since the product from this stage is fed into leaching, this analysis therefore was conducted based on throughput and energy consumption using data obtained from this work and that of stirred media mills from literature. Factors to consider are reduction ratio (RR), U, time and specific energy consumption, translated to average costs per annual operation of 8000hours. Maintenance costs and other operational costs were deemed neutral to both scenarios i.e. water expenditure, operator personnel and other maintenance costs.

4. Characterisation of the Copper concentrate.

4.1 Introduction

Sample characterisation is an important step before any further work is conducted on the sample as it determines the right pathway that will render positive results sought after, also the extent to which the objective of the research embarked can materialise. For this purpose, this was achieved by conducting analysis for morphology, mineralogy and elemental composition. The portable XRF (PXRF) gun was used upon receiving the sample for basic understanding of the concentrate sample at hand and the results obtained are displayed in Table 4-1.

Table 4-1: PXRF results for as received concentrate.

Element	Composition (wt%)
Al	2.15
Si	5
P	ND
S	5.0652
Cl	ND
K	0.6046
Ca	2.8683
Ti	ND
Cr	0.0557
Mn	0.1113
Fe	6.03
Co	ND
Ni	ND
Cu	19.12
Zn	0.7509
As	0.0984
Mo	0.0685
Ag	0.0126
Pb	0.9612

This analysis, though not qualitative, highlighted the sample as indeed a copper concentrate, rich in iron, sulphur, silicon, aluminium, calcium and lead, defining the sulphide nature of it. Further characterisation was henceforth conducted to qualitatively analyse the sample.

4.2 Particle size distribution

The sample received was characterised through wet analysis and the results obtained from the Malvern Microanalyzer 3000Uk are displayed in the volume distribution Figure 4-1.

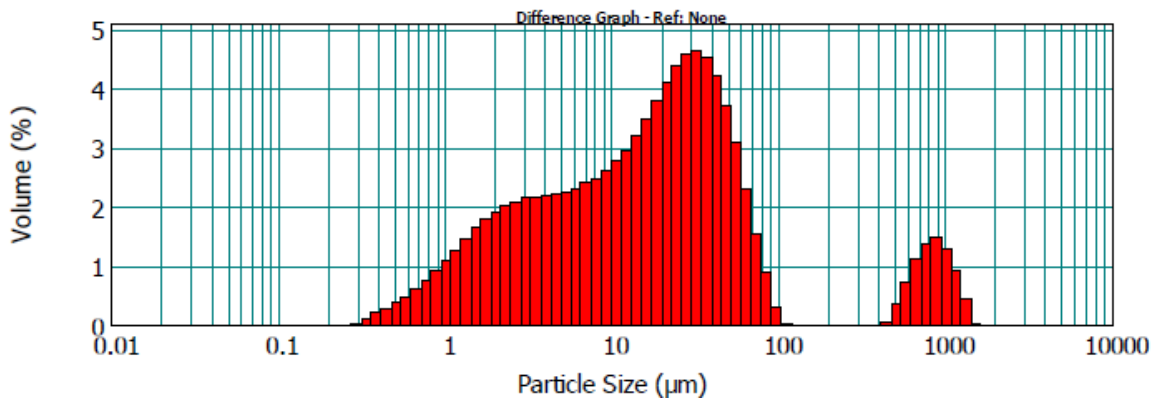


Figure 4-1: Particle size distribution for the received concentrate (vol %).

It is evident that the particle size distribution is trimodal, with mode peaks observed at 2 vol% for size range of 1.9 μm – 5 μm, with the highest peak observed at approximately 4.5% in the range of 20 μm – 40 μm and the least peak observed at 1.5% between 800 μm – 1000 μm. The lowest vol% peak for coarse particles suggests that the number of coarse particles in the sample is fewer compared to fine and ultra-fine particles. This is understood as so from the understanding of particle diameter assumption to spherical diameter as such the higher the diameter, the bigger the volume hence the low vol% distribution of the coarser particles then highlights their low number distribution. A cumulative undersize particle distribution for this sample was plotted to deduce the 80% undersize particle size and it was observed to be 40 μm, Figure 4-2.

Mineralogy and grain sizing analysis were then conducted to determine the extent to which particle size reduction should be implemented, to improve mineral liberation for enhanced downstream recovery process efficiencies.

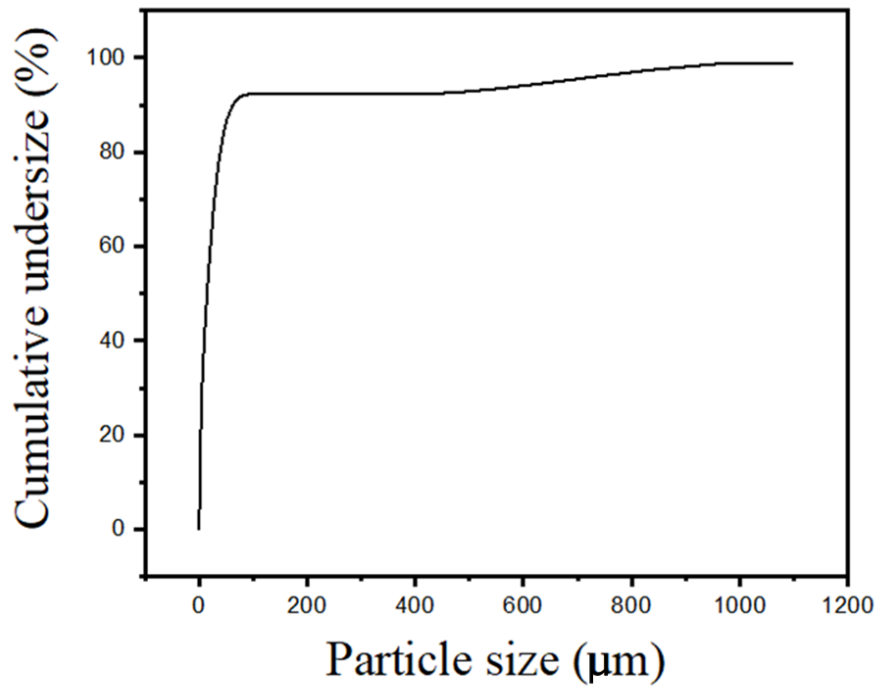


Figure 4-2: Cumulative undersize particle size distribution for concentrate as received.

4.3 Bulk density

The bulk density of the sample was found to be 2.5 g/ml while that of grinding media was found to be 3.57 g/cm³ and 4.5 g/cm³ for ceramic-fine and steel-coarse media, respectively. This suggested that the media selected was effective for grinding as more empty pockets (interstitial filling zone) would be available for the feed charge to be ground. This translated to production rate as well, i.e. higher production rate was expected for coarse media as compared to fine media.

4.4 Hardness test

The sample hardness through scratch test was determined to lie between the ranges of 3.5-4.0 Moh's, therefore both media types were suitable for grinding as they are both of hardness above 9 Moh's. Also, the harder the material, the more energy would be required for effective grinding, and so for the sulphide material under study, moderate specific energy consumption was expected based on their hardness test.

4.5 Mineralogy and crystal nature

This analysis was carried out to define the mineralogy of the sample, crystal nature of these minerals and mineral grain sizing, an important exercise in determining the extraction technology to be used to benefit these minerals. This was done using the XRD and the SEM-EDS analysis equipment.

4.5.1 XRD

This analysis determined the mineral phases (crystals) present in the sample as shown in Figure 4-3.

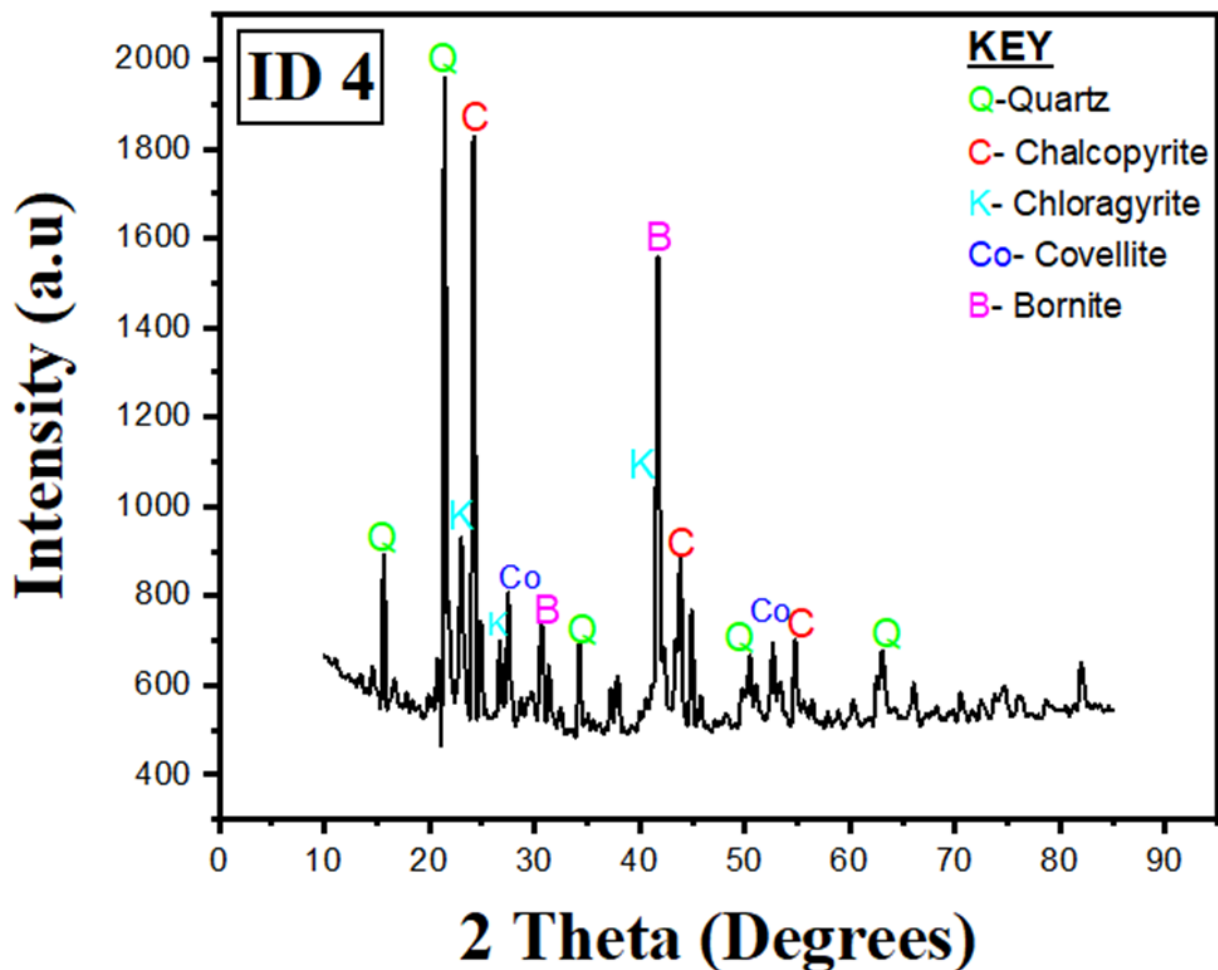


Figure 4-3: X-ray Diffraction pattern for received concentrate.

It is evident that the bulk copper phase is chalcopyrite then bornite and covellite, of which they are all sulphide compounds, Table 4-2, tallying with the sulphur presence observed in Figure 4-8 and Table 4-4. Chalcopyrite and bornite are copper-iron sulphide compounds which

therefore equate to the high iron content observed above too. Quartz, an oxide form of silicon also exists in abundance, which is a typical gangue material, therefore it will need to be considered when determining the extraction flowsheet. These results cement the suitability of the sample to be extracted using Activox technology, also it generally agrees with reported results by other researchers.

Table 4-2: XRD data obtained from Match 4 software.

Crystal name	Chemical formula	Entry number
Chalcopyrite	CuFeS ₂	96-900-7573
Bornite	Cu _{1.75} Fe _{0.275} S	96-900-3724
Quartz	SiO ₂	96-901-0145
Covellite	CuS	96-900-7573
Chloragyrite	AgCl	96-900-8598

4.5.2 SEM

Two different sections were analysed as displayed in Figure 4-4, a and b, at levels 1, 2 and 3 representing different magnification levels, 250X, 500X and 1000X, respectively. From this analysis, the morphology of the particles was observed to be random, with particle surfaces rather corrugated.

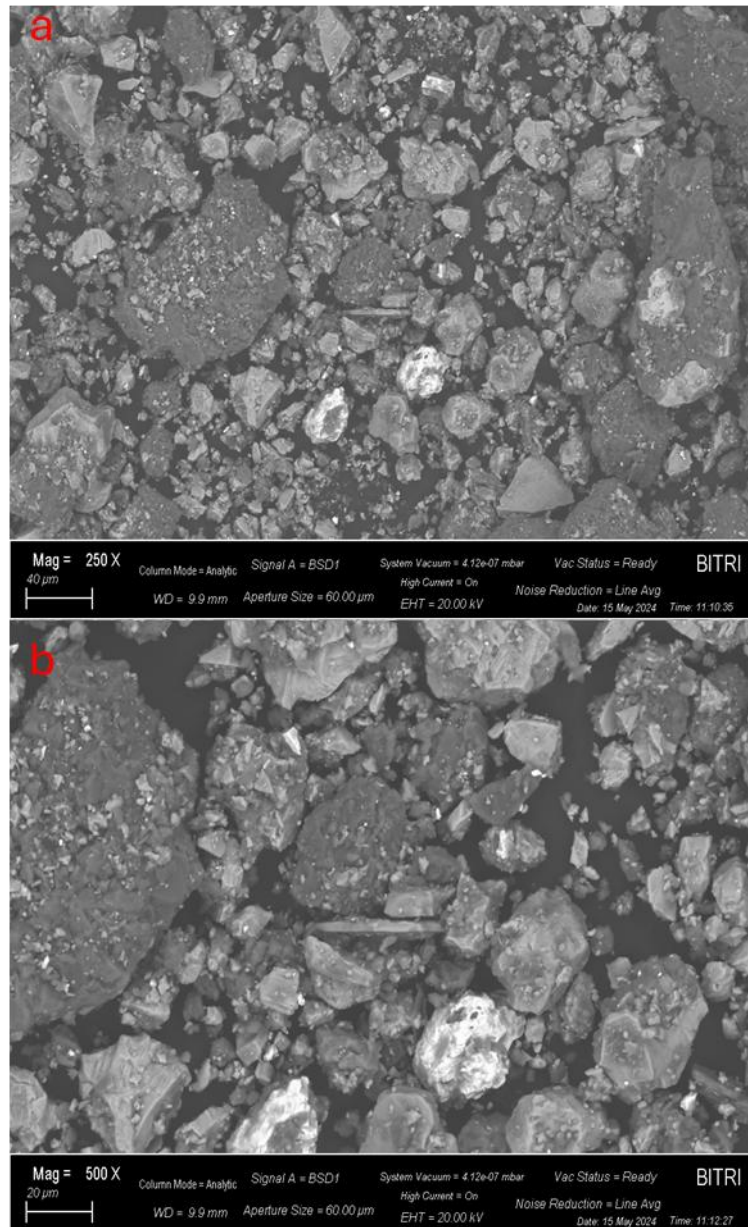


Figure 4-4: SEM imaging denoted a and b, of different magnifications.

As discussed in section 2.3.4, imaging shows a spectrum from dark grey-light grey-shinny particles, evidence of mineral elements from low to high atomic elements and non-metallic gangue material. The different magnifications assist in observing these variations as observed in Figure 4-4, also by inspection, there are more low shinny particles as compared to very shinny ones, suggesting two things. 1- low liberation of the mineral grains as this is a surface analysis technique hence possibility of coverage by gangue material and 2- increase in concentration of low atomic metals. This therefore necessitates sizing the mineral grains in relation to the whole particle, i.e., to determine the extent to which ultra-fine grinding should be done.

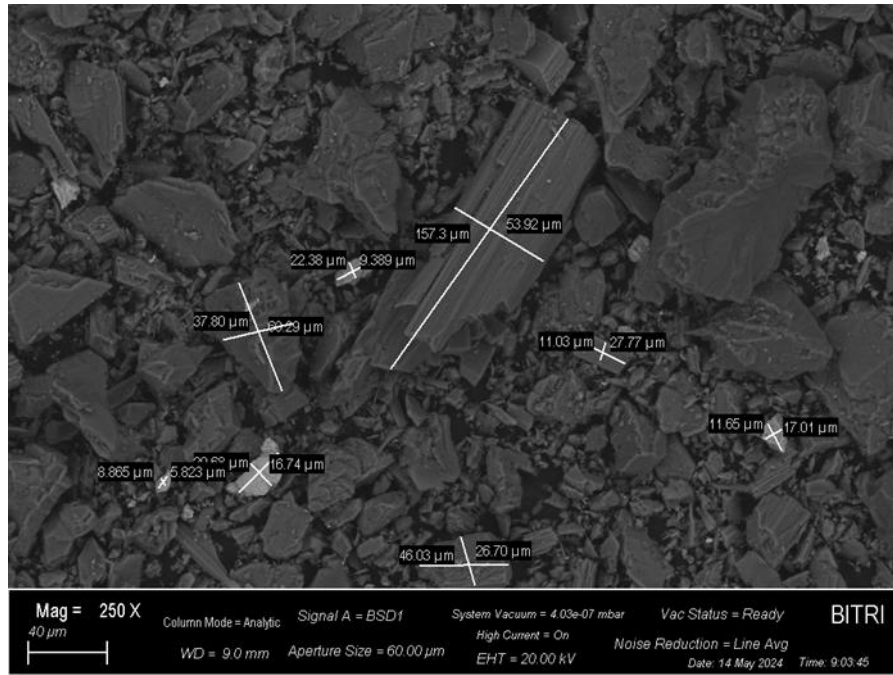


Figure 4-5: Mineral grain sizing of the sample as received.

From the grain sizing analysis, it is observed that unliberated mineral grains range from size between 12 μm to 18 μm , while mineral grains below 10 μm appear to have been fully liberated therefore for optimum liberation, a P_{80} of 20 μm seems to be the optimum extent of UFG. This analysis was the first step in optimising UFG as Activox technology defines the extent of UFG at a P_{80} of 10 μm , therefore there is significant energy saving by grinding up to a P_{80} of 20 μm , as such, it became the target zone in model development of this work.

4.5.3 EDS

This surface analytical technique was employed to determine the mineralogy, qualitative and to some extent quantitative elemental composition of the sample. Mapping imaging of the entire sample area is displayed in Figure 4-6 according to respective phases of each element as labelled. It is observable that copper, silicon, iron, aluminium and lead rich phases are well distributed across the entire sample, while zinc and titanium are fairly to poorly distributed. Figure 4-6 suggests the low undetected levels of cobalt in the sample as only one grain rich in the cobalt phase is observed and taking into consideration the proximity of copper to cobalt in the periodic table, the grain may not necessarily be cobalt rich.

The mapping spectrum of the entire sample is shown by Figure 4-7, with the respective elemental compositions with their error analysis in Table 4-3. These big errors signal the potential mismatch of the phases relative to the accorded elements e.g. cobalt, nickel- oxide

phases etc. This highlights the non-qualitative nature of EDS for elemental analysis, which then is justified using the qualitative and quantitative, destructive technique, XRF.

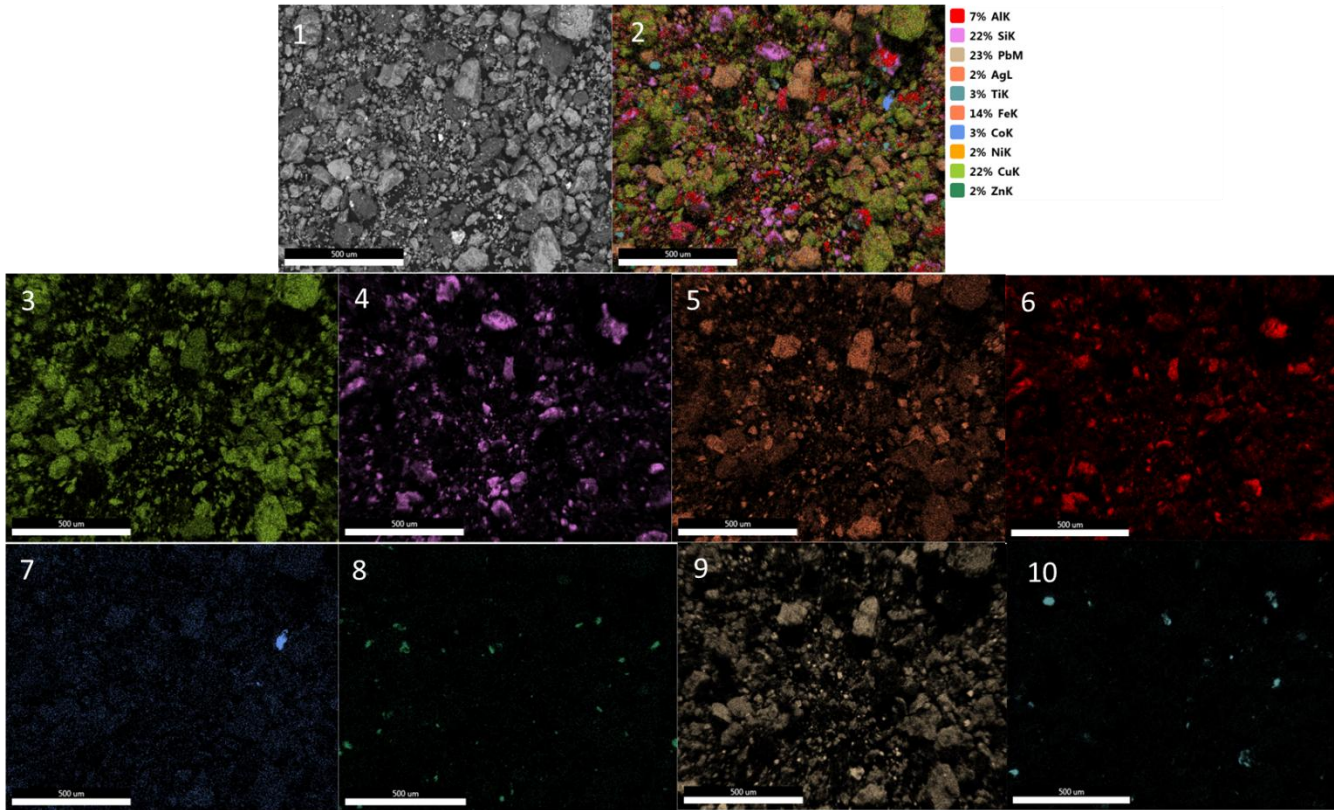


Figure 4-6: EDS mapping images, 1 as sample image/ area to be mapped, 2 as overall mineralogy mapping of the sample. IDs 3 to 10 represent areas of the sample rich in the following phases: Cu, Si, Fe, Al, Co, Zn, Pb and Ti, respectively.

