

School of Pure and Applied Sciences

Department of Physics and Astronomy

**RELATIVISTIC PLANE WAVE PREDICTIONS OF $(\vec{p}, 2\vec{p})$
POLARIZATION TRANSFER OBSERVABLES ON SPHERICAL NUCLEI**

by

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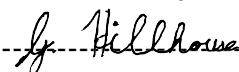
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Abstract

This study focuses on the application of the Relativistic Plane Wave Impulse Approximation (RPWIA) model for the predictions of $(\vec{p}, 2\vec{p})$ polarization transfer observables and the unpolarized triple differential cross sections on stable spherical nuclei: ^{208}Pb , ^{40}Ca and ^{12}C . The predictions are for knockout from different single-particle states, namely: $3s_{1/2}$, $2d_{3/2}$, $1h_{11/2}$, $2d_{5/2}$ and $1g_{7/2}$ in ^{208}Pb , $1d_{3/2}$ and $2s_{1/2}$ in ^{40}Ca , and $1p_{3/2}$ state in ^{12}C . The predictions are compared to experimental data from the Research Center for Nuclear Physics (RCNP) at incident proton energies of 392 MeV and coplanar scattering angles: 32.5° and -50.0° .

Within the context of the RPWIA model, the relativistic transition amplitude is employed with the incident and outgoing proton wave functions treated as relativistic free Dirac plane waves (ignores the distortion effects). The Relativistic Impulse Approximation (RIA) is adopted for the nucleon-nucleon (NN) interaction between the incident and the bound proton, the bound-state proton wave functions are extracted through the application of Quantum Hadrodynamics (QHD) and different Relativistic Mean Field (RMF) models: QHDII, NL3, FSUGold and FSUGold2.

A computer code was developed and implemented in FORTRAN, to calculate the unpolarized triple differential cross sections and the complete set of polarization transfer observables for proton knockout from single-particle states in ^{208}Pb , ^{40}Ca and ^{12}C . The predictions of the unpolarized triple differential cross sections and the analyzing powers were compared to the experimental data from RCNP measured as functions of the kinetic energy of the outgoing proton $[T_{a'} \text{ (MeV)}]$. Some of the key points that this study seeks to address are to: test the predictive power of the RPWIA model for the predictions of unpolarized triple differential cross sections and the analyzing powers for $(\vec{p}, 2\vec{p})$ knockout reactions compared to data, identify kinematic regions where the RPWIA predictions are closer to data and compare these predictions with the Relativistic Distorted Wave Impulse Approximation (RDWIA) predictions, investigate the possibilities of applying a model that does not rely on optical potentials, and to study the sensitivity of the polarization transfer observables to different RMF models applied in this study.

It shows that the RPWIA predictions quantitatively describe the data for the unpolarized triple differential cross sections and the analyzing powers for knockout from most single-particle states. The RPWIA model describe the data within a range of $\pm 10 \text{ MeV } [T_{a'} \text{ (MeV)}]$ from the peak of the unpolarized triple differential cross section for the analyzing power data. Within those regions, the RPWIA and the RDWIA predictions share intersection points for various polarization transfer observables and nuclear states, showing that the RPWIA model is sufficient to predict the observables for those energies. The variations between the predictions of the complete set of polarization transfer observables using different RMF models are small which implies that the choice of the RMF model has a limited impact on the overall predictions of these observables, i.e., the observables are insensitive to the different RMF models.

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List of Abbreviations

Commonly appearing abbreviations in this study are listed below:

RCNP Research Center for Nuclear Physics.

FAIR Facility for Antiproton and Ion Research.

RIBF Radioactive Isotope Beam Factory.

$(\vec{p}, 2\vec{p})$ Exclusive proton-induced proton-knockout reaction.

RPWIA Relativistic Plane Wave Impulse Approximation.

RDWIA Relativistic Distorted Wave Impulse Approximation.

NRDWIA Non-Relativistic Distorted Wave Impulse Approximation.

RIA Relativistic Impulse Approximation.

NN Nucleon-nucleon interaction.

D_{ij} Polarization transfer observables.

QHD Quantum Hadrodynamics.

RMF Relativistic Mean Field.

Chapter 1

Introduction

Research, both experimental and theoretical, helps us understand the systems of nature. Nuclear physics offers the possibility of understanding the forces which govern the existence of both stable and unstable nuclei [1]. Experiments have been conducted which probe the properties and structure of nuclei [2–5]. Some of these experiments involve the use of electron and proton beams to knockout protons from certain nuclear states, allowing for the study of single-particle properties of nuclei [2–4, 6, 7].

In recent years, the study of atomic nuclei far from stability lines has gained more attention leading to more financial investments into building facilities such as the Facility for Antiproton and Ion Research (FAIR) in Germany and Radioactive Isotope Beam Factory (RIBF) in Japan, which will help in studying these nuclei [8, 9]. Once fully online, these facilities are expected to offer opportunities to explore the properties of unstable neutron-rich nuclei with inverse kinematics scattering experiments [6, 8, 10]. For these particular reactions, $(\vec{p}, 2\vec{p})$, unstable beams are used as incident beams to scatter off protons from hydrogen targets [6, 8, 11, 12]. Through the scattering of high-energy beams of unstable nuclei, one can expand the knowledge of nuclear structure to a greater region in the nuclear chart [2, 8, 10–12].

1.1 Background

Scattering experiments play a crucial role in the quest to study and understand the dynamics of nuclei as they provide valuable insights into the intricate arrangement of nucleons within the nucleus [2]. Amongst these experiments, proton knockout reactions prove to be a powerful tool for probing the properties of nuclei [3]. Protons are ejected from the target nucleus by electron or proton beams [2]. By measuring the energies and scattering angles of the outgoing protons, researchers obtain valuable information about the structure and dynamics of the nucleus [2, 13].

The $(p, 2p)$ knockout reaction involves an incident unpolarized proton beam knocking out a bound proton from the target nucleus such that two outgoing protons are detected in coincidence. For a polarized incident proton beam, the reaction is represented by $(\vec{p}, 2\vec{p})$, of which one of the outgoing protons is polarized [14]. When the use of polarized beams is coupled with precise measurement techniques, they allow for the investigation of the spin-dependent properties of nuclei [6, 8].

The unpolarized triple differential cross section is related to the probability for a knockout reaction to occur [15]. Most $(p, 2p)$ experiments have measured unpolarized triple differential cross sections to allow for model extractions of nucleon occupation probabilities, the energies and momentum distributions of protons in single-particle states of stable nuclei [2, 6, 16, 17].

Polarized proton beams are used to study spin-dependent phenomena in nuclei. In particular, polarization transfer observables, D_{ij} , are related to the probability that a proton beam with initial spin direction j changes into the final spin direction i' [14, 18, 19]. Recently, $(\vec{p}, 2\vec{p})$ measurements have been extended to include polarized proton beams to measure a polarization transfer observable called the analyzing power on a variety of spherical nuclei at incident proton energies ranging between 200 MeV and 500 MeV [2].

Different theoretical models have been used to predict the unpolarized triple differential cross sections and the polarization transfer observables. The models based on the Non-Relativistic Distorted Wave Impulse Approximation (NRDWIA) in which the incident and outgoing proton wave functions are the solutions to the non-relativistic Schrödinger equation have been employed to predict these observables [4, 20, 21]. In another model, known as the Relativistic Distorted Wave Impulse Approximation (RDWIA), the wave functions of the incident and outgoing protons are the solutions to the relativistic Dirac equation [4, 7]. For the RDWIA model, the global Dirac optical potentials from elastic scattering experiments are used [7, 22, 23].

This study is focused on the application of the Relativistic Plane Wave Impulse Approximation (RPWIA) model for the predictions of the unpolarized triple differential cross sections and the complete set of polarization transfer observables for stable spherical nuclei and different single-particle states of interest.

1.2 Problem statement

The interpretation of $(\vec{p}, 2\vec{p})$ knockout reaction data has focused on applying models based on the NRDWIA [4]. In these models, the incident and outgoing scattering wave functions as well as the boundstate proton wave functions are obtained by solving the Schrödinger equation [4, 5, 24]. NRDWIA models fail to produce a consistent description of analyzing power data for proton knockout from various single-particle states in stable spherical nuclei at incident proton energies ranging between 200 MeV and 500 MeV [4, 21].

It is believed that the plausible reason for this failure is that the Schrödinger equation does not “naturally” describe spin [23, 25]. The analyzing power, which is a polarization transfer observable, is a spin-dependent quantity, therefore, a relativistic framework is required to properly describe this observable [23].

The NRDWIA and RDWIA models depend on optical potentials to describe the interaction between the incident and the outgoing protons with the residual nucleus [5, 25]. The potentials are fitted to elastic scattering data and various fits of the optical potentials may lead to different predictions and interpretations for the same observable [26].

To address these challenges, this study focuses on the application of a theoretical nuclear model based on the RPWIA, whereby instead of the Schrödinger equation, the relativistic Dirac equation will be implemented. Due to the uniqueness of the Dirac equation to incorporate spin, it is well suited for describing spin-dependent phenomena. The RPWIA model ignores the distortion effects on the incident and outgoing proton wave functions.

1.3 Motivation

Proton knockout reactions provide an excellent tool for studying the single-particle properties and medium modifications on the nucleon-nucleon (NN) interaction which describes the interaction between the incident and the bound proton [3, 21]. Exclusive proton knockout reactions provide the means to knockout protons from deep to low-lying single-particle orbitals within the nucleus. This enables a systematic investigation of the effect of nuclear density on the NN interaction [3, 14]. In particular, $(\vec{p}, 2\vec{p})$ reactions are expected to provide information on the properties of unstable nuclei [3, 6, 8, 10]. For this study, direct kinematics are employed, whereby polarized proton beams are used to knockout bound protons from single-particle states from different target nuclei (^{208}Pb , ^{40}Ca and ^{12}C). This application leads to the study of the complete set of polarization transfer observables [3, 18, 27]. Compared with the unpolarized cross section, polarization transfer observables offer stringent tests to theoretical models since spin-dependent quantities are sensitive to different model ingredients [14, 27].