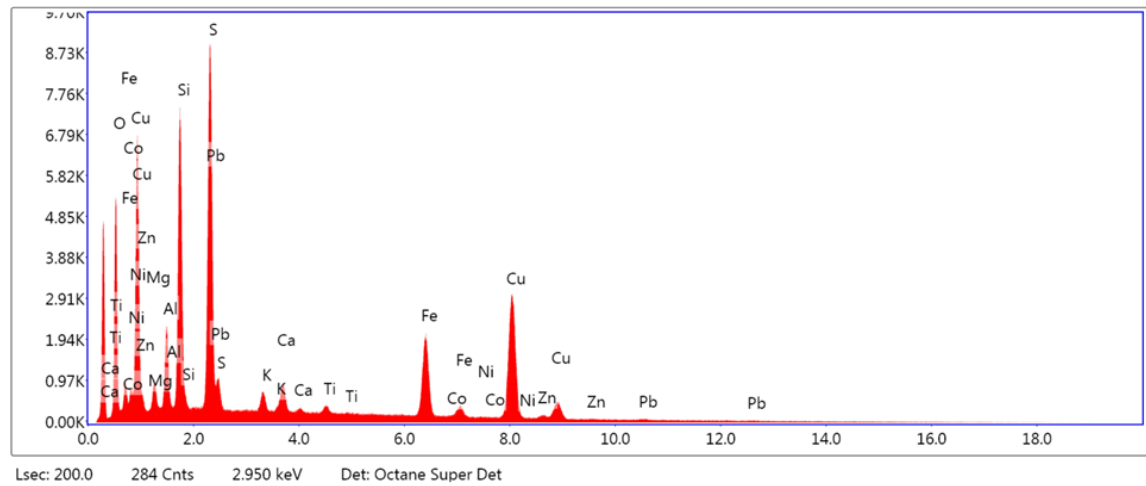


Figure 4-7: Overall mapping spectrum of the entire sample.

Table 4-3: Semi-quantitative elemental composition from EDS analysis of the concentrate as received.

Element	Weight %	Atomic %	Error %	
MgO		2.4	4.7	9.6

Al ₂ O ₃	4.9	3.8	7.6
SiO ₂	16.7	22.1	5.8
SO ₃	25.3	25	4.1
PbO ₂	0.9	0.3	11.5
K ₂ O	0.9	0.8	8.2
CaO	1.9	2.7	4.8
TiO ₂	0.9	0.9	8.5
Fe ₂ O ₃	12.1	6	3
CoO	0.5	0.5	22.2
NiO	0.2	0.2	55.3
CuO	32	31.9	2.4
ZnO	1.2	1.1	16.2

4.6 Elemental composition

For a definite analysis of the elemental composition of the concentrate sample, the following results were obtained through XRF analysis technique.

Table 4-4: Quantitative elemental composition of the concentrate sample from XRF analysis.

Element	Composition (wt%)
Mg	0.611
Al	2.2841
Si	8.5315
P	0.0371
S	7.7117
K	1.0449
Ca	3.5131
Ti	0.3474
Cr	0.0285
Mn	0.2107
Fe	9.2203
Co	0.0283
Ni	0.0205
Cu	35.9141
Zn	1.1518
As	0.2157

Mo	0.1457
Pb	0.6579

Table 4-4 validates the following notations:

1. The sample is indeed a copper-sulphide concentrate, which was observed also from the XRD and SEM-EDS results.
2. There is a lot of silica material in the concentrate confirming the quartz observed in XRD and the mapping results obtained as per Figure 4-7.
3. The concentrate has high concentration of iron as observed from the EDS mapping data, and the close association of the iron and copper observed from mapping images undoubtedly confirms the rich copper phases in the concentrate to be of copper-iron sulphide nature i.e. chalcopyrite and bornite as observed in XRD plot.
4. The sample has significant amounts of aluminium, potassium, calcium and zinc, which can be observed from mapping data as well.
5. It is interesting to note the accuracy level of the PXRF to a certain degree of proportionality hence indicating its validity in use as primary analysis of samples for indicative purposes. This will assist in informed decisive measures leading to cost savings as it is fairly a cheap analysis to conduct.

Note all phases are leachable according to their stability when exposed to practical leaching environments therefore to understand the extent to which dissolution of the minerals can be done, the digestion analysis was conducted. Iron and silica will co-leach with copper in acid leaching (Faris et al., 2023; Xie et al., 2005; Zheng et al., 2023b), therefore in selecting the recovery method for the minerals, it is very important to consider their presence in the design of the purification circuit so as to produce high grade metal products or precipitates (Robles et al., 2009).

4.7 Total leachable copper

The chemical digestion analysis incorporating the ICP-OES technique to determine total leachable copper from the concentrate was found to be 160 930 ppm, converting to a measly 45.98% leached copper from total bulk composition. This equates to more than 50% loss of copper through leaching that would be experienced, highlighting the difficult challenges faced with leaching sulphide minerals.

Covellite and bornite both have good dissolution rates under acidic conditions, with covellite readily leaching, however they both do not fully leach out copper owing to sulphur effects as discussed in section 2-1. Chalcopyrite, a refractory mineral on the other hand generally leaches poorly under acidic environments, to improve its leaching kinetics, oxidising environments are required (A. Baba et al., 2012; Devi et al., 2000; Havlik, 2008; Padilla et al., 2008; Zheng et al., 2023a). From the XRD plot, the chalcopyrite phase appeared to be the dominating copper phase, justifying therefore the observed results from ICP-OES analysis owing to its resistance to leaching. Another factor would be poor liberation of the mineral grains as observed from the SEM images and equilibrium conditions may have been reached, which then would require careful design of leaching to favour dissolution of copper from the phases present.

These discoveries therefore indicate the need for further grinding of the concentrate to improve mineral liberation, also employing oxidative pressure acid leaching to enhance the dissolution kinetics and benefit more copper from this concentrate i.e. Activox technology (A. Baba et al., 2012; Palmer & Johnson, 2005a; R. et al., 2012).

5. Modelling and Optimisation of UFG using RSM-BBD, ANN and ANFIS

5.1 Introduction.

Ultra-fine grinding (UFG) experiments were conducted to mechanically activate the mineral surfaces of the sulphide concentrate to enhance the extraction of Cu through oxidative acid leaching (ACTIVOX) technology. To successfully conduct the optimisation of the UFG process, Response Surface Methodology (RSM) was initiated to combat energy consumption either through unorthodox practice, over-grinding or wrong selection of process factors. The Box-Behnken Design (BBD) was chosen for RSM analysis of experimental data, statistically using the analysis of variance (ANOVA) package, model development and diagnostics, and lastly, optimisation for both P₈₀ size and energy consumption.

The critical speed of the mill based on 18.5 mm average diameter of the steel grinding balls was determined to be:

$$N_c = \frac{42.3}{\sqrt{(170-18.5)e^{-03}}} = 109 \text{ rpm} \quad (5.1)$$

The primary experimental design (coarse media) was therefore designed based on 3 factors: Θ_{Nc} (fraction of critical speed), L/S (ratio of liquids to solids) and grinding time (T), and the response factors were still product size [P₈₀ (μm)] and energy consumption [E (kWh/t)]. Table 5.1 shows the average statistical data obtained from the experimental runs.

Table 5-1: Experimental design with coarse media and results obtained.

Run	Θ_{Nc}	S/L	Time (hr)	E (kWh/t)	P ₈₀ (μm)
1	0.5	1.5	1	120.00	22
2	0.9	1.0	2	228.44	17
3	0.5	2.0	2	312.00	27
4	0.7	1.0	1	114.67	29
5	0.7	2.0	3	379.04	12
6	0.7	1.5	2	276.67	17

7	0.9	2.0	2	256.39	17
8	0.5	1.0	2	208.00	22
9	0.7	1.0	3	344.00	11
10	0.5	1.5	3	386.93	19
11	0.7	1.5	2	271.64	18
12	0.7	1.5	2	277.39	17
13	0.9	1.5	1	143.33	23
14	0.7	1.5	2	283.91	19
15	0.7	2.0	1	165.33	20
16	0.7	1.5	2	280.38	18
17	0.9	1.5	3	369.17	16

Figure 5.1 shows the cumulative PSD for results obtained which summarise the representative results as observed in Table 5.1. R0 in fig. 5.1 denotes the feed's cumulative PSD whose F_{80} was observed to be 40 μm . Figure 5.2 shows the power draw trend during operation, which highlights the swift response action of power drawn by the mill. Basically, the power draw across each run was observed to be at steady state, except for R4 which somehow had minor deviations.

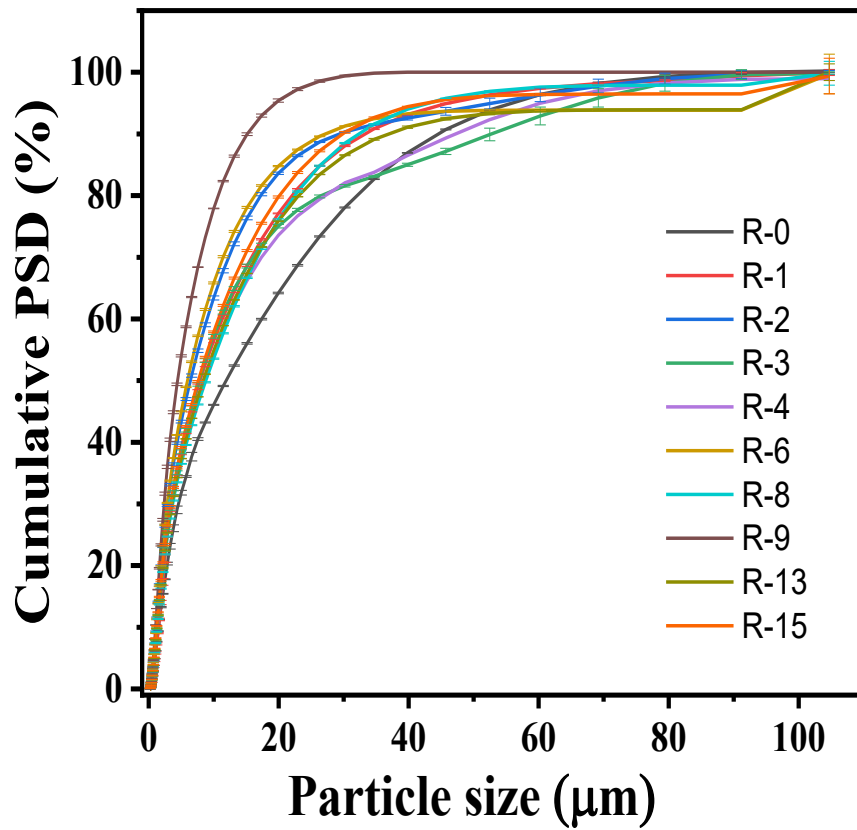


Figure 5-1: Cumulative PSD for feed (R0) and results from selected runs as defined.

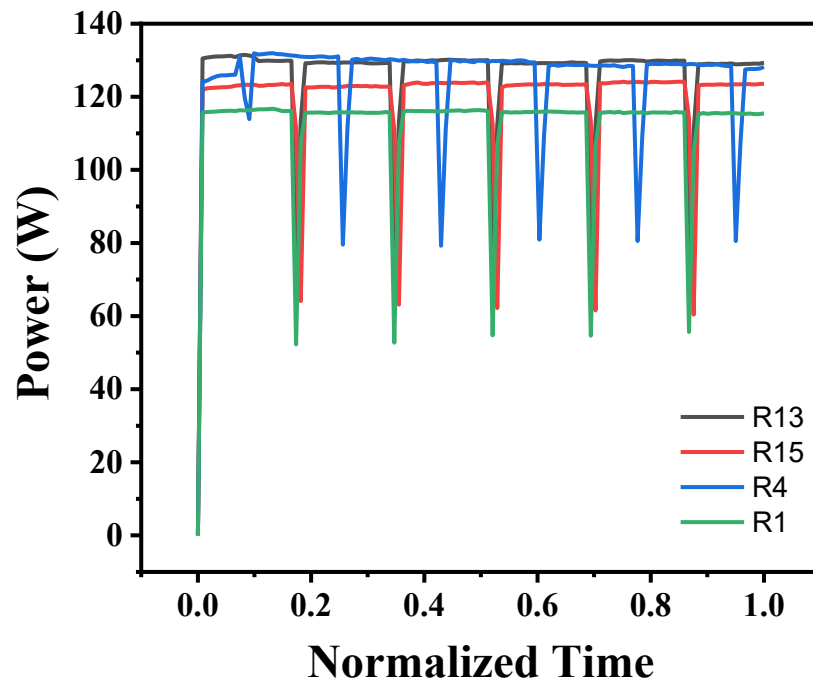


Figure 5-2: Typical power draw trend extracted from the Fluke data logger for the respective runs labelled.

The secondary design based on fine ceramic media was done based on mill speed, grinding time and media filling ratio as the factors while maintaining the same responses, inspired by the impact of stirred mills in fine and ultra-fine milling (Bernhardt et al., 1999; Celep et al., 2011; Harbort et al., 1996). The speed was varied between 250 rpm to 450 rpm (mill speed) as high speeds are favoured to impart shear forces on the feed powder, time between 0.5 hours to 1.5 hours, to check its impact particularly with energy consumption, lastly, media filling rate between 20% and 60% of mill volume. Varying media filling rate was subject to shear forces enhancement i.e. increase of surface area where particle breakage takes place, since no stirrer is present with ball mills as well as the potential interstitial filling optimisation (Hlabangana, Danha, Mguni, et al., 2018a; Tembo et al., 2019b). Table 5.2 shows the experimental design, and the results obtained.

Table 5-2: UFG with fine media DOE using Design Expert software with results.

Run	Speed (rpm)	Time (Hr)	Media filling (%)	P ₈₀ (μm)	E (kWh/t)
1	450	0.5	40	69	15.5821
2	350	0.5	20	19	11.8410
3	350	1.5	60	38	47.7090
4	250	1.5	40	9	40.4776
5	350	1.0	40	45	27.8806
6	350	0.5	60	40	11.0970
7	450	1.5	40	61	48.7164
8	350	1.0	40	41	27.5064
9	450	1.0	20	43	32.0597
10	250	1.0	60	11	31.4627
11	450	1.0	60	76	31.1642
12	250	1.0	20	10	27.5821
13	350	1.5	20	11	43.1642

14	250	0.5	40	13	9.40299
15	350	1.0	40	35	28.0597

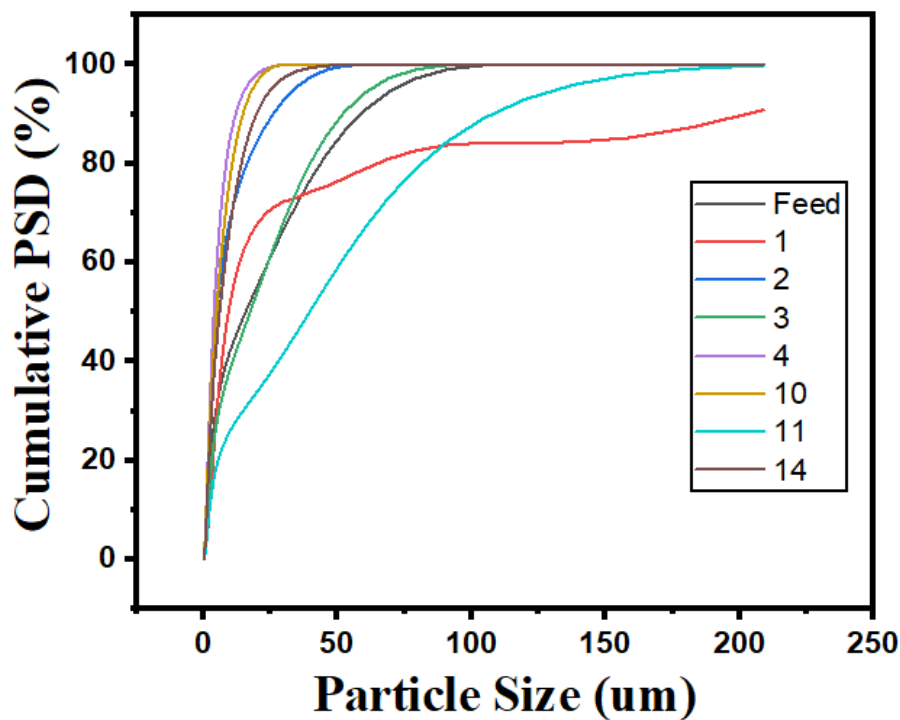


Figure 5-3: Cumulative PSD for UFG with fine media for selective runs.

In general, it was observed that the target range for P_{80} was obtained at lower rotational speeds and low mill media filling rate. Figure 5-3 displays uncharacteristic behavior of some runs, observed to have a resultant P_{80} above that of the feed as observed in Figure 5.1, and this was attributed to the effect of fine particle agglomeration due to extended periods of storage before analysis. Increasing the speed above 350 rpm negatively impacts the milling process, as powder particles together with milling media got stuck onto the walls of the mill, thus no effective milling took place (Figure 5.4). This was attributed to the particles gaining a centrifugal force that exceeds the gravitational pull as such they tend to stick to walls of the mill. For energy consumption, with graphical trend typical of Figure 5.2, the observation is that time is the most critical factor to the other ones as such, its variance is directly proportional to the response of energy consumption.



Figure 5-4: Mill contents sticking on internal mill surface, due to the impact of high milling speeds on efficiency of ball mills.

Comparing Table 5-1 and Table 5-2, the outright observation was that though P_{80} below $20\ \mu\text{m}$ could be achieved with some runs, however the energy consumption was too exorbitant as lengthy milling times were required, as such the outright conclusion was that fine media is best for ultra-fine grinding even for ball mills. This observation was also reported by (Öksüzoglu & Uçurum, 2016; Partyka & Yan, 2007) in their work of fine milling using horizontal ball milling. Henceforth to critically assess the impact by these factors in the response performances, the second DoE Table 5.2 was considered, and an in-depth statistical analysis was conducted using the software, leading to modelling and optimisation of the process.

5.2 RSM-BBD results analysis.

To kick start this analysis, the focus was channeled towards P_{80} response variable, which is the indicative measure of mechanical activation of our sample. The data was fit into a multiple models and the quadric model fit the data more profoundly to the rest, as observed in the fit summary, Table 5.3. Table 5.5 gives the fit statistics of the model, giving a coefficient of variation of 12.09% for P_{80} , slightly above the recommended 10%, indicating low variability of the data set to signify good model performance. Generally, both models had good statistical significance i.e. 0.9873 and 0.9803 for P_{80} and SE respectively, thus the model can be adopted. From the model fit analysis, the analysis of variance on the factors to both responses was conducted.

Table 5-3: Model fit summary for P_{80} analysis using Design Expert software.

Source	Sequential	Lack of fit	Adjusted	Predicted	
	p-value	p-value	R ²	R ²	
Linear	<0.0001	0.2754	0.8655	0.7944	
2FI	0.265	0.2929	0.8829	0.7307	
Quadratic	0.0297	0.7226	0.9644	0.8971	Suggested
Cubic	0.7226		0.9489		Aliased

Table 5-4: Model fit analysis for specific energy consumption (E) using Design Expert software.

Source	Sequential	Lack of fit	Adjusted	Predicted	
	p-value	p-value	R ²	R ²	
Linear	<0.0001	0.0159	0.9750	0.9595	Suggested
2FI	0.3758	0.0153	0.9761	0.9343	
Quadratic	0.2703	0.0158	0.9814	0.8950	
Cubic	0.0158		0.9995		Aliased

Table 5-5: Fit statistics for both P_{80} and E model analysis.

	P₈₀	E
Std. Dev.	4.20	2.02
Mean	34.73	28.91
C.V %	12.09	6.98
R ²	0.9873	0.9803
Adjusted R ²	0.9644	0.9750

Predicted R ²	0.8971	0.9595
Adeq. Precision	20.9996	36.1669

5.2.1 Analysis of Variance (ANOVA).

Employing the Kennard-Roger p-values, this analysis was conducted to assess the statistical significance of the factors to the response variable i.e. factors with p-values less than 0.05 were deemed significant to the model. This was done to understand the effect of factors; mill rotational speed, milling time and media filling rate, on the response P₈₀ size and Energy consumption (kWh/t). The analysis depicted both models as significant, with a lack of fit p-value more than 0.05 i.e. 0.7226 for P₈₀, however significant p-value with SE i.e. 0.0159 meaning some data misfit of the model is significantly observed. However, both models are adopted owing to the other performance criteria i.e. regression analysis and coefficients of variation respectively, Tables 5-6 and 5-7. The adequate precision value of 20.9996 and 36.1669 for the respective models is a desirable signal, as such they can be utilised within the design range.

ANOVA analysis of the respective factors to the P₈₀ response highlighted that mill speed, media filling rate, the synergistic effects from the combination of the two afore-mentioned factors, and the quadratic effect of the media filling ratio had significant influence on the response behavior, Table 5-5. Contrary to the stirred mills which perform best at high rotational speeds, this principle on speed can be attributed to the nature of ball mills that require the centrifugal force of its contents to be lower than their weight for effective milling (Hacifazlıoğlu, 2018). With ball mills, high speeds reduce the impact of abrasion i.e. efficiency, and since the weight of the media particles is small, impact force also is not significant hence the reduced milling at high speeds. Therefore, it shows that low speeds in the range could enhance shear forces more effectively. The significance due to media filling may be attributed to the interstitial filling, which relates to the exploitable surface area through which milling can be induced via abrasion forces. This observation was also reported by (Patino et al., 2022) who optimized ultra-fine grinding of coal, stating the importance of feed pulp to fill the interstitial filling of the mill. This agrees with the general observations highlighted above from Table 5-1. As for E, only speed and time appear to be significant Table 5-6.

Table 5-6: ANOVA analysis for P₈₀.

Source	Sum of Squares	df	Mean Square	F-value	p-value.	
Model	6850.77	9	761.20	43.17	0.0003	significant
A-(Speed)	5304.50	1	5304.50	300.82	<0.0001	
B-(Time)	60.50	1	60.50	3.43	0.1232	
C-(Media filling)	840.50	1	840.50	47.67	0.0010	
AB	4.00	1	4.00	0.2268	0.6540	
AC	256.00	1	256.00	14.52	0.0125	
BC	9.00	1	9.00	0.5104	0.5069	
A2	29.64	1	29.64	1.68	0.2514	
B2	98.56	1	98.56	5.59	0.0644	
C2	246.26	1	246.26	13.97	0.0135	
Residual	88.17	5	17.63			
Lack of Fit	37.50	3	12.50	0.4934	0.7226	Not significant
Pure Error	50.67	2	25.33			
Cor. Total	6938.93	14				

Table 5-7: ANOVA analysis for SE.

Source	Sum of Squares	df	Mean Square	F-value	p-value.	
Model	2231.74	3	743.91	182.71	<0.0001	significant
A-(Speed)	43.23	1	43.23	10.62	0.0076	
B-(Time)	2182.6	1	2182.76	536.11	<0.0001	

C-(Media filling)	5.76	1	5.76	1.41	0.2595	
Residual	44.79	11	4.07			
Lack of Fit	44.63	9	4.96	62.21	0.0159	significant
Pure Error	0.1594	2	0.079			
Cor. Total	2276.53	14				

NB: Energy was modelled using a linear model and hence it has only values for linear parameters (see Table 5-4).

The direct impact of time to E is as expected and normal in milling conditions and as such, it is always imperative to reduce milling time especially with ball mills to optimise the process costs. The mill electric driving motor as expected will draw in more energy to produce higher revolutions per minute as such the impact of speed to the energy consumption is as so, however media filling rate is insignificant due to the reasoning that the weight impact of media increase on the rollers is not significant enough to drive more power draw from the drive motor.

Equation 5.1 is the developed quadratic model for this analysis and equation 5.2 is the linear model for E-prediction. It is imperative to note that the coefficients to terms in the equation do not depict the relative significance of the respective factors, i.e. equation of actual factors.

$$P_{80} = -28.41667 - 0.080833A + 36.83333B + 0.595833C - 0.02AB + 0.004BC + 0.15BC - 20.66667B^2 - 0.020417C^2 \quad (5.1)$$

$$E = -13.95496 + 0.023246A + 33.03602B + 0.042412C \quad (5.2)$$

These equations, according to their high adequate precision values, promote the use of these models for predictions when varying the three factors, however within their design boundaries. To further ascertain the acceptability of the model, the diagnostics analysis of the model was conducted.

5.2.2 Diagnostic analysis.

This was the final assessment done on the model's acceptance criterion, and it was done using the normal plot of residuals, Box-Cox plot, Residuals vs. Predicted plot and finally the Cook's Distance plot.

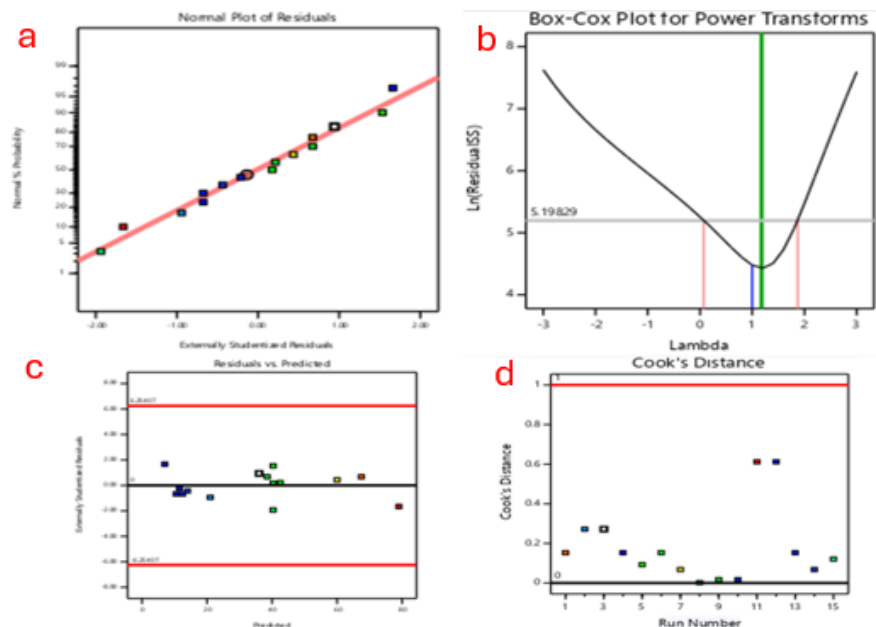


Figure 5-5: Model diagnostics graphs for P_{80} model. (a) is the normal plot of residuals, (b) is the Box-Cox plot, (c) is the residuals vs predicted and (d) is the Cook's Distance plot.

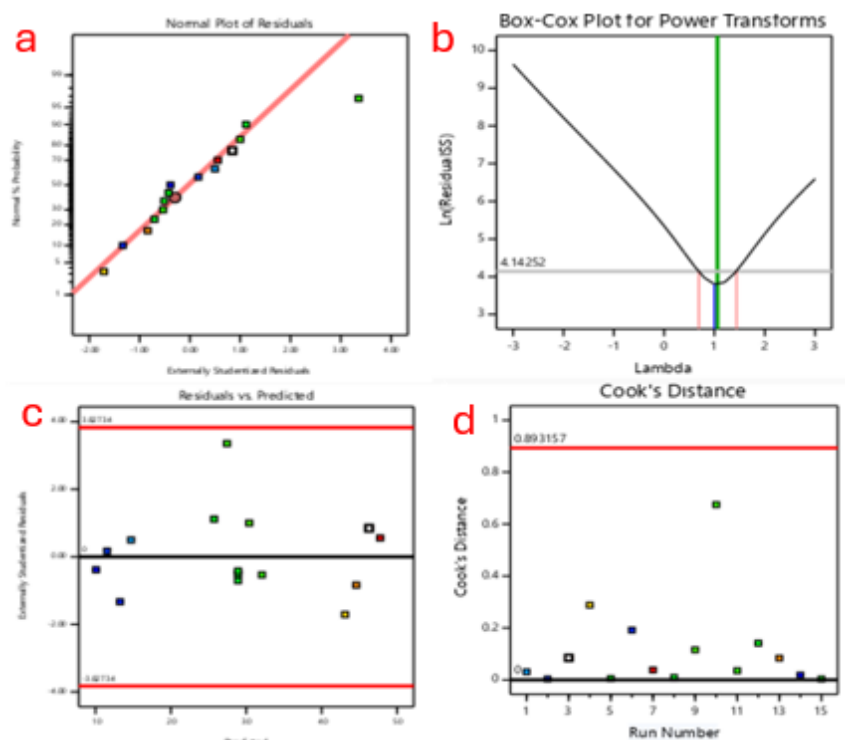


Figure 5-6: Model diagnostics graphs for SE model. (a) is the normal plot of residuals, (b) is the Box-Cox plot, (c) is the residuals vs predicted and (d) is the Cook's Distance plot.

All the four plots Figure 5-5 applaud the P_{80} model's performance, the regression analysis from the normal plot of residuals is close to 1, highlighting the model's ability to map out all data perfectly without many residuals, an undertaking also discussed from the insignificant lack of fit of the model. The Box-Cox plot displays the model's accuracy, with the zero-differential point within range and close to 1 thus model preciseness without the need of any adjustments. Figure 5-5 (c) and (d) show that all data points of the model are within acceptable range, highlighting good model prediction performance. From Figure 5-5 (d), we can also observe that further improvement of the model is possible with runs 11 and 12, however this was not done in this work as they still were within acceptable Cook's Distance

This led to further analysis of the impact of the factors on the response variable, and this was done through the model graphs. The same was observed with SE in Figure 5-6 to show that the model performance is adequately good, with run 10 showing the highest Cook's Distance, highlighting possible improvement of the model with re-engineering of the run.

5.2.3 Response Surface Analysis.

This was conducted through contour graphs and 3D surface graphs. They highlight the correlation present between the process factors to its responses.

5.2.3.1 Contour graphs.

Figure 5-7 shows contour analysis of the two factors, time and speed at varying media filling ratios, with the blue region signaling the target area of P_{80} i.e. lower range and the red at highest range. From the SEM sizing analysis, it was observed that to liberate the mineral grains, particle size reduction to 20 μm is optimum, as such the target zone is beneath the 20 μm contour line (dark blue region). It is observed from Figure 5-7 that low media filling at mill speed below 350 rpm is optimum within the time range for design specs. With increasing media filling rate, there is a reduction in mill efficiency as such lower mill speed below 280 rpm are favorable within the time zone. This revelation is as realised and discussed in the ANOVA analysis.

On the energy side, as was observed in ANOVA analysis that media filling had not any significant impact on energy consumption rate, the contour graphs analysis also displayed the same, as such its effect is excluded in this analysis Figure 5.8. Here it was observed that there is a direct proportional relationship between time and SE, and an inverse proportionality

relationship between SE vs mill speed. To understand the extent of varying these variables on the responses, 3D response surface graphs were utilised as shown below.

5.2.3.2 3D graphs.

The 3D surface response graphs map out the impact to the response variable, by various level combinations of the design factors (Ebadnejad et al., 2013b). Figure 5-9 shows the same revelations observed from the contour lines. However, of interest is the observation from Figure 5-9(c) which shows that the target range for P_{80} can be achieved at mill speed ranges within the range 250-280 rpm, within the shortest milling time frame, suggesting the opportunity to increase the mill throughput, which optimises the costs for the overall process. To understand the impact of these respective design factors, leading to optimisation, sensitivity analysis using perturbation graphs was done. Figure 5-10 shows that though the contour graphs highlighted the inverse relationship between speed and E, it is observed herein that the impact of the factor is almost linearly overall.

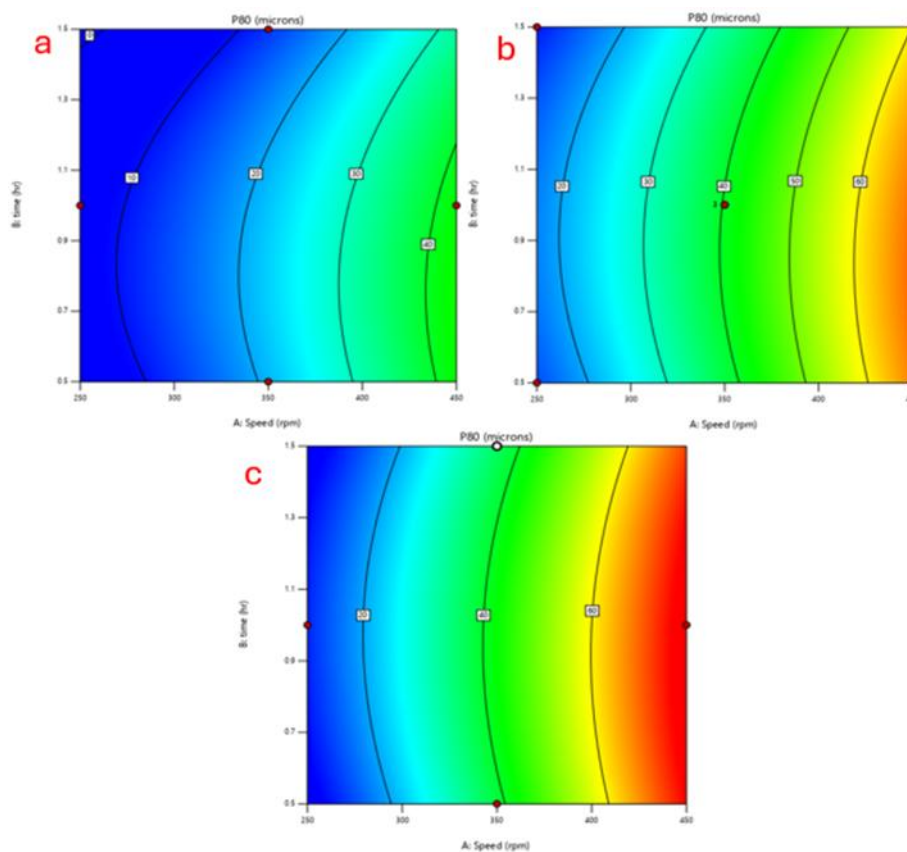


Figure 5-7: Contour graphs analysis for impact of the factors on P_{80} . (a) analysis at 20% media filling, (b) is at 40% filling and (c) at 60% filling.

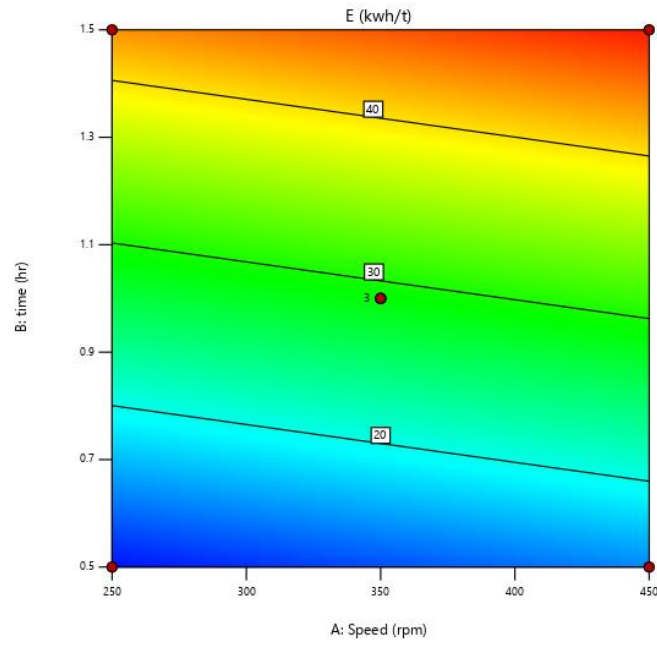


Figure 5-8: Contour analysis on the impacts of mill speed and time on specific energy consumption, SE.

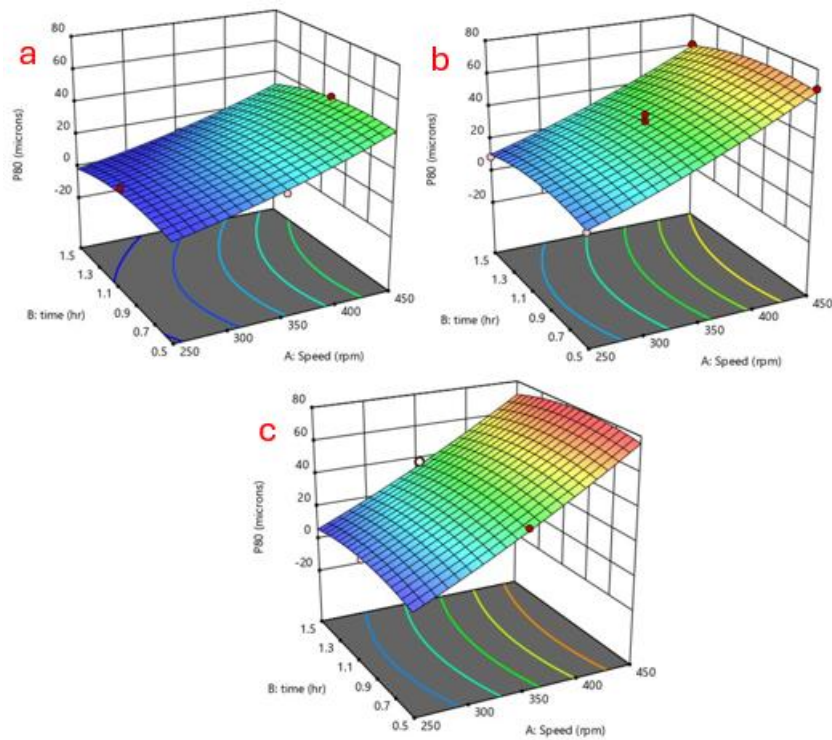


Figure 5-9: 3D surface response graphs of time and speed on P_{80} , starting with (a) at 20% media filling, (b) 40% and (c) at 60% filling respectively.

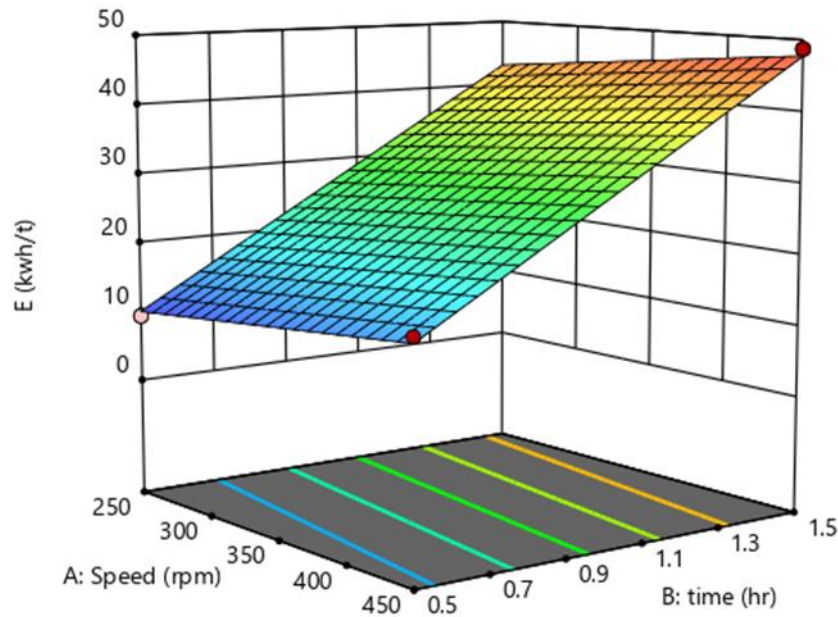


Figure 5-10: 3D surface response plot on speed and time to the response SE.

5.2.4 Sensitivity Analysis.

5.2.4.1 Perturbation graphs

The significance of each design factor to the response variable can be observed by the slope/curvature or straightness of the factors, with slope/curvature signaling the greater significance of that factor to the response variable, and the opposite is true for the flat/horizontal/less curvature graph of the factor. Figure 5.11 shows that in general, speed was the most significant factor followed by media filling and last time i.e. $A > C > B$. As disturbances were introduced, i.e. variation of factors within their respective design ranges, the following conclusions were drawn:

- ❖ Figure 5.11, as speed increases, the significance of media filling rate (C) increases, to maintain the target grinding P_{80} of 20 μm , low levels of media filling must be selected. Time, however, remains slightly significant particularly at high media filling although it was observed that at low media filling and high speeds, extended grinding time bears a significant impact on P_{80} towards the target range.
- ❖ Figure 5.12, since the impact of low speed was concluded as optimum for the P_{80} response, however it was noted herein that at low speed and low media, time is relatively as important too, as extended grinding times were shown to cause

overgrinding, which is a consequence of wasteful energy consumption, as such it must be controlled. At high media filling, however, the conclusion was different, as the time generally did not bear much effect on the outcome response.

With energy analysis however, Figure 5.13, for all possible disturbances to the process, it was observed that time was the most critical factor of them all, a note similar to the work by (Kumar et al., 2023). Speed slightly implicated the energy consumption rate but almost no change could be interpreted with media filling variations. As such, to conduct sustainable ultra-fine grinding using this set-up, time will be at the centre of optimisation of the design factors.