Attempts to interpret $(\vec{p}, 2\vec{p})$ analyzing power data using NRDWIA models have not been successful for proton knockout from different single-particle states in stable nuclei at incident proton energies from 200 MeV to 500 MeV where there is an availability of large experimental data sets [4, 7]. The latter failure could be attributed to the fact that these models do not automatically incorporate spin. In contrast, the Dirac equation which incorporates spin, offers a relativistic path to determine the relativistic scattering and boundstate wave functions [14].

Since this study aims to investigate a theoretical model for the predictions of the complete set of polarization transfer observables which are spin-dependent quantities, it is crucial to work within a relativistic framework. The use of the term “relativistic” here implies the application of the Dirac equation due to its relativistic treatment of both the kinematics and dynamics of the reaction. A Lorentz covariant formalism is important for extrapolating nuclear models to extreme conditions of momentum transfer or density such as those encountered in relativistic heavy-ion collisions and high-momentum transfer experiments [23]. The four-component Dirac equation offers a description of the nuclear spin and the nuclear spin-orbit force [23, 28]. A fundamental component of the nuclear force, nuclear saturation, causes all nuclei to have similar values of binding energies per nucleon and saturation densities. The mechanism for nuclear saturation is obtained from the Relativistic Mean Field (RMF) approximation.

In this study, a “simple” relativistic plane wave model (RPWIA) is developed and applied. It is termed a simple model since the distortion effects on the incident and scattered proton wave functions are ignored. The optical potentials are gauged relative to elastic scattering and different phenomenological optical potentials can yield the same results [14, 26]. Within the context of the RPWIA model, there are no optical potentials and this simplifies the interpretation of the polarization transfer observables. The application of reliable nuclear scattering and structure models is vital to provide consistent quantitative predictions of a variety of experimental observables on stable nuclei. These models can then be reliably extended to study nuclear reactions on unstable nuclei [6, 8]. There is a need for theoretical models that provide consistent descriptions of polarization transfer observables for stable nuclear systems that do not depend on optical potentials.

1.4 Rationale of the study

Current theoretical models do not provide a consistent description of $(\vec{p}, 2\vec{p})$ knockout reactions from stable nuclei. Therefore, before studying $(\vec{p}, 2\vec{p})$ reactions on unstable nuclei using inverse kinematics, it is important to first understand these reactions on stable nuclei using direct kinematics. For this study, the focus is on $(\vec{p}, 2\vec{p})$ knockout reactions where direct kinematics are employed to knockout a proton from the target nucleus. These types of reactions are applied to different stable spherical target nuclei, as a theoretical nuclear model based on the RPWIA is applied for the predictions of the unpolarized triple differential cross section and the complete set of polarization transfer observables. To verify the validity of the model, the theoretical predictions will be compared to experimental data.

1.5 General objective

The general objective of this study is to develop and apply a theoretical nuclear model based on the RPWIA model for predicting unpolarized triple differential cross sections and the complete set of polarization transfer observables for exclusive proton-induced proton-knockout reactions from stable spherical nuclei.

1.6 Specific objectives

The specific objectives of this study are to:

1. Determine kinematic regions where a simple plane wave model describes the data.
2. Test the predictive power of the RPWIA model for $(\vec{p}, 2\vec{p})$ analyzing powers compared to experimental data at incident proton energy of 392 MeV.
3. Investigate the sensitivity of the complete set of polarization transfer observables and the unpolarized triple differential cross sections to different RMF models.
4. Predict the complete set of $(\vec{p}, 2\vec{p})$ polarization transfer observables using the RPWIA and compare to RDWIA predictions.
5. Analyse the effects of various RMF models on the complete set of polarization transfer observables and the unpolarized triple differential cross section.

1.7 Organization of the thesis

This thesis is structured as follows: the literature review summarizing research related to this study is given in Chapter 2. Chapter 3 describes the RPWIA formalism, where the relativistic transition amplitude and polarization transfer observables are defined. The extraction of the boundstate proton wave functions is given in Chapter 4 and the results are provided in Chapter 5. Chapter 6 presents the conclusions, summary and future perspectives for this study.

Chapter 2

Literature Review

2.1 Introduction

Within this Chapter, the literature survey of previous research studies focused on the development of the RPWIA model for the predictions of polarization transfer observables is provided.

2.2 Proton-induced proton-knockout reaction models

Since the late 1950s, proton-induced proton-knockout reactions have been intensively studied, as these types of reactions were found to lead to improved knowledge of the nucleus [29]. Originally, the models used to study the $(\vec{p}, 2\vec{p})$ reactions were based on NRDWIA [4, 20] until Maxwell and Cooper developed the RDWIA model [22]. Improvements were made to this model around the late 1990s by Mano and Kudo, who worked in the framework of the self-consistent and relativistic finite-range Distorted Wave Impulse Approximation (RDWIA) [16]. The dynamics of the $(\vec{p}, 2\vec{p})$ reactions were treated in a self-consistent manner. The single-particle boundstate wave functions were then calculated using RMF models [16, 29]. The RDWIA model has enjoyed great success in terms of describing the analyzing power data and the unpolarized triple differential cross sections for the nuclei of interest namely: ^{208}Pb , ^{40}Ca and ^{12}C throughout the years, however, this study is concerned with applying the RPWIA model. The reader may refer to Refs. [2, 4, 5, 14, 17, 27, 29, 30] and the references therein for more information about the NRDWIA and RDWIA models.

2.3 Relativistic plane wave models

A study conducted by Hillhouse *et al.* indicated that the RPWIA model described the experimental data for the analyzing power and also the RDWIA in a certain energy region [14]. They predicted the complete set of polarization transfer observables on single-particle states: $3s_{1/2}$, $2d_{3/2}$ and $2d_{5/2}$ of ^{208}Pb with an incident proton energy of 202 MeV and at coplanar scattering angles (28.0° and 54.6°). Here, the consistency of the RPWIA model is investigated for the predictions of the complete set of the polarization transfer observables for ^{208}Pb , ^{40}Ca and ^{12}C for incident proton beam energies of 392 MeV for different nuclear states within selected energy ranges where the predictions are closer to the data.

Two authors, Wyngaardt and Kanting, developed the RPWIA model for the predictions of the complete set of polarization transfer observables. In Wyngaardt's study [18], he applied direct kinematics on ^{40}Ca at 392 MeV to calculate the unpolarized triple differential cross section and polarization transfer observables. Kanting [10] applied a similar formalism using both

direct and inverse kinematics on oxygen isotopes at incident proton energies of 504 MeV. Both authors did not publish their predictions on scientific journals and their models have not been properly gauged to experimental data. The following details the work of these authors.

Wyngaardt developed a relativistic plane wave model for the prediction of the complete set of $(\vec{p}, 2\vec{p})$ polarization transfer observables and used the relativistic Horowitz-Love-Franey (HLF) model for the NN amplitudes. This approach expresses the IA1 representation as a sum of meson exchanges similar to Yukawa potentials [31]. Wyngaardt then applied the Horowitz-Murdock approach, which made use of the self-consistent Dirac Hartree model to generate the boundstate wave functions of the struck nucleon. The distortion effects from proton-nucleus optical potentials on the incoming and outgoing protons were neglected for the relativistic plane wave model, hence, the scattering wave functions were treated as free plane waves [18].

The relativistic transition amplitude, which expresses all proton-induced proton-knockout reaction scattering observables, was used to determine the polarization transfer observables and the unpolarized triple differential cross section [14, 27, 29]. The triple differential cross section defines the probability of a proton knockout reaction, where an incoming proton ejects a bound proton from a certain orbital in the target nucleus, and both protons are detected in the exit channels [10, 18]. An advancement in technology brought about high-resolution spectrometers and polarized proton beams offering the possibility to determine the complete set of polarization transfer observables for different nuclear reactions [8, 18]. For the relativistic plane wave model developed by Wyngaardt, distortions had no effect on both the incident and outgoing protons [18]. The scattering wave functions satisfy the relativistic free Dirac equation [32]. A program called TIMORA based on the Dirac-Hartree approximation was used to generate the boundstate wave function of the bound proton within the target nucleus [18].

Two techniques, the trace and the brute force method, were employed to evaluate the transition amplitude. The relativistic plane wave functions for the scattered protons were obtained by applying the Wigner transformations to the Dirac spinors using the brute force approach [18]. The transition amplitude was then determined by multiplying it by its complex conjugate transpose. As opposed to the brute force method, the trace method, which expresses the transition amplitude as a contraction between two tensor covariants allows for a direct computation of the transition amplitude. The brute force method was used as a numerical verification of the trace method [18]. Wyngaardt developed a formulation that enables the direct computation of the polarization transfer observables through the trace method. This technique allows for the separation of the interaction's spin-dependent and spin-independent components. The complete set of polarization transfer observables and the unpolarized triple differential cross section were calculated with a relativistic plane wave code based on the trace technique [18].