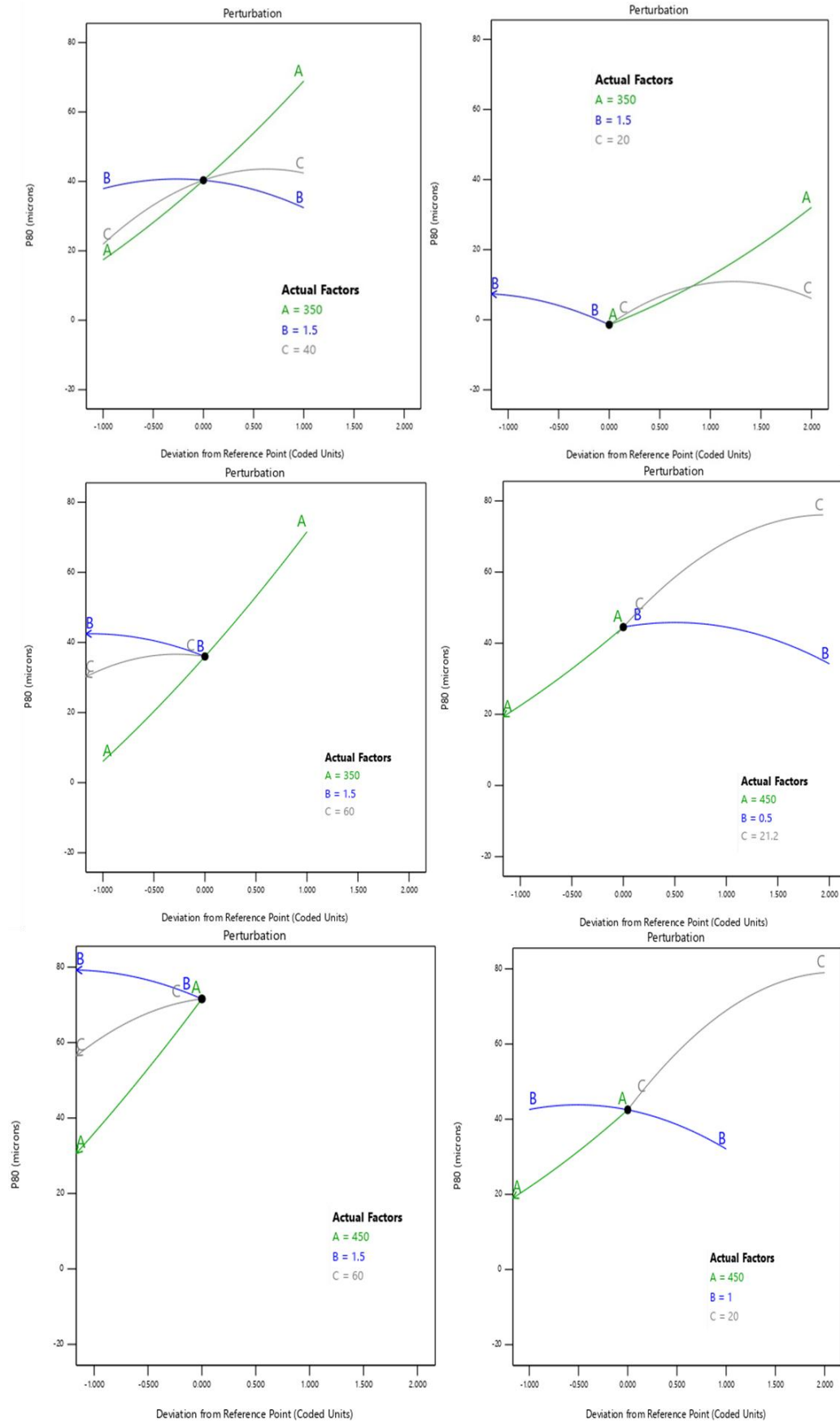


Figure 5-11: Perturbation analysis graphs.

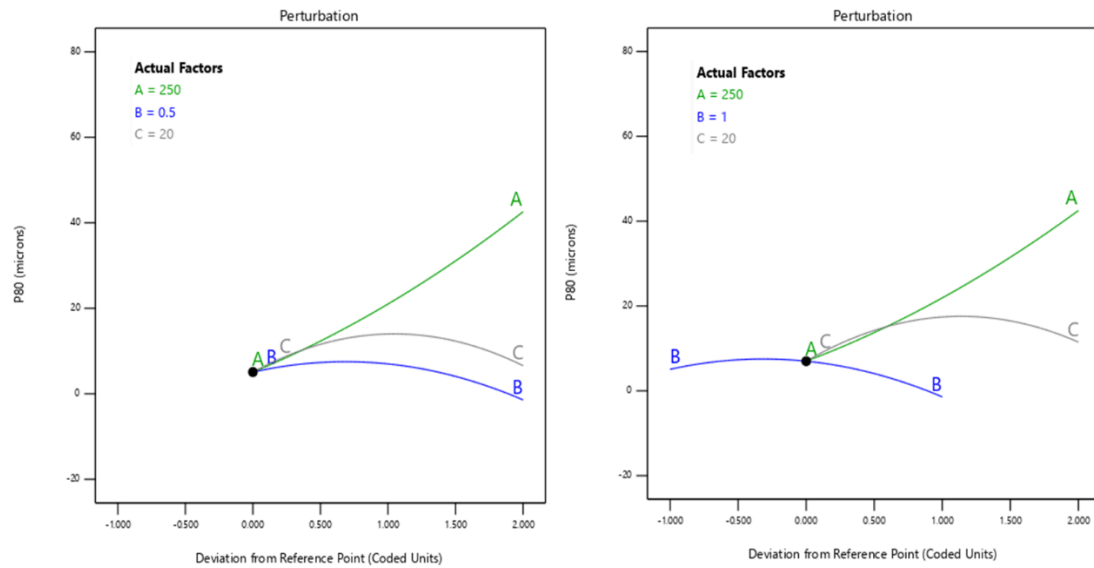


Figure 5-12: Impact of grinding time to overgrinding, at low speed and low media filling.

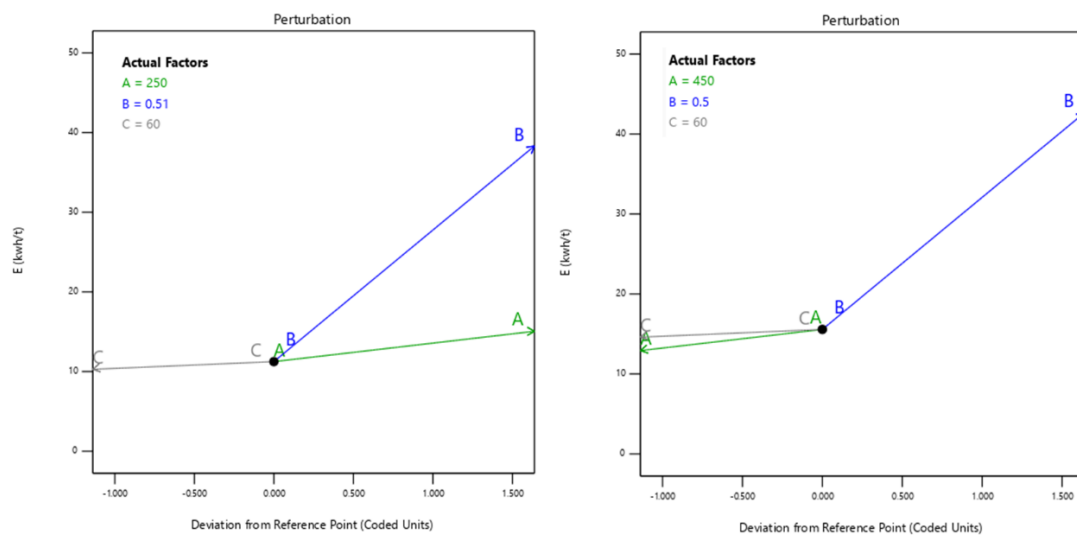


Figure 5-13: Perturbation analysis on SE.

5.2.5 Optimisation.

From the graphical analysis, it was observed that there is a close interaction between the design factors and the response variables, as such, there is always need in such scenarios to deduce the optimum conditions of the factors to suit the target goal of the response variable. This can be achieved by defining the response variables and the factors either through attaining a certain target, minimisation, maximisation or to be within a certain range. For this study's purposes, the goal was to minimise both the P_{80} and energy consumption variables, i.e. keeping the P_{80}

to a target of 20 μm while at the lowest possible energy consumption, hence the optimisation constraints were set as so, while for the factors, they were kept within the design ranges so as to determine the optimum conditions. Table 5.8 shows the constraints for the objective function while Table 5.9 gives the top 5 optimum solutions out of a generated 78, to the optimisation function and Figure 5.14 gives the desirability plot.

Table 5-8: Optimisation criteria for the UFG using ball mill.

Name	Goal	Lower limit	Upper limit	Lower weight	Upper weight	Importance
A-speed	In range	250	450	1	1	3
B-time	Minimise	0.5	1.5	1	1	3
C-media	In range	20	60	1	1	3
P ₈₀	Is target-20	15	20	1	1	3
SE	Minimise	9.40299	48.7164	1	1	3

Table 5-9: Top 5 solutions to the optimisation problem, obtained from the analysis.

Number	Speed	Time	Media	P ₈₀	SE	Desirability	
1	282.561	0.5	35.508	20	10.638	0.989	Selected
2	281.910	0.5	35.912	20	10.640	0.989	
3	285.597	0.5	33.876	20	10.639	0.989	
4	279.203	0.5	37.965	20	10.664	0.989	
5	278.009	0.5	39.229	20	10.689	0.989	

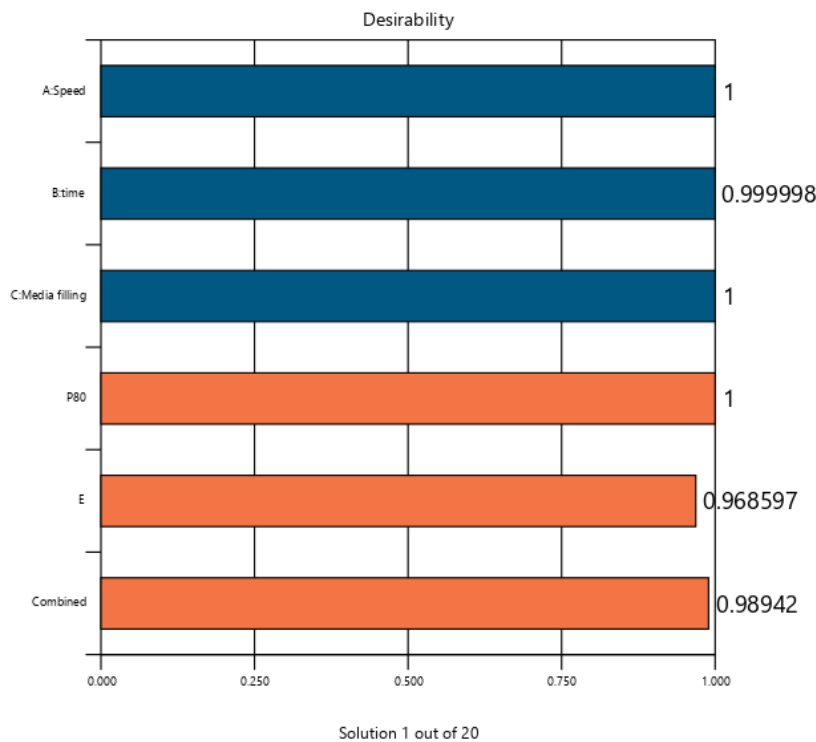


Figure 5-14: Desirability plot for the optimization task of the design.

The desirability as displayed by Figure 5.14 suggests that media filling and speed have met their optimum best levels except for time, as such to improve the performance of UFG based on the two responses, P_{80} and E, there is room for improvement with time. This suggests operating even in time periods below 0.5 hours, which was set to be the focus in our next work. However, a combined desirability level of 0.989 is accepted as it is closer to one, therefore, the optimum design parameters were adopted for further analysis tests.

5.3 Hybrid Optimization Using ANN and ANFIS.

To further assess the performance of the developed models, machine learning tools Artificial Neural Networks (ANN) and Artificial Neural Fuzzy Inference System (ANFIS) were employed. ANN technique was employed for its pro-active nature to uncover solutions of non-linear functions (inverse modelling) (Jovanović et al., 2015), as noted from the ANOVA analysis that UFG process is non-linear. ANFIS was employed for its learning and reasoning capabilities (Banza & Seodigeng, 2023a), an extended advantage to ANN. Therefore, with these machine learning tools, the intrinsic interactions of the factors are deemed to be uncovered and thus optimise the process. These tools were utilised using the embedded user interfaces in MATLAB 2021b.

5.3.1 ANN.

5.3.1.1 Methodology

The input factors and response variables were maintained as in RSM, and the design followed a 1-hidden layer design with 3 inputs and 2 outputs. The model design parameters: transfer function and training algorithm i.e. Log-Sigmoid and Levenberg-Marquardt respectively were selected, backed by literature. The Log-Sigmoid transfer function performs best compared to the pure-linear and tan-sigmoid transfer functions due to its ability to narrow down on every data set therefore reduce the lack of fit of the model (Orero et al., 2023a), while the Levenberg-Marquardt is renowned for its robustness and heuristic optimization as it employs both the Gaussian Newton Method and the Gradient Descent thus its ability to learn faster with minimal error (Using & Surface, 2022). The study was carried out by varying the effect of the number of hidden neurons in the hidden layer as this impacts the fastness, learning capabilities, overall, the economics of employing the tools as such the variations were tested at 5, 7 and 10 neurons for the individual response models. 100 data sets were generated using the prediction model equations from RSM, Eq. 5.1 and 5.2, then it was randomly divided into 70/15/15 datasets for training, testing and validation, respectively. Figure 5.15 shows the typical design framework for the 5-neuron model.

The performance criterion used to test these models were regression analysis, mean square error and number of epochs (iterations). Table 5.10 shows the analysis results from these tests.

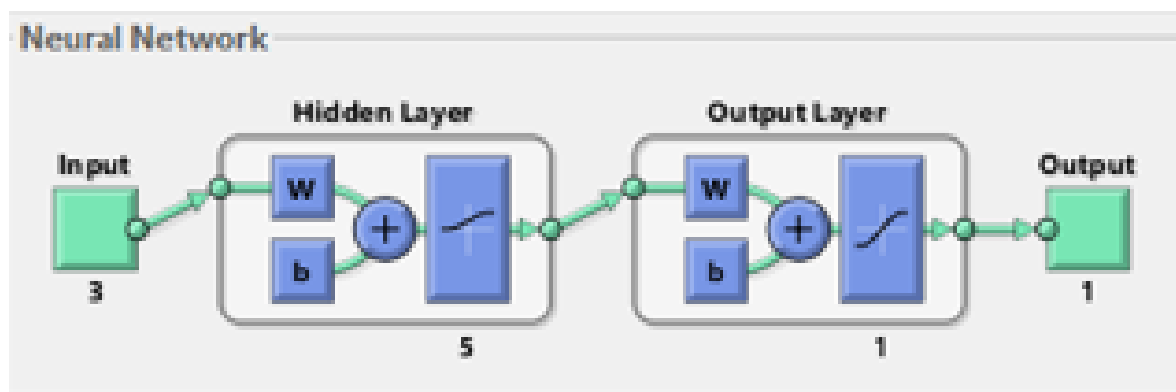


Figure 5-15: Typical ANN model design

Table 5-10: Performance analysis of the designed ANN models using regression, mean square error and required iterations.

Neurons	P ₈₀			SE		
	R ²	MSE	Epochs	R ²	MSE	Epochs
5	0.99198	1e-05	13	0.99712	1e-05	19
7	0.99634	1e-06	17	0.99699	1e-06	26
10	0.98259	1e-06	17	0.99736	1e-06	28

From the results above, all models performed exceptionally well, with regression correlations above 99% highlighting good model learning, as the simplest model architecture of them all i.e. the model with 5 neurons, was adopted for both P₈₀ and E. As the model displayed over 99% regression analysis for all training, testing and validation data with the predicted data i.e. close to ideal, with error values of the magnitude 1×10^{-5} (1e-05), as such it can be used to effectively predict process performance i.e. P₈₀ or E, employing process factors; time, mill speed and media filling rate, averting the need for optimisation through experimental work with every change in process conditions. Figure 5.16 and Figure 5.17 show the performance of the model via regression analysis for both response variables.

After vigorous learning by the models, the resultant neuron weights of the selected model were thus used to conduct a sensitivity analysis test to determine the significant impact by each factor on the respective responses.

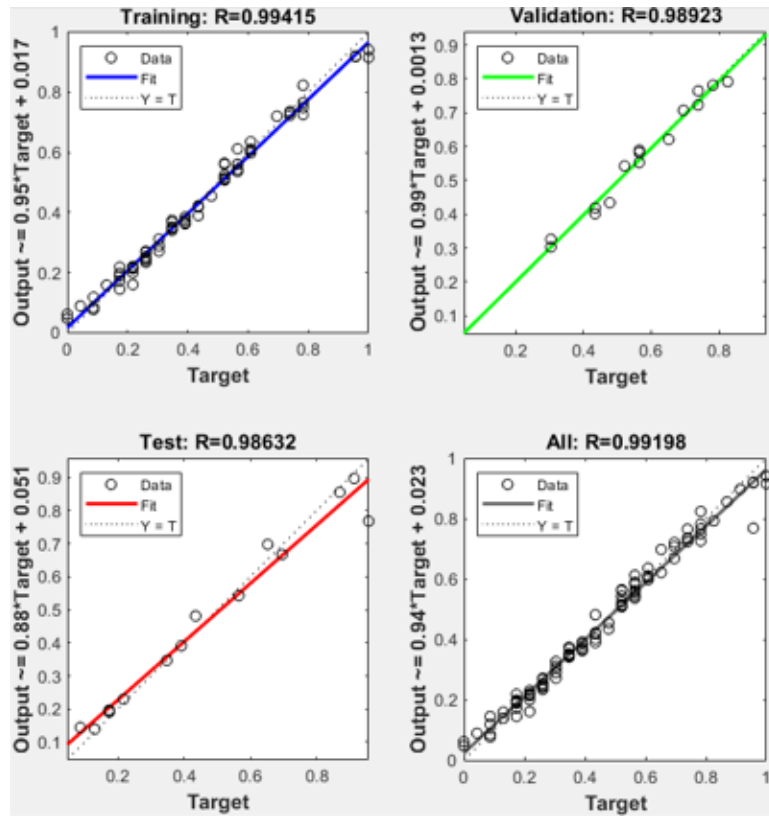


Figure 5-16: Regression analysis of the optimum model of 5 neurons for P_{80} .

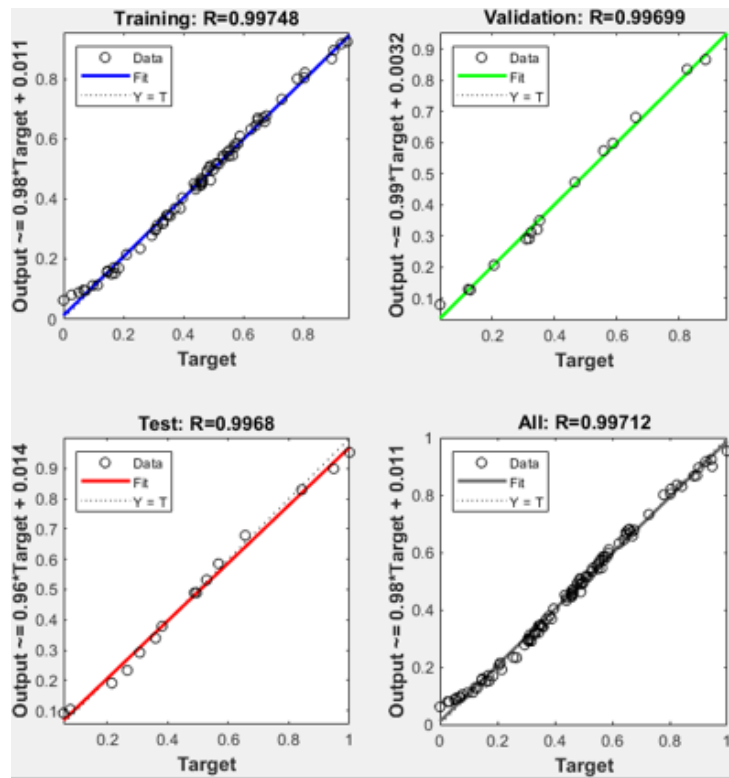


Figure 5-17: Regression analysis for optimum model of 5 neurons for SE.

5.3.1.2 Sensitivity Analysis.

(Sithole et al., 2024a) used the Garson method of utilising the weight coefficients generated from the model network to uncover the interactive effects of the input parameters to understand the relative importance of each input variable to the response outputs. Employing this principle, Figures 5.18 and 5.19 display the relative significance of the input variables for P_{80} and SE, respectively.

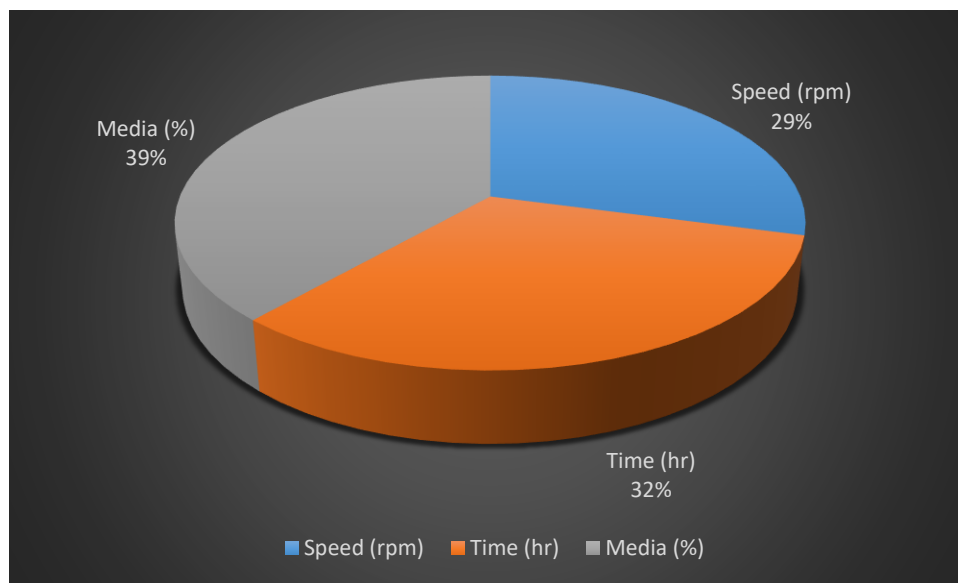


Figure 5-18: Sensitivity analysis for P_{80} using ANN weights.