The kinematic conditions were based on the $(\vec{p}, 2\vec{p})$ knockout reaction experiment conducted at RCNP, where the analyzing powers and the unpolarized triple differential cross sections for ^{40}Ca were measured around 392 MeV [18]. The results of the $(\vec{p}, 2\vec{p})$ scattering observables were compared between the initial and final energy prescriptions, to examine the variations in the prescriptions concerning the NN kinematics reflected in the observables [14, 18]. The medium modifications for the NN interaction, along with the incoming and scattered proton wave functions governed by the Brown-Rho scaling law, were employed in the calculations [33]. A comparison of the results between the medium-modified observables and the calculated free quantities was performed. Wyngaardt developed a new approach to directly calculate the modulus square of the transition amplitude in Ref. [18]. The modulus square of the transition amplitude was reduced to the product of two traces, enabling the separation of information

concerning the polarized proton beams and the ejected protons. The incident and scattered wave functions were replaced by expressions that included four momentum and spin vectors, incorporating the properties of energy-momentum and spin projection operators [18].

Calculations were made for incident proton energies of 400 MeV and scattering angles, 32.5° and -50.0° with the knockout from the $1d_{5/2}$, $1d_{3/2}$, $2s_{1/2}$ states for an experiment conducted at RCNP. Wyngaardt demonstrated that the calculations of polarization transfer observables using the trace and brute force method gave the same results thus proving that the trace method indeed works. Other than the cross section, insignificant differences were observed on the results obtained with the initial and final energy prescriptions [18]. However, Wyngaardt did not compare his predictions to experimental data, and the work was never published in scientific journals.

In another study conducted by Kanting [10], the complete set of exclusive proton-induced proton-knockout, $(\vec{p}, 2\vec{p})$, polarization transfer observables of closed shell oxygen isotopes were calculated. These calculations were performed using direct and inverse kinematics applying the RPDIA model [10]. For direct kinematics, a polarized proton beam was used to knockout a proton from a fixed target nucleus in a certain orbital [10, 27]. Inverse kinematics makes use of unstable nuclei to knockout a target proton. The nucleus of interest is used as a projectile, while the proton is used as a target, and in all cases the protons are polarized [10]. The NN interaction matrix was expressed in terms of five Fermi invariants (**SVPAT**) discussed in section 3.3. To compute the boundstate wave functions of the nucleus, the RMF approximation was employed [10, 18]. Kanting studied the polarization transfer observables in the oxygen isotopes and identified observables which are most likely to discriminate between these isotopes of interest. The same kinematical conditions were used for direct and inverse kinematics to maintain consistency and to make appropriate comparisons. The energy of the incoming energy of the proton was set to 504 MeV with angles fixed at 22.0° and -40.3° [10].

Two Python codes were written to calculate the spin transfer observables and another to compute the nuclear ground state properties using NL3 [34] and FSUGold [35] parameter sets within the context of RMF theory [10]. Numerical checks were performed to verify that the Python programs produced good results. These checks involved comparisons with a well-established RMF code called TIMORA and the codes gave the same results. Radial wave functions were calculated for the occupied states of $^{14,16,22,24,28}\text{O}$ [10]. The invariant transition matrix was calculated in the momentum space, implying that the radial wave functions also had to be transformed into the momentum space. The NL3 parameter set, which was developed to address the various inadequacies of the QHD models (QHDI and QHDII) was used [10, 35]. Another calibrated parameter set, the FSUGold parameter set, was implemented [10, 35]. The results showed three observables: A_y , P and D_{nn} , are capable of differentiating between various oxygen isotopes for the inverse kinematics. Kanting did not compare the predictions to the experimental data, and the work was never published in scientific journals.

2.4 Summary

A review of the research work similar to this study was made to help pave the way for applying the relativistic plane wave model. Even though the two authors, Kanting and Wyngaardt, were able to develop relativistic plane wave models for predictions of polarization transfer observables, the validity of their models have not been gauged to experimental data. Kanting presented predictions for unstable nuclei using inverse kinematics without first validating the model for knockout from stable nuclei. Furthermore, there has been no systematic study of the effect of different RMF models on polarization transfer observables. This justifies the need to apply a relativistic plane wave model which can be used to make theoretical predictions and compare to the available experimental data of the unpolarized triple differential cross section and analyzing power. The relativistic plane wave model forms a basis for future experimental work in measuring the complete set of polarization transfer observables from employing either the direct or inverse kinematics. In Chapter 3, the formalism of the RPWIA model which will be used for the predictions of the complete set of the polarization transfer observables is presented.

Chapter 3

Relativistic Plane Wave Model

3.1 Introduction

This Chapter focuses on the formalism of the RPWIA model for proton knockout, $(\vec{p}, 2\vec{p})$, reactions, for the unpolarized triple differential cross sections and the complete set of polarization transfer observables from stable spherical nuclei, namely: ^{208}Pb , ^{40}Ca and ^{12}C . The $(\vec{p}, 2\vec{p})$ knockout reaction within the laboratory frame is illustrated in Fig. 3.1. Notations for the $(\vec{p}, 2\vec{p})$ knockout schematic are presented in Subsection 3.1.1, with the relativistic transition amplitude provided in Section 3.2. Section 3.4 describes the polarization transfer observables, and the NN interaction for the model is discussed in Section 3.3.

3.1.1 Notation

In a $(\vec{p}, 2\vec{p})$ knockout reaction, a projectile proton (a) with momentum $\vec{k}_a$ ejects a bound proton (b) from a specific single-particle state of a stationary target nucleus ($A = C + b$) resulting in the corresponding single-proton hole state labelled nlj in the residual nucleus C . The momenta of the two ejected protons, a' and b' , are denoted by $\vec{k}_{a'}$ and $\vec{k}_{b'}$, respectively, and these protons are detected simultaneously at coplanar scattering angles, $\theta_{a'}$ and $\theta_{b'}$, with nlj representing the single-particle states within the target nucleus A and the corresponding proton hole states in the residual nucleus C . The principal quantum number, the orbital angular momentum and the total angular momentum (nuclear spin) are denoted as n , l and $j = l \pm \frac{1}{2}$, respectively. Another prevalent notation style for the single-particle or hole states within atomic nuclei is represented as j^π where the term $\pi = (-1)^l$ denotes the intrinsic parity of the state. Both notation styles (nlj or j^π) are used interchangeably in the literature for $(\vec{p}, 2\vec{p})$ reactions [2, 36].

The momentum ($\vec{k}_i$) and kinetic energies (E_i), whereby $i = a, a', b'$, are defined in terms of the rest masses of particles (m_i), the kinetic energies of protons a and a' and the scattering angles $\theta_{a'}$ and $\theta_{b'}$ within the laboratory frame and the excitation energy (E_x) of the proton hole state relative to the ground state of the residual nucleus. The expressions for the relevant kinematic quantities are given in the Appendix section.

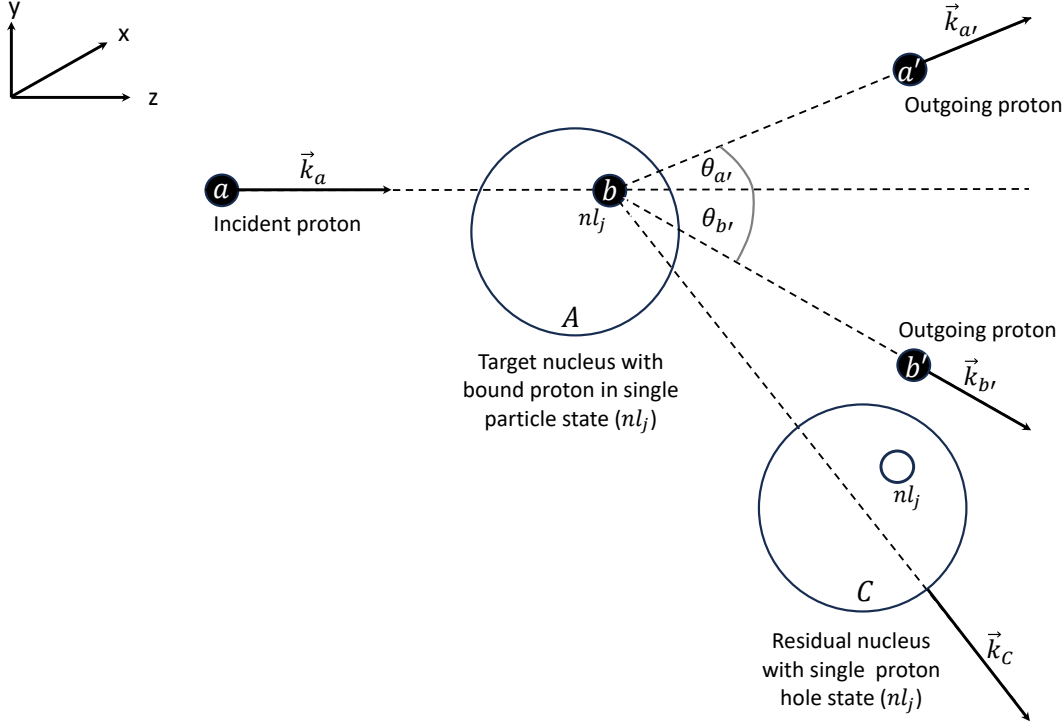


FIG. 3.1. Schematic illustration of a $(\vec{p}, 2\vec{p})$ knockout reaction in the laboratory frame. The reference axes are defined such that the incoming proton beam is directed along the z -axis, while the x -axis lies in the scattering plane. The y -axis is oriented perpendicular to the scattering plane.

All kinematic quantities and polarization transfer observables can be represented using the following orthogonal coordinate systems in the incident and outgoing laboratory frames [27]:

$$\hat{\ell} = \hat{z} = \hat{k}_a, \quad \hat{n} = \hat{y} = \hat{k}_a \times \hat{k}_{a'}, \quad \hat{s} = \hat{x} = \hat{n} \times \hat{\ell}, \quad (3.1)$$

$$\hat{\ell}' = \hat{z}' = \hat{k}_{a'}, \quad \hat{n}' = \hat{n} = \hat{y}, \quad \hat{s}' = \hat{x}' = \hat{n} \times \hat{\ell}', \quad (3.2)$$

where the momenta of the incident and scattered protons are represented by $\vec{k}_a$ and $\vec{k}_{a'}$, respectively.

The key aspect of the RPWIA model revolves around the transition amplitude which carries information about the wave functions of the incident and the two scattered protons, the NN interaction and the boundstate proton wave function in the target nucleus. The Relativistic Impulse Approximation (RIA) is adopted, which suggests that the interaction of the incoming and the bound proton is similar to the interaction of two nucleons in free space. In this study, natural units are employed where $\hbar = c = 1$ [32].

3.2 Relativistic transition amplitude

All scattering observables of a $(\vec{p}, 2\vec{p})$ knockout reaction are expressed in terms of the relativistic transition amplitude, $T_{ljm_j}(s_a, s_{a'}, s_{b'})$, corresponding to the scattering process shown in Fig. 3.1, which is given by:

$$T_{ljm_j}(s_a, s_{a'}, s_{b'}) = \int d^3\vec{x} [\bar{\psi}^{(-)}(\vec{x}, \vec{k}_{a'}, s_{a'}) \otimes \bar{\psi}^{(-)}(\vec{x}, \vec{k}_{b'}, s_{b'})] \hat{F}[\psi^{(+)}(\vec{x}, \vec{k}_a, s_a) \otimes \phi_{ljm_j}(\vec{x})], \quad (3.3)$$

The scattered proton wave functions of protons a' and b' are represented by $\bar{\psi}^{(-)}(\vec{x}, \vec{k}_{a'}, s_{a'})$ and $\bar{\psi}^{(-)}(\vec{x}, \vec{k}_{b'}, s_{b'})$ respectively with the projectile proton (a) wave function denoted by $\psi^{(+)}(\vec{x}, \vec{k}_a, s_a)$ [37]. The four component scattering wave functions, $\psi(\vec{x}, \vec{k}_i, s_i)$ for $i = a, a', b'$, are solutions to the Dirac scattering equation [5, 38]. The wave functions, $\bar{\psi}^{(-)}(\vec{x}, \vec{k}_j, s_j)$ where $j = a', b'$, is associated with the adjoint of the wave function, $\bar{\psi} = \psi^\dagger \gamma^0$ [18]. $\otimes$ is the Kronecker product, the NN interaction is represented by $\hat{F}$ and $\phi_{ljm_j}(\vec{x})$ represents the boundstate wave function of the target proton identified by the single-particle quantum numbers: l, j and m_j , where m_j specifies the projection of the total angular momentum j .

3.2.1 Scattering wave functions

In this work, the scattering wave functions are described as relativistic Dirac plane waves given by [18, 32, 38]:

$$\begin{aligned}\psi^{(+)}(\vec{x}, \vec{k}_a, s_a) &= e^{i\vec{k}_a \cdot \vec{x}} U(\vec{k}_a, s_a), \\ \psi^{(-)}(\vec{x}, \vec{k}_{a'}, s_{a'}) &= e^{-i\vec{k}_{a'} \cdot \vec{x}} U(\vec{k}_{a'}, s_{a'}), \\ \psi^{(-)}(\vec{x}, \vec{k}_{b'}, s_{b'}) &= e^{-i\vec{k}_{b'} \cdot \vec{x}} U(\vec{k}_{b'}, s_{b'}),\end{aligned}\tag{3.4}$$

The NN scattering matrix ($\hat{F}$) is represented using a parameterization based on the Fermi invariant amplitudes referred to as IA1 parameterization and described in Section 3.3. The boundstate proton wave functions are extracted through the application of RMF theory discussed in Chapter 4. where $U(\vec{k}_i, s_i)$ is the four component Dirac spinor expressed as:

$$U(\vec{k}_i, s_i) = \sqrt{\frac{E_i + m_i}{2m_i}} \begin{pmatrix} 1 \\ \frac{\vec{\sigma} \cdot \vec{k}_i}{E_i + m_i} \end{pmatrix} \chi_{s_i}, \tag{3.5}$$

with the Pauli spin matrices represented by $\vec{\sigma}$, the rest mass and energy of the nucleon are denoted as m_i and E_i respectively. The Dirac spinors, $U(\vec{k}_i, s_i)$, are normalized such that [32, 38]:

$$\bar{U}(\vec{k}_i, s_i) U(\vec{k}_i, s_i) = 1, \tag{3.6}$$

The two component Pauli spinors of projection s are denoted by:

$$\chi_{s=+\frac{1}{2}} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \chi_{s=-\frac{1}{2}} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \tag{3.7}$$

The spin polarization of the outgoing protons is determined by the choice of two-component Pauli spinors represented as χ_s . When considering the scattering of a particle with momentum $\vec{k}$ in the $\hat{x} - \hat{z}$ plane (scattering plane), at a scattering angle θ_k , $\vec{\sigma} \cdot \vec{k}$ can be written as:

$$\begin{aligned}\vec{\sigma} \cdot \vec{k} &= \sigma_x k_x + \sigma_y k_y + \sigma_z k_z, \\ &= k \sin \theta_k \sigma_x + k \cos \theta_k \sigma_z, \\ &= k \sin \theta_k \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} + k \cos \theta_k \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \\ &= k \begin{pmatrix} \cos \theta_k & \sin \theta_k \\ \sin \theta_k & -\cos \theta_k \end{pmatrix},\end{aligned}\tag{3.8}$$

Substitution of Eqs. (3.4) and (3.5) into Eq. (3.3) yields:

$$T_{ljm_j}(s_a, s_{a'}, s_{b'}) = \int d^3\vec{x} [e^{-i\vec{k}_{a'} \cdot \vec{x}} \bar{U}(\vec{k}_{a'}, s_{a'}) \otimes e^{-i\vec{k}_{b'} \cdot \vec{x}} \bar{U}(\vec{k}_{b'}, s_{b'})] \hat{F} [e^{i\vec{k}_a \cdot \vec{x}} U(\vec{k}_a, s_a) \otimes \phi_{ljm_j}(\vec{x})], \tag{3.9}$$

Since the momentum of the residual nucleus is given by $\vec{k}_C = -(\vec{k}_{a'} + \vec{k}_{b'} - \vec{k}_a)$, Eq. (3.9) can be re-written as:

$$T_{ljm_j}(s_a, s_{a'}, s_{b'}) = \int d^3\vec{x} e^{i\vec{k}_C \cdot \vec{x}} [\bar{U}(\vec{k}_{a'}, s_{a'}) \otimes \bar{U}(\vec{k}_{b'}, s_{b'})] \hat{F}[U(\vec{k}_a, s_a) \otimes \phi_{ljm_j}(\vec{x})], \quad (3.10)$$

Re-arranging these terms results in:

$$T_{ljm_j}(s_a, s_{a'}, s_{b'}) = [\bar{U}(\vec{k}_{a'}, s_{a'}) \otimes \bar{U}(\vec{k}_{b'}, s_{b'})] \hat{F}[U(\vec{k}_a, s_a) \otimes \int d^3\vec{x} e^{i\vec{k}_C \cdot \vec{x}} \phi_{ljm_j}(\vec{x})], \quad (3.11)$$

Making use of the relation:

$$\phi_{ljm_j}(\vec{k}_C) = \int d^3\vec{x} e^{i\vec{k}_C \cdot \vec{x}} \phi_{ljm_j}(\vec{x}), \quad (3.12)$$

Eq. (3.11) can be expressed as:

$$T_{ljm_j}(s_a, s_{a'}, s_{b'}) = [\bar{U}(\vec{k}_{a'}, s_{a'}) \otimes \bar{U}(\vec{k}_{b'}, s_{b'})] \hat{F}[U(\vec{k}_a, s_a) \otimes \phi_{ljm_j}(\vec{k}_C)], \quad (3.13)$$

3.2.2 Relativistic boundstate proton wave functions

The relativistic boundstate proton wave functions for the spherical nuclei (^{208}Pb , ^{40}Ca , ^{12}C) of interest are extracted through the use of RMF models discussed in Chapter 4 and are given by [38]:

$$\phi_{ljm_j}(\vec{x}) = \frac{1}{x} \sum_{s_B} \left(\begin{array}{c} u_{lj}(x) \langle l, m_j - s_B, \frac{1}{2}, s_B | j, m_j \rangle Y_{l, m_j - s_B}(\hat{x}) \chi_{s_B} \\ i w_{lj}(x) \langle 2j - l, m_j - s_B, \frac{1}{2}, s_B | j, m_j \rangle Y_{2j-l, m_j - s_B}(\hat{x}) \chi_{s_B} \end{array} \right), \quad (3.14)$$

where u_{lj} and w_{lj} are the upper and lower proton radial wave functions of the bound proton which are obtained through self-consistent solutions of the RMF equations, Y and the brackets $\langle \rangle$ represent the spherical harmonics and the Clebsch-Gordon coefficients, respectively. The partial plane wave expansion defined as [23, 39]:

$$e^{i\vec{k}_C \cdot \vec{x}} = \frac{4\pi}{k_C x} \sum_{lm} i^l u_l(k_C x) Y_{lm}^*(\hat{k}) Y_{lm}(\hat{x}). \quad (3.15)$$

is applied, and upon the substitution of:

$$u_l(k_C x) = (k_C x) j_l(k_C x), \quad (3.16)$$

into Eq. (3.15) yields:

$$e^{i\vec{k}_C \cdot \vec{x}} = 4\pi \sum_{lm} i^l j_l(k_C x) Y_{lm}^*(\hat{k}) Y_{lm}(\hat{x}), \quad (3.17)$$

where u_l represents the radial part of the wave function which is related to the spherical Bessel function $j_l(kx)$.