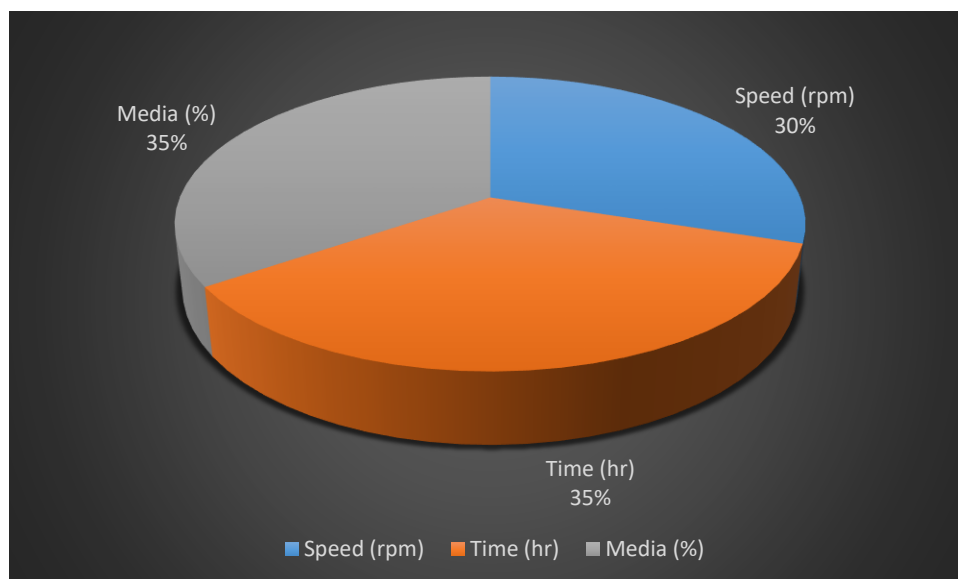


Figure 5-19: Sensitivity Analysis for SE using ANN weights.

It is evident that media filling has relative significance followed by milling time then mill speed on P_{80} (Figure 5.18), while with energy consumption (SE), media filling and milling time appear to bear equal significance then followed by mill speed (Figure 5.19). This is contrary to observations from perturbation analysis Figure 5.11 and Figure 5.13 where speed was considered the most critical factor for P_{80} and time for SE. On the note of the Garson method, media filling is concluded to be the primary factor of significance in both cases, suggesting the dependence of the other factors on media filling. This is another example of the prowess of ANN learning articulates, showing that it does not assess based on the set conditions as RSM does, however from the entire data, it concludes in a comprehensive and collective manner.

Also, what is to note is the relativeness of the significances, for SE, all are close to each other, with just 5% difference while with P_{80} , the difference increases particularly between media filling and mill speed. This close association therefore tells us that in ultra-fine grinding process using ball mills, all the 3 factors are of importance as such optimisation of all is important. The margin increment with media filling in P_{80} can be attributed to the importance of surface area of media in fine milling, which agrees with the reports by (Partyka & Yan, 2007). In E analysis, the relative significance of media and time may be attributed to the impact of weight increase or decrease on mill rollers which impact power drawn, a discovery which was, however not reported by RSM analysis but that of time is as reported in RSM. This technique inherently shows the advantage of ANN thus its applications as an expert system in optimisation.

5.3.2 ANFIS

The ANFIS tool is praised for not just its learning capabilities but also its reasoning manner as such its widespread application also in extractive metallurgy (Banza & Seodigeng, 2023b; Long et al., 2020; Orero et al., 2023a). The tool is dependent on the inference system employed, which is basically the engine of the tool i.e. Mamdani and Sugeno. Mamdani is mastery for introductory basis, however for complex optimisation and adaptive techniques, the Sugeno possesses better performance overall to Mamdani in terms of processing time, output type and optimum performance especially with dynamic and non-linear processes, as such it was selected for this work (Blej & Azizi, 2016).

Designing a model using the input variables is dependent on the selection of membership function types for both input and output i.e. triangular (trimf), trapezoidal (trapmf), Gaussian (gaussmf) and generalised bell (gbellmf); linear and constant respectively, as well the optimisation algorithm, either hybrid or back-propagation. As such 16 models were developed with varying of these factors on each of the response variables and applied the ANOVA and Post-Hoc Analysis to scrutinise and select the best model in comparison to experimental data.

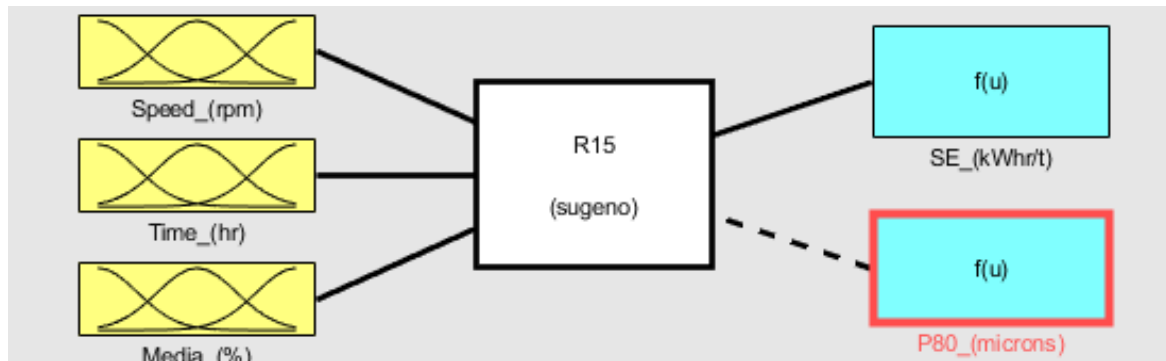


Figure 5-20: Model design for typical ANFIS-Sugeno layout.

Figure 5.20 displays a typical outlook for Sugeno model, herein displaying model 15 for P_{80} analysis. ANOVA analysis, Table 5.11, of all models deemed them significant i.e. fail to reject the null hypothesis (H_0) since the p-values for P_{80} and SE were 1.00 and 0.9671, respectively. As such the Post-Hoc Analysis was invoked to test the statistical differences between the models and the experimental data Figure 5.21, and as noted in Figure 5.22, all models had means, statistically significant to that of the experimental data, as such, to separate on these models, the combined regression and Post-Hoc analysis was used, Table 5.12. For P_{80} , model 15 was selected as the optimum model i.e. (sugeno-gaussmf-constant-hybrid), while for SE, the optimum model was the 1st model i.e. (sugeno-trimf-linear-hybrid). From these results, it was noted that for optimisation purposes, the hybrid algorithm performs best. Triangular membership function follows in general a piecewise linear approach i.e. simple and straight forward, hence that was believed to be the reason for its optimum performance as SE followed a linear system, unlike P_{80} which is non-linear thus more likely to be interpreted more effectively by the Gaussian which in essence captures more complex, gradual changes in systems.

Table 5-11: ANOVA analysis for both model groups to experimental group.

Response	Source	SS	dF	MS	F	p-value
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P ₈₀	Groups	0.086	16	0.00537	0.1	1
	Error	25.347	493	0.05141		
	Total	25.433	509			
SE	Groups	0.3578	16	0.02236	0.45	0.9671
	Error	24.3418	493	0.04937		
	Total	24.6996	509			

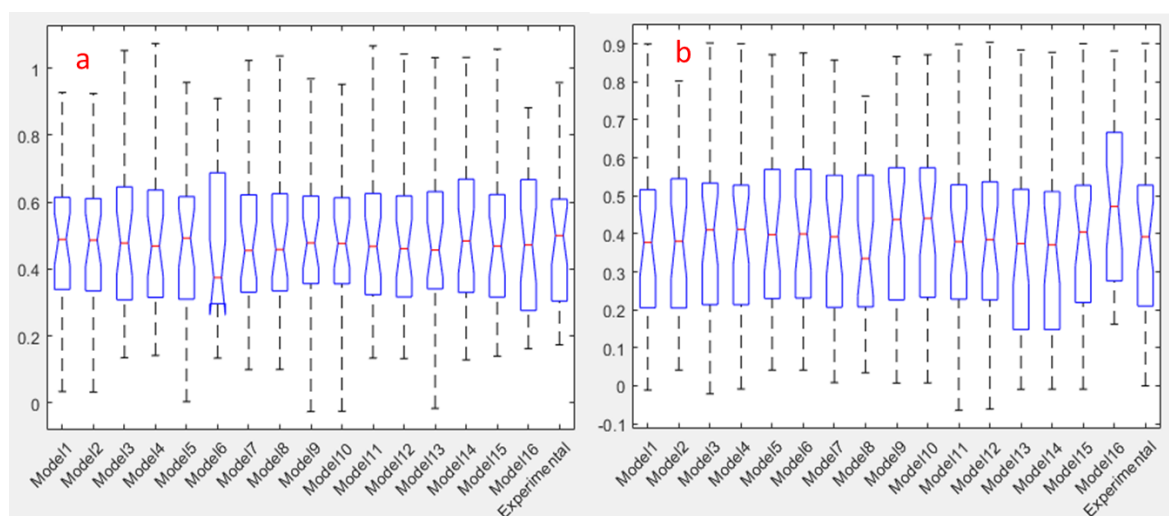


Figure 5-21: Model means analysis for models and experimental data, a- P₈₀ and b-SE.

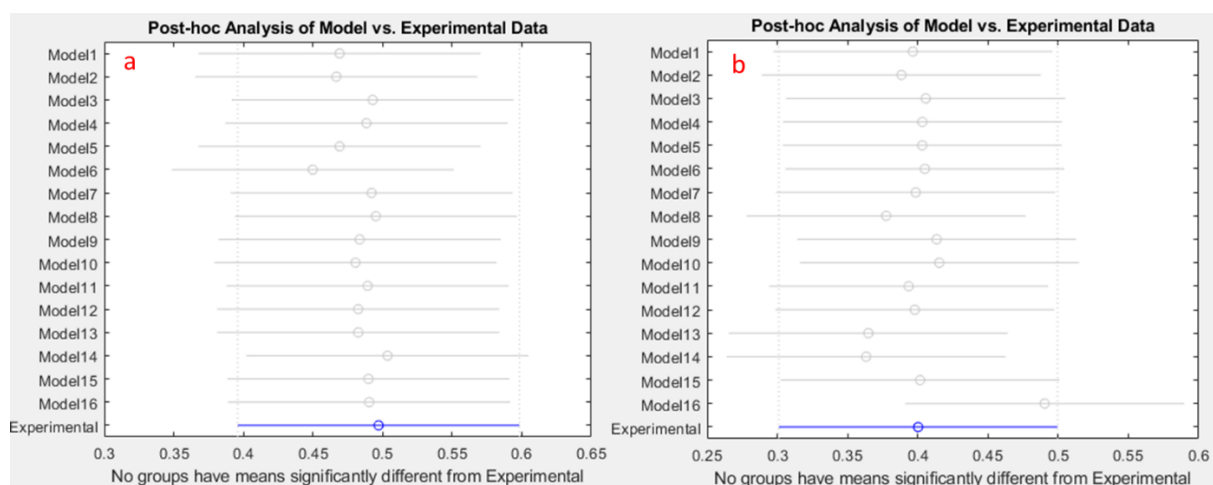


Figure 5-22: Post-hoc Analysis of model means to experimental mean, a- P₈₀ and b- SE.

Table 5-12: Combined analysis results for both response models.

Response	Model #	Post-Hoc	Regression	Comment
P ₈₀	1	0.0280	0.9017	
	2	0.0303	0.90	
	3	0.0042	0.9738	
	4	0.0087	0.9722	
	5	0.0279	0.5071	
	6	0.0473	0.5293	
	7	0.0050	0.9561	
	8	0.0019	0.9539	
	9	0.0136	0.8835	
	10	0.0166	0.8843	
	11	0.0079	0.9712	
	12	0.0146	0.9710	
	13	0.0146	0.8447	
	14	0.0064	0.9324	
	15	0.0073	0.9771	Selected
	16	0.0068	0.8260	
SE	1	0.0038	0.9972	Selected
	2	0.0119	0.8349	
	3	0.0054	0.9943	
	4	0.0030	0.9964	
	5	0.0029	0.8608	

6	0.0047	0.8592
7	0.0017	0.5411
8	0.0227	0.6298
9	0.0132	0.8957
10	0.0151	0.8970
11	0.0068	0.9787
12	0.0024	0.9812
13	0.0355	0.8527
14	0.0370	0.8558
15	0.0014	0.9954
16	0.0901	-0.9624

Figure 5.23 shows typical model rules generated for P_{80} and SE analysis using their respective models, uncovering the interactive effects of the input parameters through the generated rules. The figure tells of the 27 fuzzy rules developed by the neural inference engine, from analysing the data sets, which govern the basis of interpretation of each input variable, and how it contributes to the response variable. The yellow area for each rule highlights the impact it bears in governing the extent to which the input variable impacts the response variable.

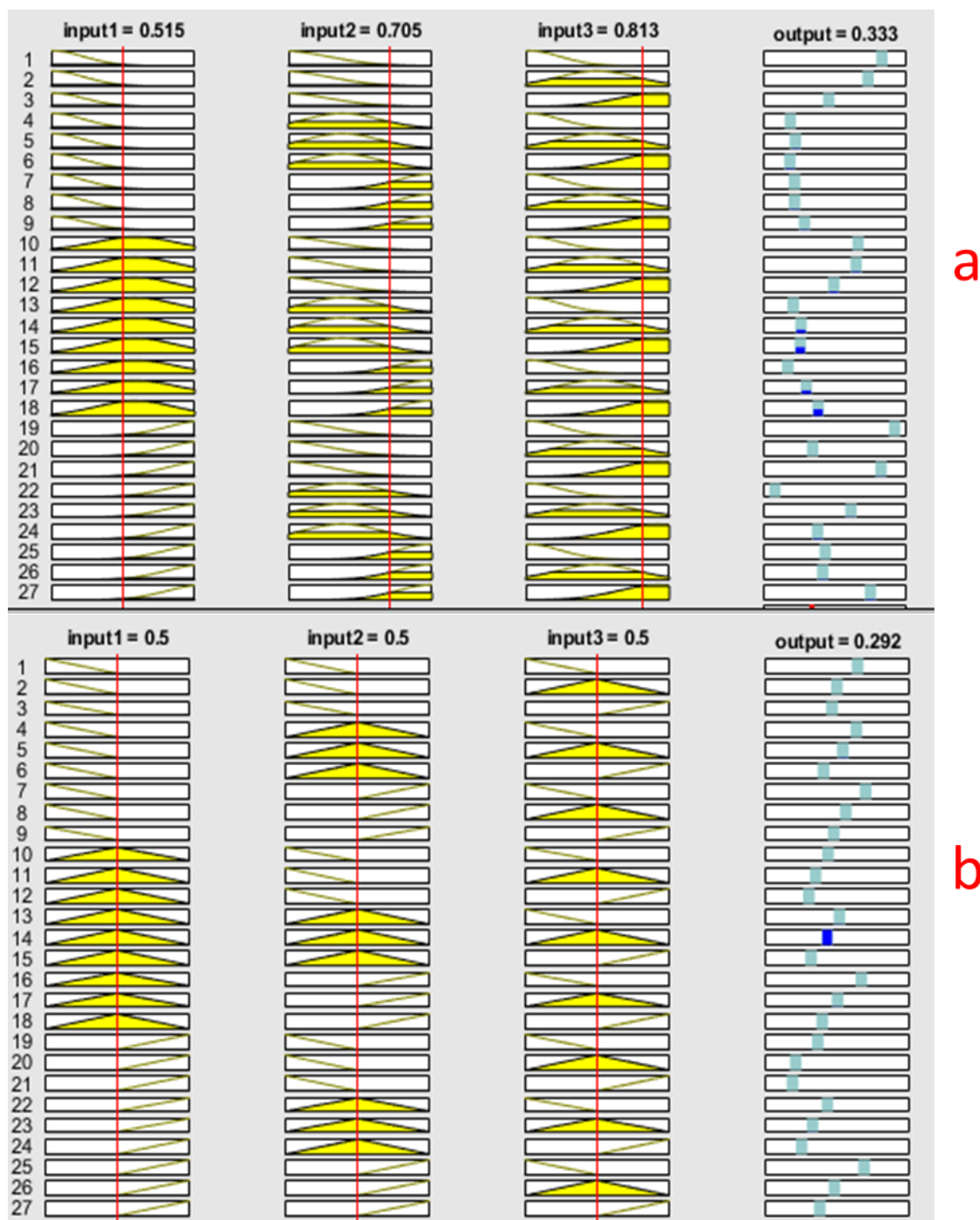


Figure 5-23: Fuzzy Inference System rules for developed models i.e. a- for P_{80} and b- for SE.

5.4 Validation.

To validate these models, they were applied on a separate sulphide sample, obtained from Tati Nickel Mining Company to assess the adoptability of the models in other ores of similar characteristics.

Table 5-13: Validation results for the different models developed.

Speed (rpm)	Time (Hr)	Media (%)	Experimental		RSM		ANN		ANFIS	
			P ₈₀	E	P ₈₀	E	P ₈₀	E	P ₈₀	E
300	0.5	20	0.998	0.475		0.486	0.981	0.533	0.996	0.406
			2	7	0.958	9	2	6	2	6
250	0.5	60		0.460	0.583	0.778	0.630	0.648	0.578	0.675
			0.56	9	3	9	7	6	4	8
300	1	40		0.340		0.282	0.354	0.276	0.350	0.291
			0.2	9	0.333	4	1	4	2	7
350	0.5	60		0.177	0.916	0.047	0.878	0.048	0.914	0.006
			0.924	2	7	7	7	6	2	7
250	1	20		0.772	0.291	0.799	0.170	0.763	0.274	0.786
			0.28	9	7	2	5	8	3	3

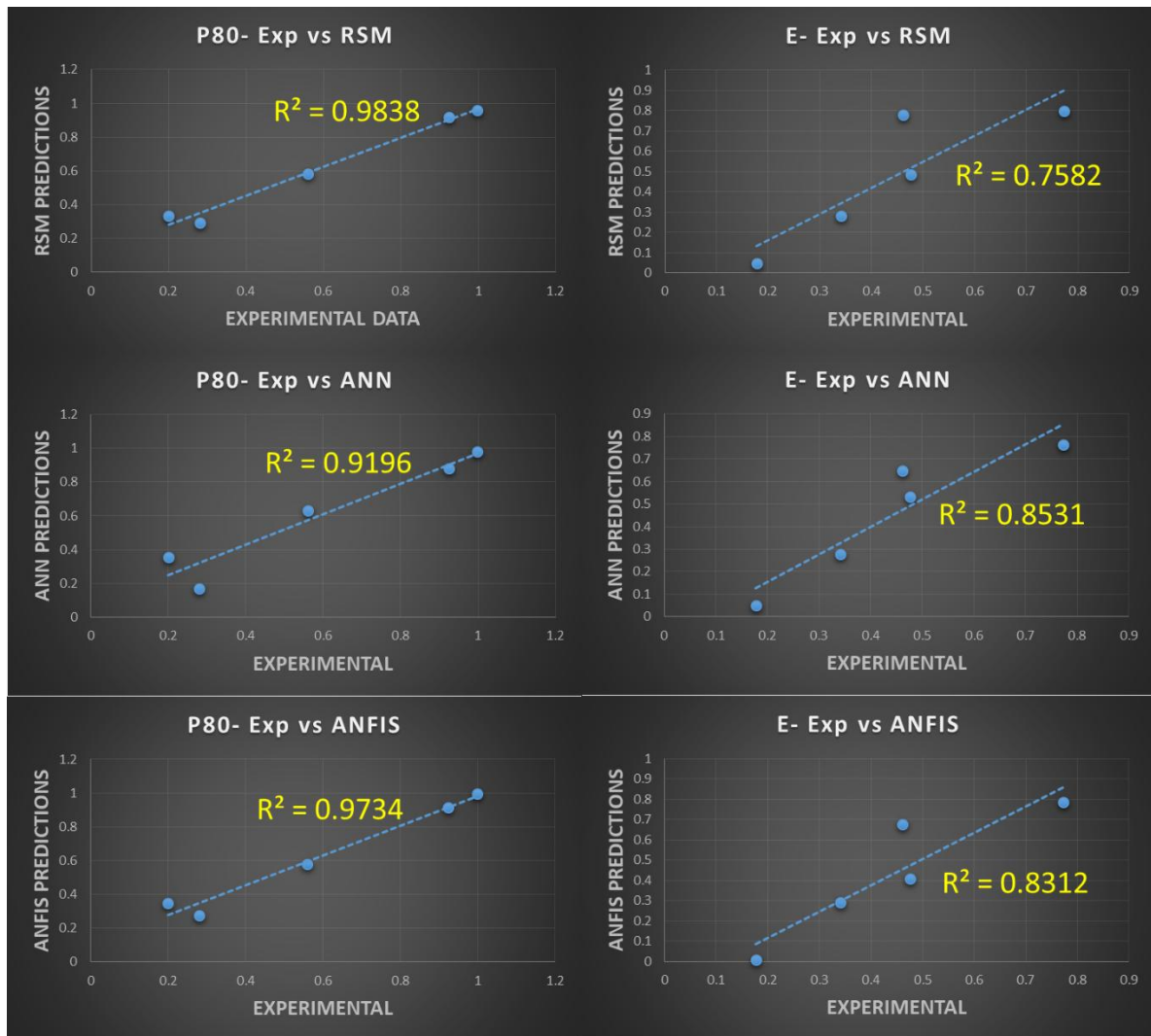


Figure 5-24: Validation regression analysis for the RSM, ANN and ANFIS models on P_{80} and E responses.