The combination of Eqs. (3.12), (3.14) and (3.17) results in:

$$\phi_{ljm_j}(\vec{k}_C) = \left(\begin{array}{c} 4\pi i^l \mathcal{Y}_{ljm_j}(\theta_{k_C}, \phi_{k_C}) u_{lj}(k_C) \\ 4\pi i^{2j-l+1} \mathcal{Y}_{2j-l+1, jm_j}(\theta_{k_C}, \phi_{k_C}) w_{2j-l, j}(k_C) \end{array} \right), \quad (3.18)$$

where [38]:

$$u_{lj}(k_C) = \int_0^\infty dx x j_l(k_C x) u_{lj}(x), \quad (3.19)$$

$$w_{2j-l,j}(k_C) = \int_0^\infty dx x j_{2j-l}(k_C x) w_{lj}(x), \quad (3.20)$$

$$\mathcal{Y}_{ljm_j}(\theta, \phi) = \sum_{s'_z = -\frac{1}{2}}^{+\frac{1}{2}} \langle l, \frac{1}{2}, m_j - s'_z, s'_z | jm_j \rangle Y_{l, m_j - s'_z}(\theta, \phi) \chi_{s'_z}, \quad (3.21)$$

Thus, the boundstate wave function in Eq. (3.18) can be expressed as:

$$\phi_{ljm_j}(\vec{k}_C) = 4\pi \sum_{s'_z} \left(i^l u_{lj}(k_C) \langle l, \frac{1}{2}, m_j - s'_z, s'_z | jm_j \rangle Y_{l, m_j - s'_z}(\theta, \phi) \chi_{s'_z} \right. \\ \left. i^{2j-l+1} w_{2j-l,j}(k_C) \langle 2j-l, \frac{1}{2}, m_j - s'_z, s'_z | jm_j \rangle Y_{2j-l, m_j - s'_z}(\theta, \phi) \chi_{s'_z} \right), \quad (3.22)$$

Upon the substitution of $\chi_{s'_z=+\frac{1}{2}} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\chi_{s'_z=-\frac{1}{2}} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ into Eq. (3.22), results in:

$$\phi_{ljm_j}(\vec{k}_C) = 4\pi \begin{pmatrix} i^l u_{lj}(k_C) \langle l, \frac{1}{2}, m_j - \frac{1}{2}, \frac{1}{2} | jm_j \rangle Y_{l, m_j - \frac{1}{2}}(\theta, \phi) \\ i^l u_{lj}(k_C) \langle l, \frac{1}{2}, m_j + \frac{1}{2}, -\frac{1}{2} | jm_j \rangle Y_{l, m_j + \frac{1}{2}}(\theta, \phi) \\ i^{2j-l+1} w_{2j-l,j}(k_C) \langle 2j-l, \frac{1}{2}, m_j - \frac{1}{2}, \frac{1}{2} | jm_j \rangle Y_{2j-l, m_j - \frac{1}{2}}(\theta, \phi) \\ i^{2j-l+1} w_{2j-l,j}(k_C) \langle 2j-l, \frac{1}{2}, m_j + \frac{1}{2}, -\frac{1}{2} | jm_j \rangle Y_{2j-l, m_j + \frac{1}{2}}(\theta, \phi) \end{pmatrix}, \quad (3.23)$$

Introducing the following notation:

$$\begin{aligned} \phi_{1ljm_j}(\vec{k}_C) &= i^l u_{lj}(k_C) \langle l, \frac{1}{2}, m_j - \frac{1}{2}, \frac{1}{2} | jm_j \rangle Y_{l, m - \frac{1}{2}}(\theta, \phi), \\ \phi_{2ljm_j}(\vec{k}_C) &= i^l u_{lj}(k_C) \langle l, \frac{1}{2}, m_j + \frac{1}{2}, -\frac{1}{2} | jm_j \rangle Y_{l, m_j + \frac{1}{2}}(\theta, \phi), \\ \phi_{3ljm_j}(\vec{k}_C) &= i^{2j-l+1} w_{2j-l,j}(k_C) \langle 2j-l, \frac{1}{2}, m_j - \frac{1}{2}, \frac{1}{2} | jm_j \rangle Y_{2j-l, m_j - \frac{1}{2}}(\theta, \phi), \\ \phi_{4ljm_j}(\vec{k}_C) &= i^{2j-l+1} w_{2j-l,j}(k_C) \langle 2j-l, \frac{1}{2}, m_j + \frac{1}{2}, -\frac{1}{2} | jm_j \rangle Y_{2j-l, m_j + \frac{1}{2}}(\theta, \phi), \end{aligned} \quad (3.24)$$

yields:

$$\phi_{ljm_j}(\vec{k}_C) = 4\pi \begin{pmatrix} \phi_{1ljm_j}(\vec{k}_C) \\ \phi_{2ljm_j}(\vec{k}_C) \\ \phi_{3ljm_j}(\vec{k}_C) \\ \phi_{4ljm_j}(\vec{k}_C) \end{pmatrix}, \quad (3.25)$$

Eq. (3.25) is a simplified form of the boundstate wave function for the proton within the target nucleus. For the NN interaction ($\hat{F}$), the RIA is applied where the IA1 parameterization known as **SVPA**T (Scalar-**V**ector-**P**seudoscalar-**A**xialvector-**T**ensor) is employed, is discussed in Section 3.3. Applying this to Eq. (3.13) yields:

$$T_{ljm_j}(s_a, s_{a'}, s_{b'}) = \sum_{i=S}^T F_i[\bar{U}(\vec{k}_{a'}, s_{a'}) \otimes \bar{U}(\vec{k}_{b'}, s_{b'})][\lambda^i \otimes \lambda_i][U(\vec{k}_a, s_a) \otimes \phi_{ljm_j}(\vec{k}_C)], \quad (3.26)$$

where λ_i 's represents the five Dirac matrices ($1, \gamma_\mu, \gamma_5, \gamma_5\gamma_\mu, \sigma_{\mu\nu}$) given in Table 3.1. Using the identity:

$$(M_1 \otimes M_2)(M_3 \otimes M_4) = (M_1 M_3) \otimes (M_2 M_4), \quad (3.27)$$

whereby M_j for $j \in 1, 2, 3, 4$ represent the $4 \otimes 1$ matrices, gives:

$$T_{ljm_j}(s_a, s_{a'}, s_{b'}) = \sum_{i=S}^T F_i [\bar{U}(\vec{k}_{a'}, s_{a'}) \lambda_1^i U(\vec{k}_a, s_a)] \otimes [\bar{U}(\vec{k}_{b'}, s_{b'}) \lambda_{2i} \phi_{ljm_j}(\vec{k}_C)], \quad (3.28)$$

Since $\bar{U}(\vec{k}_{a'}, s_{a'}) \lambda_1^i U(\vec{k}_a, s_a)$ are complex numbers, one can write Eq. (3.28) as:

$$T_{ljm_j}(s_a, s_{a'}, s_{b'}) = \sum_{i=S}^T F_i [\bar{U}(\vec{k}_{a'}, s_{a'}) \lambda_1^i U(\vec{k}_a, s_a)] [\bar{U}(\vec{k}_{b'}, s_{b'}) \lambda_{2i} \phi_{ljm_j}(\vec{k}_C)], \quad (3.29)$$

The relativistic NN interaction is given in Section 3.3.

3.3 Relativistic NN scattering matrix

In this section, an expression for the relativistic NN amplitudes from the non-relativistic amplitudes is given.

The RIA is employed in the transition matrix element which assumes that the interaction of the incident and bound proton inside the nucleus is the same as two protons interacting in free space [5, 23]. RIA is applicable for incident beam energies that are significantly higher than the binding energies of the target nucleons, as it disregards the binding energy and considers the interaction to occur between free nucleons [23]. This is true for the case of incident beam energies of 392 MeV and the magnitude of typical binding energies for the nuclear states of interest is less than about 16 MeV.

The McNeil-Ray-Wallace (MRW) framework presents a non-relativistic parameterization of the on-shell NN scattering operator, which accounts for the isospin invariance, parity, rotation, and time-reversal. It is represented as follows [40–42]:

$$(2i|\vec{k}_{cm}|)^{-1} f \equiv A + B\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 + i|\vec{q}| [C(\boldsymbol{\sigma}_1 \cdot \hat{n} + \boldsymbol{\sigma}_2 \cdot \hat{n}) + |\vec{q}|^2 D\boldsymbol{\sigma}_1 \cdot \hat{q}\boldsymbol{\sigma}_2 \cdot \hat{q} + E\boldsymbol{\sigma}_1 \cdot \hat{z}\boldsymbol{\sigma}_2 \cdot \hat{z}], \quad (3.30)$$

where the operator f denotes a 4×4 matrix that operates across the spin space of the initial and final nucleons. The complex functions: A, B, C, D and E , depend on the three-momentum transfer $|\vec{q}| = |\vec{k}'_{cm} - \vec{k}_{cm}|$ and the collision energy $E_{cm} = (|\vec{k}_{cm}|^2 + m^2)^{1/2}$, with m representing the nucleon rest mass. These functions are derived from phase shift analyses. An isospin dependence is introduced by [23]:

$$A = A_0 + A_1 \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2, \quad B = B_0 + B_1 \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2, \quad \text{etc.} \quad (3.31)$$

This approach then yields the parameterization for the non-relativistic proton-proton (pp) scattering operators, f^{pp} :

$$(2i|\vec{k}_{cm}|)^{-1} f^{pp} \equiv (A_0 + A_1) + (B_0 + B_1) \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 + i|\vec{q}| [(C_0 + C_1)(\boldsymbol{\sigma}_1 \cdot \hat{n} + \boldsymbol{\sigma}_2 \cdot \hat{n}) + |\vec{q}|^2 (D_0 + D_1) \boldsymbol{\sigma}_1 \cdot \hat{q}\boldsymbol{\sigma}_2 \cdot \hat{q} + (E_0 + E_1) \boldsymbol{\sigma}_1 \cdot \hat{z}\boldsymbol{\sigma}_2 \cdot \hat{z}], \quad (3.32)$$

The scattering amplitudes of specific spin orientations are obtained by applying operations to the initial and final Pauli spinors. The scattering amplitudes for the particular spin orientations

are determined by operating on the initial and final Pauli spinors. The perpendicular directions $\hat{z}$, $\hat{q}$ and $\hat{n}$ are given by:

$$\hat{z} = \hat{k}_a, \quad \hat{q} = \hat{k}_{cm} - \hat{k}'_{cm}, \quad \hat{n} = \hat{q} \times \hat{z}, \quad (3.33)$$

whereby $\vec{k}_a$:

$$\vec{k}_a = \frac{1}{2}(\vec{k}_{cm} + \vec{k}'_{cm}), \quad (3.34)$$

The normalisation of the scattering matrix results in the expression for the polarised differential cross section for the free NN scattering:

$$\frac{d\sigma}{d\Omega} = |\langle \chi_{s'_1}^\dagger \chi_{s'_2}^\dagger | f | \chi_{s_1}^\dagger \chi_{s_2}^\dagger \rangle|^2, \quad (3.35)$$

where χ 's represents the standard Pauli spinors for various spin projections of the incident and scattered particles.