The observed regression analysis in Figure 5.24 validates the hybrid optimisation of the UFG using ball mills, by employing RSM, ANN and ANFIS. The performance of the models RSM and ANFIS on P_{80} are superior to that by ANN suggesting overtraining with ANN, a challenge that is prone as such training with fewer neurons might improve the models' validity, however they are all above 90% as such possess close relationship to ideality. With energy modelling, ANN model learned and predicted better than the other two models, however the relationship to ideality is still less than appreciable levels thus using more neurons might achieve desired levels for adoptability of the model.

6. Conclusion and Recommendations.

6.1 Introduction

The aim of this work was to model the ultra-fine grinding process of copper sulphide ore and optimise the energy consumption and the grinding extent. This was achieved through critically conducting the four set objectives, to assist in answering the set research questions.

The following conclusions were thus drawn.

1. UFG mechanically activates the mineral grain surfaces, improving the dissolution rate of the target base metals by the action of the lixiviant. It therefore aids in efficient sulphide mineral leaching, leading to high recoveries of the base metals as observed at Botswana Metal Refinery (BMR) at TNMC.
2. The effect of grinding media diameter was identified to impact the efficiency of the UFG, from product quality (P_{80}) to the specific energy consumed (SE). In general, grinding efficiency in UFG improved with fine grinding media i.e. more energy was consumed in the process, and this was in sync with other reports from literature. Changing from coarse to fine media led to energy savings of more than 50%. Apart from the grinding media size, grinding media filling rate was also determined as the base parameter to which the two other parameters, milling time and mill speed heavily relied on, in governing the efficiency of the UFG process, in both perspectives.

The drawbacks in utilising the traditional ball mill i.e. the laboratory TENCAN ball mill, in conducting efficient UFG were noted, as had been highlighted in literature. However, the game changer to these negative effects were countered by employing fine media that is typically used in stirred media mills, over the traditional ball mill media. This managed to render desirable product quality and reduced specific energy consumptions, as compared to the base case scenario. In general, fine grinding media is more costly as compared to coarse media therefore, their selection is sure to increase process expenditure. To the contrary, they render much improved efficiency as compared to coarse media and this points to the potential of counteracting the procurement cost of it as grinding media constitutes one of the main costs of milling.

3. To employ ball mills in UFG, the milling circuit would need to be re-configured, from coarse to fine milling, then fine to ultra-fine milling, incorporating the media size effect

i.e. coarse and fine size respectively. Abrasion or shear forces were thus determined to be greatly behind the impactful size reduction with fine media milling.

4. Predictive modelling by coupling the RSM-BBD method with ANN and ANFIS machine learning models was deemed important in deciphering the intricacies involved in the complex UFG process, which is even more complex when utilising the ball mill. From the modelling exercise, the hybrid model RSM-BBD-ANFIS displayed the best predictive capabilities particularly on the product quality (P_{80}), while RSM-BBD-ANN for specific energy consumption (SE) predictions.
5. Characterisation was noted as the baseline of this work, from where the optimisation criterion was constituted.

To quantify the extent of optimisation, a cost benefit analysis for this process was carried out, discussed next.

6.2 Cost-benefit-analysis.

During the Activox technology demonstration at TNMC (BMR), the milling circuit comprised of three stirred media mills, one horizontal (Netszch IsaMill) and two vertical ones (Bradken Metprotech mill and the Metso Minerals stirred media detritor mill), wherein 310kg/hr rougher concentrate was processed either by one or two of the three mills. Feed cumulative undersize PSD (F_{80}) for the concentrate was 60 μ m and the target P_{80} at 10 μ m.

Taking these as the baseline to the analysis for an 8000hours annual operation of the plant, the implications behind incorporating the optimized process in terms of energy consumption and product quality grade were assessed.

Table 6.1 shows the impact of grinding media size on the energy efficiency of the process, comparing the optimum results obtained from the two UFG experimental designs. When using coarse media, the optimum P_{80} was 11 μ m while 10 μ m was achieved with fine media. From these results, a 55.26% specific energy savings could be realized, further cementing the conclusion drawn for the second research question.

Table 6-1: Effect of media size on milling efficiency and SE.

Media size	P_{80} (μ m)	SE (kWh/t)
coarse	11	31.4627
fine	10	14.0777
	20	10.635

From this basis, fine media was used in analysing the BMR plant milling circuit potential specific energy conservation. Assumption was that employing one ball mill incorporating the optimised design configurations at 5% uncertainty level would suffice. Table 6.2 details the economic analysis based on the process conditions as adopted. The power unit was obtained from (BPC, 2025), the current rating for large businesses.

Table 6-2: Economic analysis of UFG based on BMR circuit.

Parameter	Unit	Value
Mill feed rate	kg/hr	310
Annual operation time	hrs	8000
Total annual production	tonnes	2480
Electricity unit cost	BWP/kWh	0.81
Traditional milling cost	BWP	28 303.72
Optimised milling costs	BWP	21 382.05
Cost savings annually	BWP	6 921.67
Savings annually	%	24.45

From this analysis, a 24.45% energy saving was achieved, and this translates to a massive reduction in the global warming potential (GWP) of the process as energy production in Botswana is solely coal based as such this reduction translates to reduced coal utilization for this process. Such a saving in industry translates to almost a quarter reduction in operation costs per tonne of ore processed, leading to a probable quarter reduction in harmful emissions to the environment, reduced waste production and ultimately more profit generation. This enhances the effective operation of the plant, and with proper management, these savings can be channeled to other developmental projects in the plant, for the community, as well as charitable acts.

This analysis effectively addressed the importance of optimization of industrial processes to avoid unnecessary expenditure. Since UFG is an energy intensive process, this work significantly justified the adoption of the optimised extent of grinding, from traditional 10 μ m to an optimised 20 μ m for the copper sulphide feed being processed, subject of course to leaching efficiencies at the different grinding levels.

6.3 Limitations

The study was faced with the following limitations.

1. To my knowledge, no previous work had been conducted on using fine media in ball mills, as such the work lacked reference work to use as a base support or critic to this work. Most of the comparison therefore would be done viz with stirred media published data.
2. A 5L laboratory mill was used as such, adoption of this research will require a carefully conducted scale up study to mimic industrial standards. Also, with industrial standards, i.e. mill sizes, the mass of media required would be too much, translating to implicating economic costs, therefore, separation techniques of the media from the feed powder is to be considered to recycle as much of the media.
3. During validation, overtraining of ANN model was observed and this translates to reduced robustness and accuracy of the model. As such, re-training of the models is key prior to any adoption of the models, to improve their accuracy.

6.4 Recommendations

The Activox process can be improved by optimizing the UFG process, therefore the following recommendations from this work were outlined:

1. Characterization of the feed sample is crucial, particularly SEM imaging and sizing of the mineral grains. As such, automated scanning systems e.g. QEMSCAN or development of visual technologies either ML or AI that can predict mineral grain size; coupled with integrated multivariable control systems to tune operational variables to meet the determined optimisation criterion must be developed and put in place.
2. Though a 24.45% specific energy saving was realised, initial energy required to rotate the ball mill is significant to the total energy consumption of the UFG process as such to counter this effect, there is need to explore enhancement of the production rate of the mill, employing techniques like the Attainable Region (AR) to maximize interstitial filling of the mill. This is to capitalise on the production rate thus save on energy consumption of a mill circuit involving multiple stirred mill units.
3. To improve the accuracy of the predictive models developed, the principle of F1 score can be further applied to ascertain the validation of the models, leading to successful adoption of the models. Also, multivariable optimisation algorithms like the multi

objective genetic algorithm (MOGA) can be used to improve the optimisation and predictive accuracy of the models.

4. The process of separating the fine media from the product is intense in water usage, as such, a more conservative way of separation of the two needs development.
5. Comparison of leaching at the traditional 10 μ m and that at optimum 20 μ m for this ore must be conducted, to validate the adoption of this work using the extraction efficiencies realised against the economic savings realised.

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8. Appendix

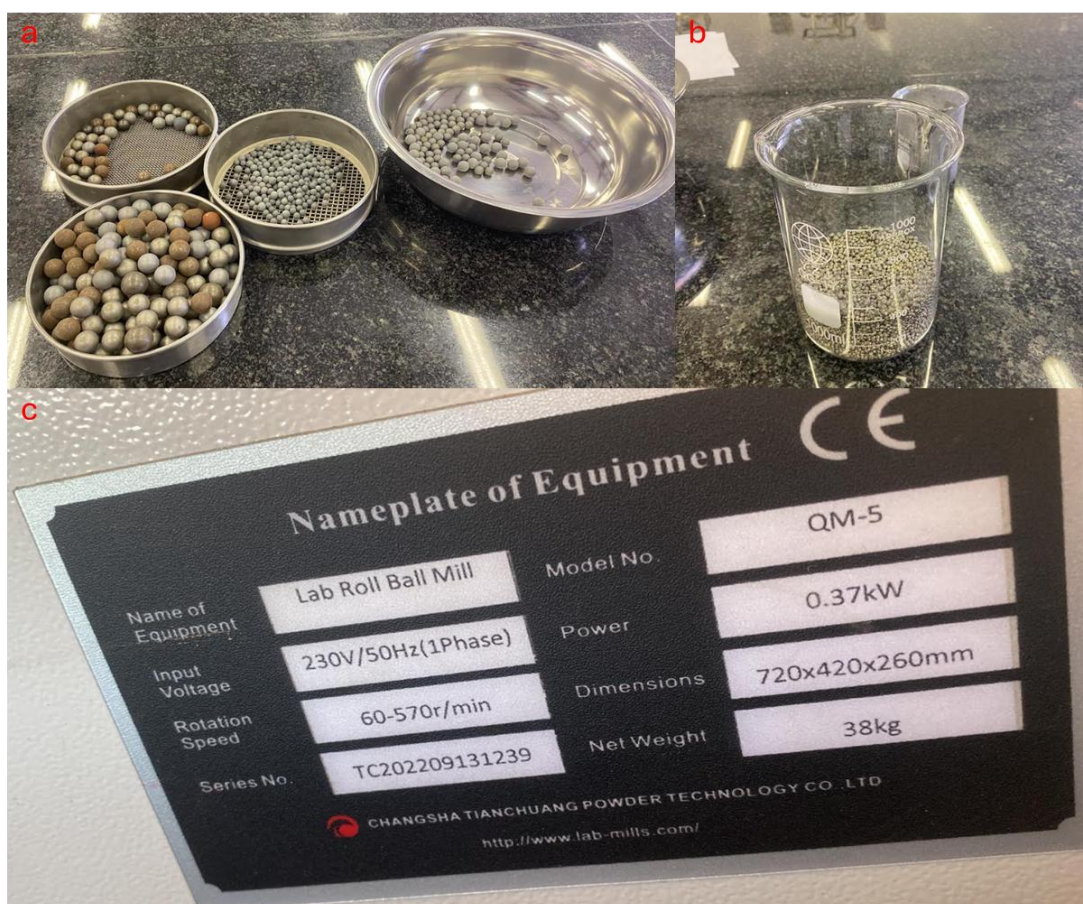
Appendix A: Research team; a- laboratory work, b- team at TNMC, and c- at BMR.



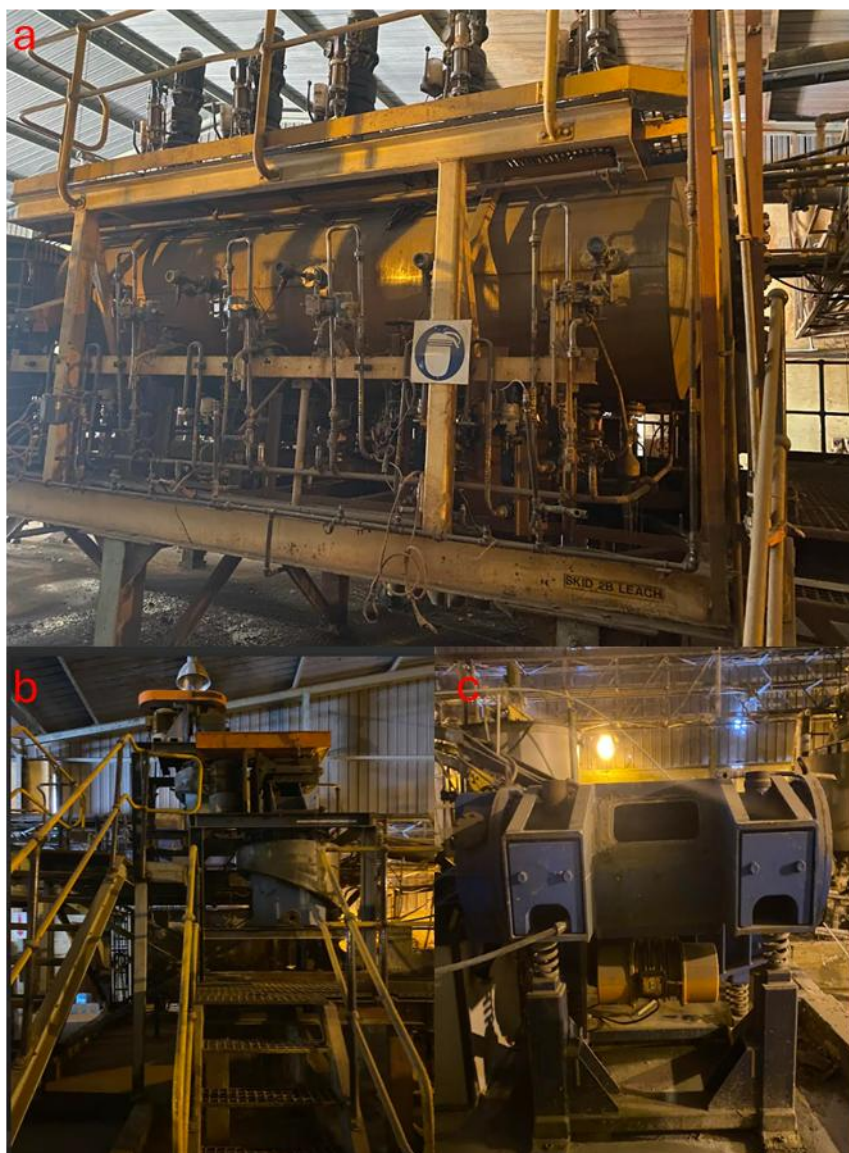
Appendix B: Major equipment used during experimental work; a- milling setup, b&d- fluke data logger wire connections and c- Malvern particle size analyser utilised.



Appendix C: a- coarse grinding media, b- fine ceramic grinding media and c- nameplate of the ball mill used.



Appendix D: BMR Activox equipment; a- leaching autoclave, b- Bradken Metprotech vertical mill and c- Netsch's IsaMill-horizontal.



Appendix E: Average PSD data for DOE #1.

Mesh size- μm	#1	#2	#3	#4	#5	#6	#7	#8	#9	#10	#11	#12	#13	#14	#15	#16	#17
0.2	0.02	0.01	0.01	0.00	0.07	0.01	0.01	0.02	0.02	0.01	0.01	0.01	0.01	0.01	0.01	0.00	0.03
0.3	0.10	0.07	0.07	0.00	0.15	0.07	0.08	0.08	0.11	0.08	0.07	0.08	0.07	0.06	0.08	0.00	0.15
0.3	0.20	0.08	0.13	0.01	0.25	0.15	0.11	0.12	0.15	0.13	0.17	0.15	0.15	0.17	0.16	0.00	0.28
0.4	0.29	0.10	0.19	0.08	0.35	0.23	0.14	0.16	0.20	0.19	0.26	0.22	0.23	0.27	0.24	0.06	0.41
0.4	0.40	0.12	0.26	0.11	0.47	0.33	0.19	0.21	0.25	0.27	0.38	0.30	0.33	0.40	0.34	0.08	0.56
0.5	0.53	0.15	0.34	0.14	0.59	0.44	0.24	0.27	0.31	0.37	0.51	0.40	0.46	0.55	0.45	0.10	0.72
0.6	0.68	0.19	0.45	0.18	0.76	0.57	0.31	0.35	0.39	0.50	0.67	0.52	0.61	0.72	0.59	0.12	0.90
0.6	0.86	0.24	0.59	0.22	0.99	0.74	0.41	0.45	0.50	0.67	0.86	0.68	0.79	0.92	0.76	0.16	1.11
0.7	1.08	0.32	0.77	0.28	1.31	0.94	0.53	0.59	0.66	0.90	1.11	0.88	1.02	1.17	0.99	0.21	1.37
0.8	1.34	0.42	1.01	0.36	1.76	1.19	0.69	0.77	0.86	1.19	1.41	1.14	1.30	1.46	1.27	0.26	1.69
1.0	1.64	0.56	1.30	0.44	2.36	1.49	0.88	0.99	1.11	1.54	1.80	1.47	1.64	1.81	1.64	0.33	2.07
1.1	1.98	0.72	1.65	0.54	3.13	1.85	1.10	1.24	1.40	1.96	2.28	1.86	2.02	2.22	2.06	0.41	2.51
1.3	2.35	0.90	2.04	0.64	4.05	2.25	1.34	1.51	1.73	2.43	2.84	2.30	2.44	2.70	2.56	0.50	3.03
1.4	2.71	1.09	2.44	0.74	5.07	2.69	1.59	1.79	2.08	2.91	3.48	2.76	2.86	3.22	3.08	0.59	3.59
1.7	3.06	1.27	2.83	0.85	6.13	3.14	1.83	2.06	2.43	3.38	4.17	3.22	3.25	3.78	3.61	0.68	4.18
1.9	3.37	1.43	3.17	0.93	7.09	3.58	2.03	2.30	2.74	3.78	4.86	3.64	3.60	4.35	4.11	0.76	4.78

2.2	3.63	1.56	3.44	1.00	7.86	3.99	2.19	2.47	3.00	4.08	5.53	4.00	3.89	4.93	4.56	0.82	5.34
2.5	3.85	1.64	3.62	1.05	8.32	4.38	2.29	2.59	3.18	4.27	6.12	4.28	4.11	5.49	4.93	0.86	5.85
2.9	4.04	1.68	3.73	1.08	8.40	4.72	2.34	2.64	3.29	4.35	6.60	4.48	4.29	6.02	5.22	0.88	6.26
3.3	4.22	1.67	3.80	1.08	8.08	5.04	2.34	2.63	3.32	4.35	6.91	4.62	4.47	6.48	5.44	0.88	6.55
3.8	4.42	1.63	3.86	1.06	7.40	5.32	2.31	2.59	3.29	4.28	7.04	4.74	4.67	6.83	5.61	0.86	6.69
4.4	4.64	1.57	3.95	1.03	6.44	5.55	2.26	2.52	3.22	4.19	6.94	4.84	4.91	7.03	5.72	0.82	6.66
5.0	4.89	1.51	4.08	1.00	5.33	5.71	2.22	2.44	3.11	4.10	6.62	4.93	5.17	7.00	5.77	0.79	6.43
5.8	5.14	1.46	4.27	0.96	4.15	5.77	2.18	2.36	2.99	4.00	6.07	5.01	5.44	6.69	5.74	0.76	6.00
6.6	5.35	1.41	4.50	0.93	3.04	5.68	2.16	2.30	2.86	3.89	5.33	5.05	5.64	6.11	5.59	0.74	5.39
7.6	5.48	1.38	4.74	0.90	2.04	5.42	2.16	2.24	2.72	3.77	4.44	5.03	5.71	5.27	5.31	0.73	4.62
8.7	5.48	1.36	4.95	0.88	1.25	5.01	2.20	2.20	2.57	3.63	3.51	4.92	5.60	4.29	4.89	0.73	3.78
10.0	5.33	1.36	5.10	0.87	0.67	4.47	2.28	2.19	2.43	3.47	2.61	4.73	5.29	3.25	4.36	0.75	2.92
11.5	5.02	1.38	5.13	0.87	0.31	3.86	2.42	2.23	2.32	3.32	1.85	4.45	4.79	2.33	3.77	0.78	2.16
13.2	4.56	1.43	5.02	0.87	0.12	3.24	2.65	2.33	2.23	3.19	1.26	4.09	4.12	1.57	3.15	0.81	1.50
15.1	3.99	1.52	4.75	0.89	0.06	2.66	2.96	2.51	2.18	3.09	0.84	3.65	3.38	1.03	2.55	0.83	0.99
17.4	3.32	1.66	4.30	0.90	0.08	2.16	3.38	2.77	2.16	3.03	0.59	3.15	2.63	0.69	1.99	0.84	0.63
20.0	2.61	1.89	3.73	0.94	0.16	1.76	3.88	3.10	2.19	3.00	0.46	2.61	1.94	0.49	1.48	0.85	0.39
22.9	1.86	2.21	3.07	1.01	0.27	1.46	4.43	3.51	2.24	2.97	0.42	2.05	1.35	0.38	1.03	0.89	0.24
26.3	1.18	2.64	2.39	1.15	0.39	1.23	4.98	3.96	2.33	2.90	0.42	1.52	0.89	0.23	0.65	1.00	0.14
30.2	0.38	3.19	1.75	1.41	0.44	1.05	5.44	4.39	2.44	2.75	0.43	1.01	0.55	0.10	0.26	1.25	0.07

34.7	0.00	3.82	1.19	1.84	0.41	0.86	5.72	4.76	2.57	2.46	0.41	0.62	0.28	0.00	0.05	1.69	0.00
39.8	0.00	4.49	0.75	2.47	0.24	0.64	5.75	4.99	2.71	2.02	0.36	0.38	0.11	0.00	0.00	2.36	0.00
45.7	0.00	5.15	0.42	3.31	0.03	0.36	5.47	5.02	2.85	1.46	0.25	0.21	0.00	0.00	0.00	3.30	0.00
52.5	0.00	5.69	0.18	4.33	0.00	0.04	4.86	4.82	2.98	0.85	0.10	0.09	0.00	0.00	0.00	4.45	0.00
60.3	0.00	6.04	0.02	5.44	0.00	0.00	3.96	4.35	3.07	0.21	0.01	0.01	0.00	0.00	0.00	5.72	0.00
69.2	0.00	6.13	0.00	6.49	0.00	0.00	2.91	3.64	3.08	0.02	0.00	0.00	0.00	0.00	0.00	6.94	0.00
79.4	0.00	5.92	0.00	7.35	0.00	0.00	1.81	2.76	3.00	0.00	0.00	0.00	0.00	0.00	0.00	7.95	0.00
91.2	0.00	5.44	0.00	7.84	0.00	0.00	0.84	1.80	2.80	0.00	0.00	0.00	0.00	0.00	0.00	8.55	0.00
104.7	0.00	4.70	0.00	7.86	0.00	0.00	0.11	0.85	2.48	0.00	0.00	0.00	0.00	0.00	0.00	8.62	0.00
120.2	0.00	3.83	0.00	7.39	0.00	0.00	0.00	0.13	2.06	0.00	0.00	0.00	0.00	0.00	0.00	8.12	0.00
138.0	0.00	2.89	0.00	6.46	0.00	0.00	0.00	0.00	1.56	0.00	0.00	0.00	0.00	0.00	0.00	7.09	0.00
158.5	0.00	2.01	0.00	5.25	0.00	0.00	0.00	0.00	1.06	0.00	0.00	0.00	0.00	0.00	0.00	5.71	0.00
182.0	0.00	1.25	0.00	3.88	0.00	0.00	0.00	0.00	0.58	0.00	0.00	0.00	0.00	0.00	0.00	4.14	0.00
208.9	0.00	0.64	0.00	2.58	0.00	0.00	0.00	0.00	0.20	0.00	0.00	0.00	0.00	0.00	0.00	2.62	0.00
239.9	0.00	0.19	0.00	1.45	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.30	0.00
275.4	0.00	0.01	0.00	0.66	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.30	0.00
316.2	0.00	0.00	0.00	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
363.1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
416.9	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
478.6	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

549.5	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
631.0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
724.4	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
831.8	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
955.0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
1096.5	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Appendix F: Average PSD data for DOE #2.