For two interacting nucleons within the Dirac spinor space, the non-relativistic scattering operator $(2i|\vec{k}_{cm}|)^{-1}f$ is replaced by a relativistic scattering operator, denoted as $\hat{F}$, whereby $\hat{F}(E_{cm}, |\vec{q}|)$ is a $4 \times 4 \otimes 4 \times 4$ matrix with 256 components [41]. The relationship linking the relativistic and non-relativistic scattering matrices is provided by [43]:

$$(2i|\vec{k}_{cm}|)^{-1} \chi_{s'_1}^\dagger \chi_{s'_2}^\dagger f(E_{cm}, |\vec{q}|) \chi_{s_1} \chi_{s_2} \\ = \bar{U}(\vec{k}'_{cm}, s'_1) \bar{U}(-\vec{k}'_{cm}, s'_2) \hat{F}(E_{cm}, |\vec{q}|) U(\vec{k}_{cm}, s_1) U(-\vec{k}_{cm}, s_2), \quad (3.36)$$

where $U(\vec{k}_{cm}, s)$ is a four component Dirac positive-energy spinor given in Eq. (3.5).

An isospin designation is assigned to $\hat{F}$, with $\hat{F}^{pp}$ for pp scattering. The operator $\hat{F}(E_{cm}, |\vec{q}|)$, within the two-particle Dirac space, has 256 elements when considering all the possible combinations of nucleon and anti-nucleon spinors at a specified momentum transfer ($|\vec{q}|$) and energy (E_{cm}) [44]. Due to symmetries which reduce the number of independent spin matrix components of f to five, and the constraints of parity, isospin invariance and time-reversal, the number of independent amplitudes for $\hat{F}$ is reduced to 44 for the on-shell kinematics [23, 44, 45]. The information obtained from the measurements of free NN amplitudes, linked to the five complex Wolfenstein parameters, reveals an insight into only a subset of $\hat{F}$. As a result, certain assumptions about the structure of $\hat{F}$ in Eq. (3.36) must be made [23].

The **SVPAT** parameterization of the relativistic NN amplitudes is presented as [43, 46–48]:

$$\hat{F}(E_{cm}, |\vec{q}|) = \sum_{i=1}^5 F_i(E_{cm}, |\vec{q}|) \lambda_{(1)}^i \cdot \lambda_{i(2)}, \quad (3.37)$$

The subscripts (1) and (2) indicate the incoming and target nucleons, respectively. The i 's denotes the five standard Dirac matrices listed in Table 3.1, whereas the scalar product shows the contraction of Lorentz indices.

Each of the **SVPAT** amplitudes are related to the Wolfenstein amplitudes (A, B, C, D, E) and evaluated using the effective laboratory kinetic energies and the effective center-of-mass (cm) scattering angles. There are two prescriptions used to evaluate the effective laboratory kinetic energies, namely, the initial energy prescriptions and the final energy prescriptions [18]. For this study, the final energy prescriptions are applied for the NN amplitudes. The derivations of the effective laboratory kinetic energies using initial and final energy prescriptions, and the effective

Table 3.1: The Dirac matrix types used in the parameterization of the free NN amplitudes.

i	λ_i
S (Scalar)	1
V (Vector)	γ_μ
P (Pseudoscalar)	γ_5
A (Axial-vector)	$\gamma_5 \gamma_\mu$
T (Tensor)	$\sigma_{\mu\nu}$

cm scattering angles, are given in the appendix section. The effective laboratory kinetic energy using final energy prescriptions is given by:

$$E_a = \frac{s_f - 4m_p^2}{2m_p}, \quad (3.38)$$

and the center-of-mass scattering angle:

$$\theta^{cm} = \cos^{-1} \left[\frac{t}{2(k^{cm})^2} + 1 \right], \quad (3.39)$$

whereby s_f and t represent the Lorentz invariant Mandelstam variables given by Eq. A.33 in the appendix section and the mass of the proton denoted as m_p .

With Eq. (3.37), the expression on the right-hand side of Eq. (3.36) can be expanded as:

$$\begin{aligned} (2i|\vec{k}_{cm}|)^{-1} \chi_{s'_1}^\dagger \chi_{s'_2}^\dagger f(E_{cm}, |\vec{q}|) \chi_{s_1} \chi_{s_2} = & F_S(\bar{U}_1 U_1)(\bar{U}_2 U_2) + F_V(\bar{U}_1 \gamma^\mu U_1)(\bar{U}_2 \gamma_\mu U_2) \\ & + F_P(\bar{U}_1 \gamma^5 U_1)(\bar{U}_2 \gamma^5 U_2) + F_A(\bar{U}_1 \gamma^5 \gamma^\mu U_1)(\bar{U}_2 \gamma^5 \gamma_\mu U_2) + F_T(\bar{U}_1 \sigma^{\mu\nu} U_1)(\bar{U}_2 \sigma_{\mu\nu} U_2) \quad , \quad (3.40) \end{aligned}$$

Through the application of the Dirac spinors and the γ matrices, the right-hand side of Eq. (3.40) can be represented in relation to the independent set of spin operators and Pauli spinors. The coefficients can then be associated with those of the right-hand side of Eq. (3.30). This approach allows the Wolfenstein amplitudes (A, B, C, D, E) to be expressed as a linear combination of the five SVPAT amplitudes (F_S, F_V, F_P, F_A, F_T) as [42, 43]:

$$\begin{pmatrix} A \\ B \\ C \\ D \\ E \end{pmatrix} = \begin{pmatrix} & & & & \\ & & & & \\ \mathcal{O}(E_{cm}, |\vec{k}_a|, |\vec{q}|) & & & & \\ & & & & \\ & & & & \end{pmatrix} \begin{pmatrix} F_S \\ F_V \\ F_T \\ F_P \\ F_A \end{pmatrix}. \quad (3.41)$$

The expressions for the 5×5 matrix ($\mathcal{O}$) are provided in Ref. [43]. The polarization transfer observables for $(\vec{p}, 2\vec{p})$ knockout reactions are discussed in section 3.4 and the link between the matrix T , the relativistic $(\vec{p}, 2\vec{p})$ transition amplitude matrix element $T_{ljm_j}(s_a, s_{a'}, s_{b'})$ and the unpolarized triple differential cross section is described.

3.4 Polarization transfer observables

Advancements in technology have led to the development of high-resolution spectrometers and polarized proton beams, which allow measurements of the complete set of polarization transfer observables for different nuclear reactions [47]. These spin-dependent quantities provide more information about the nature of interactions inside the nucleus, since they offer the probability that a projectile proton with an initial spin orientation j will change into a final spin direction i' [23, 38].

The unpolarized triple differential cross section is given by [27, 38]:

$$\sigma = \frac{d^3\sigma}{dT_{a'}d\Omega_{a'}d\Omega_b} = S_{lj}\sigma_{\text{calc}}, \quad (3.42)$$

$$\sigma_{\text{calc}} = \frac{F_{kin}}{(2s_a + 1)(2j + 1)} \sum_{m_j, s_b} \text{Tr}(TT^\dagger), \quad (3.43)$$

whereby S_{lj} represents the spectroscopic factor related to the single-particle state occupation numbers [16, 27] and F_{kin} is a kinematical factor given by Eq. (A.1) in the Appendix section. The polarization transfer observables, $D_{i'j}$, are defined as [19, 27, 38]:

$$D_{i'j} = \frac{\text{Tr}(T\sigma_j T^\dagger \sigma_{i'})}{\text{Tr}(TT^\dagger)}, \quad (3.44)$$

where the 2×2 Pauli spin matrices are given by:

$$\begin{aligned} \sigma_0 &= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, & \sigma_{\hat{x}} &= \sigma_{\hat{s}} = \sigma_{\hat{s}'} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \\ \sigma_{\hat{y}} &= \sigma_{\hat{n}} = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, & \sigma_{\hat{z}} &= \sigma_{\hat{\ell}} = \sigma_{\hat{\ell}'} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \end{aligned} \quad (3.45)$$