Mesh size- μm	#1	#2	#3	#4	#5	#6	#7	#8	#9	#10	#11	#12	#13	#14	#15
0.2	0.00	0.02	0.00	0.00	0.00	0.00	0.02	0.00	0.00	0.03	0.01	0.01	0.02	0.04	0.00
0.2	0.03	0.07	0.02	0.02	0.02	0.03	0.09	0.03	0.02	0.11	0.06	0.05	0.08	0.14	0.03
0.3	0.13	0.10	0.11	0.11	0.11	0.14	0.11	0.16	0.12	0.16	0.10	0.12	0.13	0.20	0.15
0.3	0.19	0.13	0.16	0.16	0.16	0.21	0.14	0.23	0.19	0.19	0.17	0.19	0.18	0.25	0.23
0.4	0.24	0.16	0.20	0.20	0.20	0.26	0.15	0.29	0.24	0.22	0.22	0.24	0.22	0.28	0.30
0.4	0.28	0.18	0.24	0.24	0.24	0.32	0.17	0.35	0.29	0.24	0.27	0.30	0.25	0.31	0.37
0.5	0.32	0.19	0.27	0.27	0.27	0.37	0.19	0.41	0.34	0.26	0.32	0.35	0.28	0.34	0.43
0.6	0.36	0.21	0.29	0.29	0.29	0.42	0.21	0.46	0.38	0.28	0.38	0.40	0.30	0.38	0.50
0.6	0.41	0.23	0.33	0.33	0.33	0.49	0.24	0.53	0.42	0.31	0.46	0.48	0.32	0.45	0.57

0.7	0.48	0.26	0.37	0.37	0.37	0.59	0.29	0.63	0.48	0.36	0.57	0.58	0.36	0.56	0.67
0.8	0.57	0.31	0.44	0.44	0.44	0.72	0.37	0.76	0.57	0.45	0.72	0.72	0.43	0.73	0.82
1.0	0.71	0.40	0.54	0.54	0.54	0.90	0.48	0.96	0.69	0.60	0.94	0.92	0.54	0.99	1.02
1.1	0.90	0.52	0.68	0.68	0.68	1.14	0.62	1.23	0.86	0.80	1.24	1.19	0.70	1.34	1.30
1.3	1.15	0.71	0.88	0.88	0.88	1.45	0.81	1.59	1.09	1.09	1.65	1.55	0.95	1.80	1.66
1.4	1.47	0.96	1.14	1.14	1.14	1.82	1.03	2.04	1.40	1.47	2.16	1.99	1.27	2.35	2.12
1.7	1.85	1.28	1.46	1.46	1.46	2.26	1.28	2.57	1.78	1.92	2.79	2.53	1.70	2.98	2.66
1.9	2.28	1.67	1.84	1.84	1.84	2.75	1.56	3.17	2.24	2.45	3.51	3.13	2.21	3.64	3.28
2.2	2.76	2.11	2.27	2.27	2.27	3.27	1.84	3.81	2.76	3.01	4.27	3.77	2.80	4.30	3.96
2.5	3.26	2.58	2.73	2.73	2.73	3.81	2.11	4.46	3.34	3.57	5.03	4.42	3.43	4.90	4.65
2.9	3.77	3.05	3.20	3.20	3.20	4.33	2.36	5.06	3.93	4.09	5.71	5.02	4.06	5.37	5.31
3.3	4.25	3.46	3.65	3.65	3.65	4.82	2.56	5.57	4.51	4.51	6.24	5.53	4.64	5.67	5.90
3.8	4.70	3.80	4.04	4.04	4.04	5.23	2.73	5.94	5.04	4.80	6.54	5.89	5.10	5.78	6.34
4.4	5.08	4.02	4.36	4.36	4.36	5.55	2.84	6.13	5.47	4.94	6.56	6.06	5.42	5.68	6.60
5.0	5.36	4.10	4.57	4.57	4.57	5.74	2.91	6.11	5.78	4.91	6.28	6.01	5.55	5.38	6.64
5.8	5.54	4.07	4.67	4.67	4.67	5.78	2.95	5.87	5.91	4.72	5.72	5.75	5.48	4.92	6.41
6.6	5.58	3.93	4.64	4.64	4.64	5.64	2.98	5.42	5.85	4.42	4.94	5.29	5.23	4.34	5.93
7.6	5.48	3.74	4.49	4.49	4.49	5.32	3.01	4.80	5.59	4.02	4.01	4.67	4.83	3.69	5.22
8.7	5.25	3.54	4.24	4.24	4.24	4.86	3.06	4.09	5.16	3.58	3.07	3.97	4.35	3.04	4.37
10.0	4.88	3.39	3.91	3.91	3.91	4.27	3.14	3.35	4.58	3.16	2.20	3.24	3.83	2.42	3.46

11.5	4.43	3.34	3.56	3.56	3.56	3.64	3.25	2.67	3.93	2.80	1.49	2.57	3.35	1.91	2.61
13.2	3.91	3.38	3.18	3.18	3.18	2.99	3.35	2.08	3.24	2.50	0.96	1.98	2.92	1.49	1.87
15.1	8.53	4.17	2.79	2.79	2.79	2.39	3.41	1.62	2.56	2.29	0.61	1.50	2.55	1.19	1.29
17.4	7.06	3.96	2.66	2.40	2.40	1.82	7.96	1.26	1.93	2.14	0.38	1.10	2.25	0.98	0.87
20.0	5.63	4.07	2.47	1.99	1.99	1.33	3.55	0.96	1.35	2.03	0.24	0.79	1.98	1.12	0.56
22.9	4.28	4.10	1.58	1.68	2.55	0.89	4.64	0.70	0.85	3.18	0.15	0.52	1.71	2.44	0.30
26.3	3.03	3.89	4.83	1.44	2.36	0.52	6.17	0.10	0.39	3.30	1.61	2.15	1.43	2.45	0.07
30.2	1.91	4.35	0.92	0.88	1.86	0.18	2.95	0.59	0.06	2.62	2.32	2.41	1.90	2.33	0.10
34.7	0.94	3.49	3.43	0.63	1.28	0.28	5.86	1.26	1.53	2.76	2.86	2.44	2.62	2.24	0.54
39.8	0.20	3.57	1.85	0.61	0.98	0.81	1.98	1.88	2.07	2.21	3.12	0.41	2.74	2.06	1.54
45.7	0.00	3.63	1.22	0.63	0.14	1.34	3.35	2.34	2.46	3.32	3.03	0.67	2.70	1.87	2.74
52.5	0.00	2.71	3.64	0.35	0.50	1.75	1.59	2.48	2.60	1.72	2.60	0.88	2.50	1.69	4.34
60.3	0.00	2.38	1.03	1.21	1.47	1.97	3.83	2.27	2.46	1.96	1.94	2.23	2.19	1.55	2.20
69.2	0.08	1.08	2.34	1.41	1.68	1.95	1.56	1.75	2.05	1.83	1.14	1.84	1.89	1.19	0.05
79.4	0.92	1.60	3.71	1.78	3.04	1.75	2.58	1.05	0.35	1.88	0.87	1.11	0.92	1.41	0.00
91.2	1.48	1.10	5.07	2.48	3.29	1.43	3.74	0.57	0.93	1.88	0.07	1.68	1.40	1.61	0.02
104.7	0.00	0.00	0.00	0.47	1.35	1.07	0.02	0.15	1.43	0.81	0.22	1.34	0.07	0.05	0.01
120.2	0.00	0.00	0.00	0.58	0.19	0.73	0.10	0.10	0.76	0.04	0.10	0.80	0.03	0.03	0.01
138.0	0.00	0.00	0.00	0.00	0.00	0.45	0.00	0.03	0.00	0.02	0.02	0.53	0.07	0.06	0.00
158.5	0.00	0.00	0.00	0.00	0.00	0.19	0.00	0.10	0.00	0.01	0.01	0.14	0.18	0.02	0.00

182.0	0.00	0.00	0.00	0.00	0.00	0.02	0.00	0.00	0.01	0.04	0.00	0.30	0.00	0.00	0.00
208.9	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.03	0.02	0.14	0.00	0.00	0.00
239.9	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.03	0.00	0.14	0.00	0.00	0.00
275.4	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.21	0.00	0.00	0.00
316.2	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.04	0.00	0.02	0.02	0.02	0.00	0.00
363.1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
416.9	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
478.6	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
549.5	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
631.0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
724.4	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
831.8	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
955.0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Appendix G: Machine learning data used i.e. normalised data

P ₈₀ norm predictors and responses					SE norm predictors and responses					P ₈₀ anfis normalised data					SEanfis normalised data			
speed	time	media ratio	P ₈₀		speed	time	media ratio	SE		speed	time	media ratio	P ₈₀		speed	time	media ratio	SE
0.790	0.600	0.300	0.391		0.606	0.600	0.575	0.267		0.790	0.600	0.300	0.391		0.606	0.600	0.575	0.267
0.920	0.800	0.400	0.522		0.828	0.600	0.350	0.344		0.920	0.800	0.400	0.522		0.828	0.600	0.350	0.344

0.53 0	0.00 0	0.300	0.95 7		0.82 8	0.90 0	0.550	0.49 9		0.53 0	0.00 0	0.300	0.95 7		0.82 8	0.90 0	0.550	0.49 9
0.81 0	0.90 0	0.775	0.69 6		0.09 1	0.40 0	0.300	0.53 1		0.81 0	0.90 0	0.775	0.69 6		0.09 1	0.40 0	0.300	0.53 1
0.56 0	0.40 0	0.350	0.43 5		0.73 7	0.80 0	0.150	0.72 7		0.56 0	0.40 0	0.350	0.43 5		0.73 7	0.80 0	0.150	0.72 7
0.75 0	0.40 0	0.125	0.43 5		0.91 9	0.10 0	0.675	0.03 2		0.75 0	0.40 0	0.125	0.43 5		0.91 9	0.10 0	0.675	0.03 2
0.00 0	0.30 0	0.625	0.43 5		0.45 5	0.00 0	0.700	0.30 7		0.00 0	0.30 0	0.625	0.43 5		0.45 5	0.00 0	0.700	0.30 7
0.48 0	0.60 0	0.350	0.26 1		0.07 1	0.30 0	0.800	0.47 8		0.48 0	0.60 0	0.350	0.26 1		0.07 1	0.30 0	0.800	0.47 8
0.25 0	0.30 0	0.350	0.52 2		0.24 2	0.80 0	0.350	0.66 0		0.25 0	0.30 0	0.350	0.52 2		0.24 2	0.80 0	0.350	0.66 0
0.31 0	0.40 0	0.050	0.34 8		0.73 7	0.40 0	0.250	0.30 6		0.31 0	0.40 0	0.050	0.34 8		0.73 7	0.40 0	0.250	0.30 6
0.97 0	0.30 0	0.875	0.78 3		0.04 0	1.00 0	0.350	0.94 5		0.97 0	0.30 0	0.875	0.78 3		0.04 0	1.00 0	0.350	0.94 5
0.21 0	0.60 0	0.450	0.17 4		0.42 4	0.60 0	0.325	0.45 6		0.21 0	0.60 0	0.450	0.17 4		0.42 4	0.60 0	0.325	0.45 6
0.52 0	0.70 0	0.075	0.08 7		0.54 5	0.30 0	0.950	0.18 4		0.52 0	0.70 0	0.075	0.08 7		0.54 5	0.30 0	0.950	0.18 4
0.22 0	1.00 0	0.925	0.21 7		0.97 0	1.00 0	0.750	0.49 0		0.22 0	1.00 0	0.925	0.21 7		0.97 0	1.00 0	0.750	0.49 0
0.57 0	1.00 0	0.275	0.21 7		0.21 2	0.90 0	0.275	0.82 5		0.57 0	1.00 0	0.275	0.21 7		0.21 2	0.90 0	0.275	0.82 5
1.00 0	0.30 0	0.625	0.78 3		0.11 1	0.90 0	0.600	0.67 6		1.00 0	0.30 0	0.625	0.78 3		0.11 1	0.90 0	0.600	0.67 6
0.13 0	0.20 0	0.675	0.52 2		0.65 7	0.20 0	0.775	0.12 3		0.13 0	0.20 0	0.675	0.52 2		0.65 7	0.20 0	0.775	0.12 3
0.96 0	1.00 0	0.200	0.43 5		0.10 1	0.30 0	0.850	0.46 5		0.96 0	1.00 0	0.200	0.43 5		0.10 1	0.30 0	0.850	0.46 5
0.12 0	0.30 0	0.975	0.26 1		1.00 0	0.80 0	0.900	0.20 6		0.12 0	0.30 0	0.975	0.26 1		1.00 0	0.80 0	0.900	0.20 6

0.18 0	0.60 0	0.675	0.17 4		0.25 3	0.70 0	0.600	0.45 8		0.18 0	0.60 0	0.675	0.17 4		0.25 3	0.70 0	0.600	0.45 8
0.65 0	1.00 0	0.550	0.43 5		0.61 6	0.50 0	0.050	0.55 4		0.65 0	1.00 0	0.550	0.43 5		0.61 6	0.50 0	0.050	0.55 4
0.12 0	0.00 0	0.875	0.69 6		0.82 8	1.00 0	0.200	0.94 8		0.12 0	0.00 0	0.875	0.69 6		0.82 8	1.00 0	0.200	0.94 8
0.08 0	0.20 0	0.550	0.60 9		0.83 8	0.50 0	0.875	0.06 7		0.08 0	0.20 0	0.550	0.60 9		0.83 8	0.50 0	0.875	0.06 7
0.66 0	0.40 0	0.000	0.34 8		0.49 5	1.00 0	0.400	0.77 9		0.66 0	0.40 0	0.000	0.34 8		0.49 5	1.00 0	0.400	0.77 9
0.02 0	0.60 0	0.475	0.17 4		0.76 8	0.50 0	0.675	0.13 0		0.02 0	0.60 0	0.475	0.17 4		0.76 8	0.50 0	0.675	0.13 0
0.99 0	0.90 0	0.825	0.95 7		0.92 9	0.50 0	0.800	0.05 1		0.99 0	0.90 0	0.825	0.95 7		0.92 9	0.50 0	0.800	0.05 1
0.54 0	0.20 0	0.750	0.56 5		0.90 9	0.80 0	0.250	0.62 4		0.54 0	0.20 0	0.750	0.56 5		0.90 9	0.80 0	0.250	0.62 4
0.37 0	1.00 0	0.575	0.21 7		0.21 2	0.40 0	0.550	0.38 1		0.37 0	1.00 0	0.575	0.21 7		0.21 2	0.40 0	0.550	0.38 1
0.71 0	0.40 0	0.000	0.34 8		0.55 6	0.40 0	0.225	0.37 1		0.71 0	0.40 0	0.000	0.34 8		0.55 6	0.40 0	0.225	0.37 1
0.95 0	0.80 0	0.725	0.78 3		0.79 8	0.10 0	0.000	0.39 6		0.95 0	0.80 0	0.725	0.78 3		0.79 8	0.10 0	0.000	0.39 6
0.38 0	0.50 0	0.450	0.30 4		0.19 2	0.80 0	0.775	0.51 0		0.38 0	0.50 0	0.450	0.30 4		0.19 2	0.80 0	0.775	0.51 0
0.69 0	0.30 0	0.675	0.56 5		0.75 8	0.60 0	0.000	0.67 1		0.69 0	0.30 0	0.675	0.56 5		0.75 8	0.60 0	0.000	0.67 1
0.60 0	0.40 0	0.750	0.47 8		0.48 5	0.70 0	0.400	0.46 4		0.60 0	0.40 0	0.750	0.47 8		0.48 5	0.70 0	0.400	0.46 4
0.78 0	0.70 0	0.800	0.56 5		0.58 6	0.80 0	0.250	0.64 8		0.78 0	0.70 0	0.800	0.56 5		0.58 6	0.80 0	0.250	0.64 8
0.59 0	1.00 0	0.025	0.00 0		0.26 3	0.20 0	0.200	0.46 4		0.59 0	1.00 0	0.025	0.00 0		0.26 3	0.20 0	0.200	0.46 4
0.37 0	0.90 0	0.775	0.26 1		0.08 1	0.90 0	0.675	0.66 6		0.37 0	0.90 0	0.775	0.26 1		0.08 1	0.90 0	0.675	0.66 6

0.85 0	0.10 0	0.575	0.82 6		0.30 3	0.90 0	0.925	0.50 8		0.85 0	0.10 0	0.575	0.82 6		0.30 3	0.90 0	0.925	0.50 8
0.22 0	0.10 0	0.075	0.91 3		0.92 9	0.30 0	0.000	0.44 0		0.22 0	0.10 0	0.075	0.91 3		0.92 9	0.30 0	0.000	0.44 0
0.80 0	0.90 0	0.250	0.30 4		0.47 5	0.70 0	0.750	0.31 9		0.80 0	0.90 0	0.250	0.30 4		0.47 5	0.70 0	0.750	0.31 9
1.00 0	0.70 0	0.125	0.34 8		0.27 3	0.30 0	0.800	0.33 3		1.00 0	0.70 0	0.125	0.34 8		0.27 3	0.30 0	0.800	0.33 3
0.28 0	0.00 0	0.575	0.87 0		0.77 8	0.20 0	0.925	0.07 3		0.28 0	0.00 0	0.575	0.87 0		0.77 8	0.20 0	0.925	0.07 3
0.71 0	0.50 0	0.550	0.47 8		0.77 8	0.10 0	0.975	0.11 4		0.71 0	0.50 0	0.550	0.47 8		0.77 8	0.10 0	0.975	0.11 4
0.67 0	0.90 0	0.050	0.08 7		0.02 0	0.70 0	0.100	0.84 2		0.67 0	0.90 0	0.050	0.08 7		0.02 0	0.70 0	0.100	0.84 2
0.41 0	0.40 0	1.000	0.30 4		0.88 9	0.50 0	0.525	0.15 9		0.41 0	0.40 0	1.000	0.30 4		0.88 9	0.50 0	0.525	0.15 9
0.50 0	0.30 0	0.250	0.52 2		0.80 8	0.90 0	0.825	0.36 0		0.50 0	0.30 0	0.250	0.52 2		0.80 8	0.90 0	0.825	0.36 0
0.95 0	0.20 0	0.300	0.73 9		0.32 3	0.20 0	0.650	0.31 3		0.95 0	0.20 0	0.300	0.73 9		0.32 3	0.20 0	0.650	0.31 3
0.96 0	0.20 0	0.300	0.73 9		0.67 7	0.10 0	0.975	0.17 1		0.96 0	0.20 0	0.300	0.73 9		0.67 7	0.10 0	0.975	0.17 1
0.74 0	0.40 0	0.025	0.39 1		0.29 3	0.40 0	0.950	0.33 2		0.74 0	0.40 0	0.025	0.39 1		0.29 3	0.40 0	0.950	0.33 2
0.38 0	0.60 0	0.525	0.26 1		0.93 9	0.50 0	0.975	0.02 7		0.38 0	0.60 0	0.525	0.26 1		0.93 9	0.50 0	0.975	0.02 7
0.04 0	0.20 0	0.250	0.73 9		0.28 3	0.80 0	0.050	0.89 4		0.04 0	0.20 0	0.250	0.73 9		0.28 3	0.80 0	0.050	0.89 4
0.58 0	0.60 0	0.400	0.30 4		0.17 2	0.10 0	0.425	0.46 0		0.58 0	0.60 0	0.400	0.30 4		0.17 2	0.10 0	0.425	0.46 0
0.57 0	0.80 0	0.950	0.43 5		0.75 8	0.00 0	0.325	0.21 0		0.57 0	0.80 0	0.950	0.43 5		0.75 8	0.00 0	0.325	0.21 0
0.35 0	0.10 0	0.925	0.56 5		0.48 5	0.60 0	0.125	0.58 8		0.35 0	0.10 0	0.925	0.56 5		0.48 5	0.60 0	0.125	0.58 8