The analyzing power is denoted as A_y , induced polarization by P , and other polarization transfer observables by $D_{n'n}$, $D_{s'\ell}$, $D_{s's}$, $D_{\ell's}$ and $D_{\ell'\ell}$. For polarization transfer observables, the spectroscopic factor S_{lj} cancels in the numerator and denominator of Eq. (3.44). T is a 2×2 matrix given by:

$$T = \begin{pmatrix} T_{lj}^{s_a=\frac{1}{2}, s_{a'}=\frac{1}{2}} & T_{lj}^{s_a=-\frac{1}{2}, s_{a'}=+\frac{1}{2}} \\ T_{lj}^{s_a=-\frac{1}{2}, s_{a'}=+\frac{1}{2}} & T_{lj}^{s_a=-\frac{1}{2}, s_{a'}=-\frac{1}{2}} \end{pmatrix}, \quad (3.46)$$

where $s_a = \pm\frac{1}{2}$ and $s_{a'} = \pm\frac{1}{2}$ denote the spin projections of both the incident (a) and the outgoing proton (a'). Through the application of time-reversal, parity and rotational invariance to the NN scattering matrix, the only seven polarization transfer observables allowed are: $D_{n'0}$, $D_{0'n}$, $D_{n'n}$, $D_{s'\ell}$, $D_{s's}$, $D_{\ell's}$ and $D_{\ell'\ell}$ [18]. The matrix elements $T_{lj}^{s_a, s_{a'}}$ are associated with the relativistic $(\vec{p}, 2\vec{p})$ transition matrix element $T_{ljm_j}(s_a, s_{a'}, s_{b'})$ through [27, 30, 38]:

$$T_{lj}^{s_a, s_{a'}} = \sum_{m_j, s_{b'}} T_{ljm_j}(s_a, s_{a'}, s_{b'}). \quad (3.47)$$

3.5 Summary

The formalism for the theoretical nuclear model based on RPWIA was presented. This model will be used for the theoretical predictions of the unpolarized triple differential cross section and the complete set of polarization transfer observables for knockout of protons from single-particle states in stable spherical nuclei (^{208}Pb , ^{40}Ca and ^{12}C). For the model (RPWIA), the scattering wave functions are the relativistic Dirac plane waves and solutions to the free Dirac equation since distortions are neglected. The NN interaction is parameterized in terms of five invariant scattering amplitudes introduced in the context of RIA. The NN amplitudes are evaluated using the final energy prescriptions for the effective kinetic energy in the laboratory frame and the center-of-mass scattering angle. To extract the relativistic boundstate proton radial wave functions, different RMF models are applied. The boundstate proton radial wave functions of different target nuclei are determined in Chapter 4.

Chapter 4

Relativistic Mean Field Theory

4.1 Introduction

This Chapter describes the models that are used to extract the boundstate proton radial wave functions in Eq. (3.14) for different single-particle states in nuclei of interest. The following four RMF models are considered: QHDII, NL3 FSUGold and FSUGold2. The background of the QHD model is given in Section 4.2. In Section 4.3, the general Lagrangian density for the QHD model is provided which captures the dynamics of the constituent particles inside the nucleus, and Subsection 4.3.1 highlights the application of different RMF models. The RMF predictions for the bulk properties of saturated nuclear matter and calculations for the fields, nucleon densities, boundstate wave functions and single-particle proton energies for different target nuclei are presented in Section 4.4.

4.2 Background

Nuclear physics aims to provide information about the nature of nucleon interactions (protons and neutrons) in many body systems for finite and infinite nuclear matter [10, 24, 49]. An effective field theory, Quantum Hadrodynamics (QHD), also referred to as the $\sigma - \omega$ model, was introduced by J. D. Walecka in 1974 as an attempt to provide a more satisfactory explanation of the behaviour of nucleons [50–52]. Within the context of the RMF approximation, the QHD model provides a relativistic description of the nucleus as the nucleons interact via the exchange of mesons [53]. Within this framework, model parameters are constrained to reproduce the properties of saturated nuclear matter, i.e., saturation density, binding energy, compressibility (compression modulus) and symmetry energy [34, 52, 54–57]. Successive iterations of the model parameters, including additional mesons and couplings, have been made to improve the reproduction of the properties mentioned since the original $\sigma - \omega$ model [52, 58]. The QHD parameter sets considered in this study are QHDII [58], NL3 [34], FSUGold [35] and FSUGold2 [57]. The different RMF models were considered to investigate their effects on the predictions of the complete set of polarization transfer observables.

QHD is a Quantum Field Theory (QFT) defined by a finite number of large coupling constants (>1) and meson masses [24]. The equations of motion of the constituent fields are solved using an approximation known as RMF [52, 59]. Within this context, the meson fields are replaced with their ground state expectation values. Due to large meson-nucleon coupling constants, these RMF equations cannot be solved through a perturbative approach, however with the RMF theory, they will be solved self-consistently as the densities are the sources for the meson fields [24, 60].

To extract the boundstate wave functions of the proton in the target nuclei: ^{208}Pb , ^{40}Ca and ^{12}C , the nuclear structure is modelled using RMF. Different RMF parameter sets will be used to calculate the bulk properties of infinite nuclear matter and properties of finite nuclei matter such as nucleon densities, potentials, boundstate wave functions and single-particle proton energies to evaluate the numerical correctness of the codes.

4.3 Lagrangian density

The baryon interactions inside the nucleus for QHD models are expressed in terms of Lagrangian density. Since the Coulomb potential is negligible compared to the magnitude of the meson potentials and their coupling constants, it was omitted from the Lagrangian [59, 61]:

$$\begin{aligned} \mathcal{L} = & \bar{\psi}(x) \left[\gamma^\mu \left(i\partial_\mu - g_v V_\mu(x) - \frac{g_\rho}{2} \boldsymbol{\tau} \cdot \mathbf{b}_\mu(x) \right) - [M - g_s \phi(x)] \right] \psi(x) \\ & + \frac{1}{2} \partial_\mu \phi(x) \partial^\mu \phi(x) - \frac{1}{2} m_s^2 \phi^2(x) - \frac{1}{4} V^{\mu\nu} V_{\mu\nu} + \frac{1}{2} m_\omega^2 V^\mu(x) V_\mu(x) \\ & - \frac{1}{4} \mathbf{b}^{\mu\nu} \cdot \mathbf{b}_{\mu\nu} + \frac{1}{2} m_\rho^2 \mathbf{b}^\mu(x) \cdot \mathbf{b}_\mu(x) - U_{\text{eff}}[\phi(x), V^\mu(x), \mathbf{b}^\mu(x)]. \end{aligned} \quad (4.1)$$

The baryon field is represented by $\psi(x)$; γ^μ represents the Dirac matrices; $\phi(x)$, $V^\mu(x)$ and $\mathbf{b}^\mu(x)$ represent the sigma (scalar), omega (vector), and rho meson fields, respectively, together with their respective coupling constants: g_s , g_v and g_ρ . $\boldsymbol{\tau}$ represents the isospin operator and M is the nucleon mass, while the meson field tensors are:

$$V_{\mu\nu}(x) = \partial_\mu V_\nu(x) - \partial_\nu V_\mu(x), \quad (4.2a)$$

$$\mathbf{b}_{\mu\nu}(x) = \partial_\mu \mathbf{b}_\nu(x) - \partial_\nu \mathbf{b}_\mu(x). \quad (4.2b)$$

U_{eff} represents the effective interactions between the isoscalar, isovector and the mesons [57, 62]:

$$\begin{aligned} U_{\text{eff}}[\phi(x), V^\mu(x), \mathbf{b}^\mu(x)] = & \frac{\kappa}{3!} [g_s \phi(x)]^3 + \frac{\lambda}{4!} [g_s \phi(x)]^4 - \frac{\zeta}{4!} [g_v^2 V^\mu(x) V_\mu(x)]^2 \\ & - \Lambda_v [g_\rho^2 \mathbf{b}_\mu(x) \cdot \mathbf{b}^\mu(x)] [g_v^2 V^\nu(x) V_\nu(x)]. \end{aligned} \quad (4.3)$$

The coefficients κ , λ , ζ and Λ_v are meson self-coupling terms. Boguta and Bodmer proposed the κ and λ isoscalar meson self-interactions to reduce the nuclear compression modulus of saturated nuclear matter [10, 62, 63]. Muller and Serot introduced the ζ isoscalar meson self-interaction which enables the pressure of a nuclear system to increase gradually with an increase in density [51, 52, 57]. The Λ_v term was introduced by Horowitz and Piekarewicz to modify the density dependence of the symmetry energy without influencing the ground state properties [62, 64].

For the original QHD model, $\sigma - \omega$, only scalar and vector mesons were considered in the interaction of the nucleons (protons and neutrons) [50–52]. Improvements were made to this model leading to the introduction of the QHDI model, by including the isovector-vector meson together with its coupling constant into the Lagrangian density [51, 58]. This was done in an attempt to reduce the value of the incompressibility of the $\sigma - \omega$ model. The NL3 model introduces the meson self-coupling terms, κ and λ , which improved the properties of saturated nuclear matter [34]. More advanced RMF models, FSUGold and FSUGold2, introduce the isoscalar meson self-interacting terms, ζ and Λ_v , to further improve the properties of finite nuclear matter shown in Table 4.2 [35, 57]. The FSUGold2 model, which is an extension of the

FSUGold model, is constrained based on the properties of finite nuclei and neutron stars [57, 65]. Regardless of the different coupling constants and self-interacting terms, all RMF models are constrained to reproduce the properties of saturated nuclei matter: saturation density, binding energy, incompressibility, symmetry energy, and the slope of the symmetry energy [62, 64].

The QHDII model has the same parameter sets which serve as inputs in the well known, “Timora” code. This model serves as a baseline for other RMF models since the model does not incorporate any additional self-coupling terms. The other RMF models (NL3, FSUGold and FSUGold2) were used since they have additional terms contributing to the Lagrangian density leading to improved properties of infinite nuclear matter. Furthermore, the RMF models were applied to stable spherically symmetric nuclei ranging from a neutron-rich nucleus, ^{208}Pb , through ^{40}Ca to a lighter target nucleus ^{12}C .