0.88 0	0.80 0	0.100	0.26 1		0.65 7	0.00 0	0.125	0.34 9		0.88 0	0.80 0	0.100	0.26 1		0.65 7	0.00 0	0.125	0.34 9
0.63 0	0.70 0	0.600	0.39 1		0.98 0	0.10 0	0.825	0.00 0		0.63 0	0.70 0	0.600	0.39 1		0.98 0	0.10 0	0.825	0.00 0
0.63 0	0.50 0	0.350	0.39 1		0.31 3	0.70 0	0.525	0.45 9		0.63 0	0.50 0	0.350	0.39 1		0.31 3	0.70 0	0.525	0.45 9
0.29 0	1.00 0	0.725	0.21 7		0.07 1	0.50 0	0.950	0.48 8		0.29 0	1.00 0	0.725	0.21 7		0.07 1	0.50 0	0.950	0.48 8
0.07 0	0.10 0	0.525	0.78 3		0.79 8	0.50 0	1.000	0.07 8		0.07 0	0.10 0	0.525	0.78 3		0.79 8	0.50 0	1.000	0.07 8
0.29 0	0.60 0	0.250	0.17 4		0.36 4	0.80 0	0.400	0.58 7		0.29 0	0.60 0	0.250	0.17 4		0.36 4	0.80 0	0.400	0.58 7
0.23 0	0.20 0	0.500	0.60 9		0.59 6	0.80 0	0.375	0.54 5		0.23 0	0.20 0	0.500	0.60 9		0.59 6	0.80 0	0.375	0.54 5
0.34 0	0.00 0	0.875	0.69 6		0.18 2	0.50 0	0.225	0.55 7		0.34 0	0.00 0	0.875	0.69 6		0.18 2	0.50 0	0.225	0.55 7
0.85 0	0.60 0	0.725	0.60 9		0.07 1	0.30 0	0.450	0.48 5		0.85 0	0.60 0	0.725	0.60 9		0.07 1	0.30 0	0.450	0.48 5
0.16 0	0.90 0	0.200	0.00 0		0.15 2	0.10 0	0.250	0.51 8		0.16 0	0.90 0	0.200	0.00 0		0.15 2	0.10 0	0.250	0.51 8
0.15 0	0.60 0	0.150	0.17 4		0.83 8	0.90 0	0.450	0.57 2		0.15 0	0.60 0	0.150	0.17 4		0.83 8	0.90 0	0.450	0.57 2
0.51 0	0.90 0	0.375	0.21 7		0.30 3	1.00 0	0.425	0.80 5		0.51 0	0.90 0	0.375	0.21 7		0.30 3	1.00 0	0.425	0.80 5
0.60 0	0.20 0	0.875	0.56 5		0.38 4	1.00 0	0.250	0.92 6		0.60 0	0.20 0	0.875	0.56 5		0.38 4	1.00 0	0.250	0.92 6
0.16 0	0.60 0	0.700	0.17 4		0.65 7	0.70 0	0.125	0.64 8		0.16 0	0.60 0	0.700	0.17 4		0.65 7	0.70 0	0.125	0.64 8
0.64 0	0.10 0	0.650	0.73 9		0.82 8	0.00 0	0.425	0.14 7		0.64 0	0.10 0	0.650	0.73 9		0.82 8	0.00 0	0.425	0.14 7
0.85 0	0.30 0	0.125	0.56 5		0.77 8	0.00 0	0.500	0.14 6		0.85 0	0.30 0	0.125	0.56 5		0.77 8	0.00 0	0.500	0.14 6
0.79 0	0.00 0	0.075	0.95 7		0.37 4	0.20 0	0.325	0.35 2		0.79 0	0.00 0	0.075	0.95 7		0.37 4	0.20 0	0.325	0.35 2

0.26 0	0.70 0	0.875	0.17 4		0.19 2	0.00 0	0.700	0.49 5		0.26 0	0.70 0	0.875	0.17 4		0.19 2	0.00 0	0.700	0.49 5
0.31 0	1.00 0	0.425	0.13 0		0.65 7	0.70 0	0.450	0.38 9		0.31 0	1.00 0	0.425	0.13 0		0.65 7	0.70 0	0.450	0.38 9
0.18 0	0.80 0	0.475	0.08 7		0.00 0	0.10 0	0.425	0.58 2		0.18 0	0.80 0	0.475	0.08 7		0.00 0	0.10 0	0.425	0.58 2
0.45 0	0.80 0	0.450	0.21 7		0.28 3	0.60 0	0.800	0.35 6		0.45 0	0.80 0	0.450	0.21 7		0.28 3	0.60 0	0.800	0.35 6
0.32 0	0.50 0	0.525	0.30 4		0.64 6	0.80 0	0.825	0.30 9		0.32 0	0.50 0	0.525	0.30 4		0.64 6	0.80 0	0.825	0.30 9
0.28 0	0.30 0	0.275	0.52 2		0.59 6	0.60 0	0.750	0.21 5		0.28 0	0.30 0	0.275	0.52 2		0.59 6	0.60 0	0.750	0.21 5
0.94 0	0.20 0	0.225	0.73 9		0.32 3	0.40 0	0.800	0.29 4		0.94 0	0.20 0	0.225	0.73 9		0.32 3	0.40 0	0.800	0.29 4
0.40 0	0.30 0	0.900	0.39 1		1.00 0	0.90 0	0.500	0.52 8		0.40 0	0.30 0	0.900	0.39 1		1.00 0	0.90 0	0.500	0.52 8
0.38 0	1.00 0	0.925	0.34 8		0.12 1	0.30 0	0.550	0.43 5		0.38 0	1.00 0	0.925	0.34 8		0.12 1	0.30 0	0.550	0.43 5
0.59 0	0.20 0	0.600	0.60 9		0.74 7	0.30 0	0.625	0.09 7		0.59 0	0.20 0	0.600	0.60 9		0.74 7	0.30 0	0.625	0.09 7
0.06 0	0.50 0	0.350	0.26 1		0.15 2	0.80 0	0.100	0.88 4		0.06 0	0.50 0	0.350	0.26 1		0.15 2	0.80 0	0.100	0.88 4
0.20 0	0.20 0	0.600	0.56 5		0.43 4	0.50 0	0.250	0.44 1		0.20 0	0.20 0	0.600	0.56 5		0.43 4	0.50 0	0.250	0.44 1
0.73 0	0.40 0	0.675	0.52 2		0.84 8	0.20 0	0.625	0.05 6		0.73 0	0.40 0	0.675	0.52 2		0.84 8	0.20 0	0.625	0.05 6
0.58 0	0.10 0	0.900	0.65 2		0.36 4	0.80 0	0.300	0.65 6		0.58 0	0.10 0	0.900	0.65 2		0.36 4	0.80 0	0.300	0.65 6
0.20 0	0.40 0	0.075	0.39 1		0.07 1	0.10 0	0.775	0.55 1		0.20 0	0.40 0	0.075	0.39 1		0.07 1	0.10 0	0.775	0.55 1
0.85 0	0.80 0	0.550	0.56 5		0.56 6	1.00 0	0.250	0.90 1		0.85 0	0.80 0	0.550	0.56 5		0.56 6	1.00 0	0.250	0.90 1
0.42 0	0.80 0	0.425	0.17 4		0.27 3	0.20 0	0.025	0.56 3		0.42 0	0.80 0	0.425	0.17 4		0.27 3	0.20 0	0.025	0.56 3

0.55 0	0.90 0	0.625	0.34 8		0.13 1	0.00 0	0.300	0.55 0		0.55 0	0.90 0	0.625	0.34 8		0.13 1	0.00 0	0.300	0.55 0
0.53 0	0.50 0	0.550	0.34 8		0.70 7	0.00 0	0.550	0.16 8		0.53 0	0.50 0	0.550	0.34 8		0.70 7	0.00 0	0.550	0.16 8
0.18 0	0.80 0	0.225	0.04 3		0.49 5	0.80 0	0.975	0.34 4		0.18 0	0.80 0	0.225	0.04 3		0.49 5	0.80 0	0.975	0.34 4
0.08 0	0.70 0	0.100	0.08 7		0.18 2	0.80 0	0.700	0.53 0		0.08 0	0.70 0	0.100	0.08 7		0.18 2	0.80 0	0.700	0.53 0
0.46 0	0.00 0	0.000	1.00 0		0.10 1	0.80 0	0.725	0.56 7		0.46 0	0.00 0	0.000	1.00 0		0.10 1	0.80 0	0.725	0.56 7
0.03 0	0.10 0	0.050	1.00 0		0.20 2	0.80 0	0.550	0.57 0		0.03 0	0.10 0	0.050	1.00 0		0.20 2	0.80 0	0.550	0.57 0
0.43 0	0.20 0	0.325	0.65 2		0.13 1	1.00 0	0.525	0.80 3		0.43 0	0.20 0	0.325	0.65 2		0.13 1	1.00 0	0.525	0.80 3
0.56 0	0.20 0	0.775	0.56 5		0.32 3	0.50 0	0.875	0.30 7		0.56 0	0.20 0	0.775	0.56 5		0.32 3	0.50 0	0.875	0.30 7
0.64 0	0.10 0	0.325	0.78 3		0.97 0	0.80 0	0.400	0.48 8		0.64 0	0.10 0	0.325	0.78 3		0.97 0	0.80 0	0.400	0.48 8
0.03 0	0.20 0	0.625	0.56 5		0.53 5	0.40 0	0.450	0.25 6		0.03 0	0.20 0	0.625	0.56 5		0.53 5	0.40 0	0.450	0.25 6
0.71 0	0.30 0	1.000	0.56 5		0.25 3	1.00 0	0.200	1.00 0		0.71 0	0.30 0	1.000	0.56 5		0.25 3	1.00 0	0.200	1.00 0
0.17 0	1.00 0	0.650	0.13 0		0.44 4	0.00 0	0.775	0.32 4		0.17 0	1.00 0	0.650	0.13 0		0.44 4	0.00 0	0.775	0.32 4
0.59 0	0.30 0	0.550	0.52 2		0.67 7	1.00 0	0.550	0.64 1		0.59 0	0.30 0	0.550	0.52 2		0.67 7	1.00 0	0.550	0.64 1

Appendix H: Combined Post-Hoc Analysis and regression data

Speed	Time	Media	Experimental		m1	m2	m3	m4	m5	m6	m7	m8	m9	m10	m11	m12	m13	m14	m15	m16
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	e																			
0.7 9	0. 6	0.3	0.3913 04348		0.39 8932	0.39 7195	0.39 0548	0.37 7825	0.65 6259	0.44 0886	0.38 3601	0.38 7325	0.43 1272	0.43 0277	0.40 3654	0.39 8153	0.37 8533	0.41 0941	0.39 6197	0.44 8086
0.9 2	0. 8	0.4	0.5217 3913		0.53 4401	0.53 3072	0.52 9706	0.51 8494	0.53 9609	0.61 0236	0.56 859	0.56 8803	0.51 8893	0.51 9639	0.54 8583	0.54 7809	0.49 2423	0.51 9418	0.53 5701	0.49 6164
0.8 1	0. 9	0.7 75	0.6956 52174		0.64 2893	0.64 2438	0.69 6456	0.66 7001	0.76 8984	0.68 7561	0.66 6453	0.66 6299	0.75 2782	0.75 233	0.65 8005	0.65 5376	0.78 1132	0.66 8233	0.68 6489	0.44 3521
0.7 5	0. 4	0.1 25	0.4347 82609		0.41 1954	0.40 972	0.40 7958	0.40 172	0.36 1645	0.35 7011	0.38 1963	0.38 4998	0.40 2875	0.40 0735	0.38 4818	0.37 6384	0.40 184	0.48 484	0.39 6125	0.51 7121
0	0. 3	0.6 25	0.4347 82609		0.45 7141	0.45 5151	0.43 351	0.41 7896	0.53 3617	0.37 6671	0.42 033	0.42 3236	0.41 6953	0.41 3392	0.40 8691	0.40 2291	0.40 5442	0.40 2442	0.41 4174	0.44 3311
0.4 8	0. 6	0.3 5	0.2608 69565		0.24 1284	0.23 821	0.23 4916	0.23 5135	0.30 1119	0.26 7591	0.27 6978	0.28 084	0.27 7995	0.27 6727	0.25 6729	0.24 9932	0.26 2045	0.29 372	0.25 8888	0.27 6073
0.3 1	0. 4	0.0 5	0.3478 26087		0.35 8975	0.35 6088	0.38 1348	0.38 5004	0.38 466	0.29 6904	0.36 5475	0.36 7642	0.37 9127	0.37 482	0.36 2231	0.35 4009	0.38 2661	0.33 0409	0.37 3396	0.32 4523
0.2 1	0. 6	0.4 5	0.1739 13043		0.18 7141	0.18 4762	0.21 1643	0.20 0098	0.13 4052	0.20 1209	0.24 0844	0.24 5195	0.24 8513	0.24 6959	0.19 7666	0.19 1626	0.25 0712	0.20 1507	0.19 8586	0.17 9506
0.5 7	1	0.2 75	0.2173 91304		0.03 4059	0.03 2847	0.13 5025	0.14 1859	0.00 4135	0.19 6045	0.09 9638	0.09 9996	- 0.02 562	- 0.02 466	0.13 429	0.13 2826	- 0.01 589	0.17 8101	0.14 0079	0.24 1578
0.6 5	1	0.5 5	0.4347 82609		0.39 3731	0.39 2704	0.38 0402	0.38 03	0.30 0061	0.37 2467	0.38 5628	0.38 5307	0.44 2613	0.44 2459	0.38 8225	0.38 6799	0.42 1404	0.40 2824	0.38 0353	0.27 3592
0.1 2	0	0.8 75	0.6956 52174		0.53 3159	0.53 0635	0.66 7314	0.67 47	0.95 7152	0.13 408	0.60 9884	0.61 2444	0.80 5265	0.79 4996	0.65 565	0.64 4376	0.85 7025	0.69 8334	0.65 7363	0.72 4058
0.0 8	0. 2	0.5 5	0.6086 95652		0.60 6393	0.60 3036	0.61 4791	0.60 336	0.60 0215	0.70 709	0.59 8852	0.60 2592	0.59 3548	0.58 6617	0.59 597	0.58 8704	0.59 8143	0.60 9011	0.59 8728	0.61 3827
0.0 2	0. 6	0.4 75	0.1739 13043		0.09 1484	0.09 0881	0.17 4114	0.16 1537	0.13 0689	0.20 375	0.23 3696	0.23 8071	0.19 1509	0.18 9625	0.17 5838	0.17 0207	0.20 1879	0.12 8674	0.17 1217	0.16 2362
0.3 7	1	0.5 75	0.2173 91304		0.25 5944	0.25 448	0.19 9807	0.21 913	0.01 7318	0.21 7321	0.21 2547	0.21 238	0.18 3659	0.18 3575	0.22 1854	0.21 9586	0.19 8665	0.21 7015	0.21 7124	0.17 3779
0.3 8	0. 5	0.4 5	0.3043 47826		0.33 9419	0.33 4763	0.28 3693	0.31 55	0.32 7225	0.31 1406	0.32 7791	0.33 1986	0.32 5931	0.32 4384	0.30 6299	0.29 8732	0.32 5073	0.31 3664	0.31 1507	0.26 4739
0.6 9	0. 3	0.6 75	0.5652 17391		0.55 3341	0.55 0183	0.59 1922	0.57 2984	0.56 7769	0.52 6331	0.57 363	0.57 6178	0.54 7019	0.54 4885	0.56 8014	0.56 2766	0.54 9668	0.57 6675	0.56 5086	0.60 6502

0.6	0.4	0.7	0.4782 6087		0.41 8189	0.41 4651	0.43 7814	0.43 0384	0.48 5028	0.36 4073	0.42 8614	0.43 2157	0.39 292	0.39 0587	0.43 5144	0.42 8889	0.39 1935	0.48 2601	0.42 8914	0.44 1994
0.8	0.1	0.5	0.8260 86957		0.87 4497	0.87 1346	0.76 2182	0.77 7169	0.86 7129	0.70 6962	0.76 924	0.76 7159	0.79 6958	0.79 2904	0.78 3619	0.78 1277	0.80 2935	0.78 3457	0.79 5743	0.85 7189
0.2	0.1	0.0	0.9130 43478		0.92 7153	0.92 4091	0.91 7636	0.93 0514	0.91 0988	0.90 9153	0.97 4765	0.98 023	0.96 8075	0.95 1127	0.96 1525	0.95 3026	1.03 121	0.93 6054	0.93 7675	0.73 3564
0.5	0.3	0.2	0.5217 3913		0.52 0017	0.51 6902	0.51 7477	0.50 6536	0.61 6302	0.51 5312	0.48 1902	0.48 4198	0.51 3592	0.51 0509	0.49 9556	0.49 2289	0.49 2804	0.54 4355	0.50 7614	0.53 0779
0.9	0.2	0.3	0.7391 30435		0.74 4573	0.74 2181	0.74 3665	0.74 0492	0.74 2446	0.73 9627	0.74 1091	0.74 4245	0.74 4561	0.74 1852	0.73 713	0.73 1787	0.74 6344	0.78 0903	0.74 0962	0.80 5119
0.7	0.4	0.0	0.3913 04348		0.36 1712	0.35 9476	0.35 8471	0.35 1984	0.35 3307	0.34 9178	0.36 1061	0.36 3979	0.36 046	0.35 8179	0.35 8637	0.34 9856	0.36 1365	0.46 967	0.35 8804	0.51 2319
0.5	0.6	0.4	0.3043 47826		0.31 9137	0.31 5961	0.30 8401	0.30 1055	0.31 6903	0.30 8542	0.33 0833	0.33 5036	0.35 666	0.35 6031	0.32 3463	0.31 6985	0.34 0532	0.34 311	0.31 6236	0.32 9628
0.2	0.2	0.5	0.6086 95652		0.61 8833	0.61 5115	0.64 5458	0.63 5684	0.61 1769	0.73 1433	0.65 718	0.66 1482	0.60 7251	0.60 1475	0.62 5559	0.61 8466	0.61 8358	0.63 9821	0.62 1796	0.62 482
0.5	0.9	0.3	0.2173 91304		0.11 804	0.11 6417	0.18 0004	0.17 8683	0.13 3433	0.19 8919	0.21 7907	0.21 7752	0.15 4179	0.15 5513	0.19 0554	0.18 8367	0.11 2161	0.21 6779	0.19 2679	0.27 4801
0.8	0.3	0.1	0.5652 17391		0.52 8831	0.52 6769	0.57 8601	0.55 3062	0.31 0283	0.42 4099	0.53 1999	0.53 7983	0.53 459	0.53 2743	0.53 4919	0.52 2475	0.53 9251	0.63 8437	0.54 3711	0.66 6659
0.7	0	0.0	0.9565 21739		0.76 4425	0.76 3865	1.05 2104	1.07 3336	0.49 9465	0.71 4205	1.02 2611	1.03 6008	0.73 141	0.72 6749	1.06 6387	1.04 1749	0.68 6559	1.03 1506	1.05 6052	0.88 1403
0.9	0.2	0.2	0.7391 30435		0.70 0654	0.69 8435	0.74 3847	0.73 1535	0.40 6647	0.35 8402	0.72 2563	0.72 994	0.69 9228	0.69 6104	0.72 8121	0.71 5153	0.70 5744	0.79 3495	0.73 0029	0.80 1404
0.5	0.2	0.6	0.6086 95652		0.61 4273	0.61 0414	0.63 9289	0.62 4727	0.64 2748	0.75 0199	0.62 1562	0.62 4732	0.61 7951	0.61 2702	0.61 7886	0.61 2406	0.63 1077	0.66 8779	0.62 2566	0.70 5658
0.7	0.3	1	0.5652 17391		0.52 3931	0.52 1329	0.56 7922	0.54 4306	0.59 3721	0.51 7189	0.55 562	0.55 8183	0.53 5355	0.53 3214	0.54 8288	0.54 2172	0.52 4957	0.34 0977	0.54 1115	0.35 5984

Appendix I: ANN regression graphs at different number of neurons in the hidden layer i.e. 5,7,10. a, b&c are for P_{80} , while d, e&f for SE.