The equations of motion for the constituent particles are extracted from the Lagrangian density, Eq. (4.1), using the Euler-Lagrange equation for a general field q , where $q \in \phi(x), V^\mu(x), b^\mu(x)$ and $\bar{\psi}$ is:

$$\partial_\mu \frac{\partial \mathcal{L}}{\partial(\partial_\mu q)} - \frac{\partial \mathcal{L}}{\partial q} = 0. \quad (4.4)$$

Applying the Euler-Lagrange equation yields the following equations:

$$\partial_\mu \partial^\mu \phi(x) + m_s^2 \phi(x) + \frac{\kappa}{2!} g_s^3 \phi(x)^2 + \frac{\lambda}{3!} g_s^4 \phi(x)^3 = g_s \bar{\psi}(x) \psi(x), \quad (4.5a)$$

$$\partial_\mu V^{\mu\nu} + m_\omega^2 V^\nu(x) + \frac{\zeta}{3!} g_v^4 V_\nu(x)^2 V^\nu(x) + 2\Lambda_v g_v^2 V^\nu(x) g_\rho^2 \mathbf{b}^\mu(x) \cdot \mathbf{b}_\mu(x) = g_v \bar{\psi}(x) \gamma^\nu \psi(x), \quad (4.5b)$$

$$\partial_\mu \mathbf{b}^{\mu\nu} + m_\rho^2 \mathbf{b}^\nu(x) + 2\Lambda_v g_v^2 V^\nu(x) V_\nu(x) g_\rho^2 \mathbf{b}^\nu(x) = \frac{g_\rho}{2} \bar{\psi}(x) \gamma^\nu \boldsymbol{\tau} \psi(x), \quad (4.5c)$$

$$\left[\gamma^\mu \left(i\partial_\mu - g_v V_\mu(x) - \frac{g_\rho}{2} \boldsymbol{\tau} \cdot \mathbf{b}_\mu(x) \right) - [M - g_s \phi(x)] \right] \psi(x) = 0. \quad (4.5d)$$

Eqs. (4.5a) - (4.5c) represent the equations of motion for the scalar, vector and isovector mesons respectively [52, 59]. Eq. (4.5d) is the free Dirac equation with scalar and vector fields [52, 54].

4.3.1 Relativistic Mean Field approximation

The RMF approximation is applied in solving these equations of motion [Eqs. (4.5a) - (4.5c)] of the different fields. Within this context, the meson fields are replaced with their respective ground state expectation values [51, 54, 59, 61]. These fields are treated as classical fields and are assumed to be uniform. Since a static spherically symmetric system (where $|\vec{r}| = r$) is assumed, the meson fields depend only on the radial distance (r) from the centre of the nucleus which accounts for the density dependence of these fields. RMF also assumes that the ground state is both rotational and translational invariant which simplifies the equations of motion of the various meson fields [59, 62]:

$$\phi(x) \rightarrow \langle \phi(x) \rangle = \phi_0(r), \quad (4.6a)$$

$$V^\mu(x) \rightarrow \langle V^\mu(x) \rangle = g^{\mu 0} V_0(r), \quad (4.6b)$$

$$\mathbf{b}^\mu(x) \rightarrow \langle b_\alpha^\mu(x) \rangle = g^{\mu 0} \delta_{a3} b_0(r). \quad (4.6c)$$

Considering that the nucleon densities are sources of the meson fields in Eqs. (4.5a) - (4.5c), these sources must also be replaced by their ground state expectation values within the RMF

approximation [62]:

$$\bar{\psi}(x)\psi(x) = \langle \bar{\psi}\psi \rangle = \rho_s(r), \quad (4.7a)$$

$$\bar{\psi}(x)\gamma^\mu\psi(x) = \langle \bar{\psi}\gamma^\mu\psi \rangle = g^{\mu 0}\rho_b(r), \quad (4.7b)$$

$$\bar{\psi}(x)\gamma^\mu\tau_i\psi(x) = \langle \bar{\psi}\gamma^\mu\tau_i\psi \rangle = g^{\mu 0}\delta_{i3}\rho_3(r). \quad (4.7c)$$

whereby ρ_s represents the scalar density, ρ_b denotes the baryon density, and ρ_3 is the isovector density showing the difference between the proton and neutron densities ($\rho_3 = \rho_p - \rho_n$). Within the RMF approximation, the equations of motion defined in Eqs. (4.5a) - (4.5d), are expressed as:

$$g_s\phi_0(r) = \frac{g_s^2}{m_s^2} \left[\rho_s(r) - \frac{\kappa}{2} [g_s\phi_0(r)]^2 - \frac{\lambda}{6} [g_s\phi_0(r)]^3 \right], \quad (4.8a)$$

$$g_v V_0(r) = \frac{g_v^2}{m_\omega^2} \left[\rho_b(r) - \frac{\zeta}{6} [g_v V_0(r)]^3 - 2\Lambda_v [g_v V_0(r)] [g_\rho b_0(r)]^2 \right], \quad (4.8b)$$

$$g_\rho b_0(r) = \frac{g_\rho^2}{m_\rho^2} \left[\frac{1}{2} \rho_3(r) - 2\Lambda_v [g_v V_0(r)]^2 [g_\rho b_0(r)] \right], \quad (4.8c)$$

$$0 = \left[i\gamma^\mu \partial_\mu - g_v \gamma^0 V_0(r) - \frac{g_\rho}{2} \tau_3 \gamma^0 b_0(r) - [M - g_s \phi_0(r)] \right] \psi(r), \quad (4.8d)$$

The solution to the free Dirac equation in Eq. (4.8d), is:

$$\psi(r) = \psi(\vec{k}, s) e^{i\vec{k}\cdot\vec{r} - iE(\vec{k}, s)t}, \quad (4.9)$$

The four component Dirac spinor with spin index s is denoted as $\psi(\vec{k}, s)$ and the Dirac Hamiltonian H describing the energy (E) is related to the momentum state $\vec{k}$ and includes the couplings to the potentials $V_0(r)$, $b_0(r)$ and $\phi_0(r)$ [54]:

$$H\psi(r) \equiv \left[-i\boldsymbol{\alpha} \cdot \boldsymbol{\nabla} + g_v V_0(r) + \frac{g_\rho}{2} \tau_3 b_0(r) + \beta(M - g_s \phi_0(r)) \right] \psi(r) = E\psi(r), \quad (4.10)$$

where the usual Dirac matrices are represented as $\boldsymbol{\alpha}$ and β . Eq. (4.10) has both positive $[\mathcal{U}(\vec{r})]$ and negative energy $[\mathcal{V}(\vec{r})]$ solutions. The positive solution of the Dirac equation for a spherically symmetric nuclear potential is given by [54, 62, 66]:

$$\mathcal{U}_\alpha(\vec{r}) \equiv \mathcal{U}_{n\kappa m t}(\vec{r}) = \frac{1}{r} \begin{pmatrix} G_{n\kappa t}(r) \mathcal{Y}_{\kappa m} \\ iF_{n\kappa t}(r) \mathcal{Y}_{-\kappa m} \end{pmatrix} \zeta_t, \quad (4.11)$$

where $\mathcal{U}_\alpha(\vec{r})$ is represented in terms of the upper and lower radial functions $G_{n\kappa t}(r)$ and $F_{n\kappa t}(r)$, respectively, depending on the radial coordinate r , the spherical harmonics ($\mathcal{Y}_{\kappa m}$ and $\mathcal{Y}_{-\kappa m}$) which are given by Eq. (3.21) from section 3.2 and a Pauli spinor (ζ_t) denoting the isospin state. α is a composite index, ($\alpha = n, \kappa, m, t$), representing the quantum numbers: the principal quantum number (n), the relativistic angular momentum quantum number (κ), magnetic quantum number m and the isospin t (where $t = +\frac{1}{2}$ for the protons and $t = -\frac{1}{2}$ for neutrons) [54].

The relationship between the relativistic angular momentum quantum number (κ), the total angular momentum quantum number (j) and the orbital angular momentum quantum number (l) and $\bar{l}$ is given by:

$$j = |\kappa| - \frac{1}{2}, \quad l = \begin{cases} \kappa, & \kappa > 0 \\ -\kappa - 1, & \kappa < 0 \end{cases}, \quad \bar{l} = 2j - l = \begin{cases} \kappa - 1, & \kappa > 0 \\ -\kappa, & \kappa < 0 \end{cases}, \quad (4.12)$$

The following baryon wave functions are produced upon substitution of Eq. (4.11) into Eq. (4.10):

$$\frac{d}{dr}G_\alpha(r) + \frac{\kappa}{r}G_\alpha(r) - [E_\alpha - g_v V_0(r) - t_\alpha g_\rho b_0(r) + M - g_s \phi_0(r)]F_\alpha(r) = 0, \quad (4.13)$$

$$\frac{d}{dr}F_\alpha(r) - \frac{\kappa}{r}G_\alpha(r) + [E_\alpha - g_v V_0(r) - t_\alpha g_\rho b_0(r) - M + g_s \phi_0(r)]G_\alpha(r) = 0, \quad (4.14)$$

where the normalization condition which ensures the unit probability of obtaining a nucleon in space is given by:

$$\int_0^\infty (|G_\alpha(r)|^2 + |F_\alpha(r)|^2)dr = 1. \quad (4.15)$$

Eqs. (4.8a - 4.8d) and (4.13 - 4.14) represent a system of coupled differential equations. To obtain numerical solutions to the Hartree equations, the Dirac equation is solved via a self-consistent iterative procedure that takes advantage of the scalar and vector potentials [62]. Woods-Saxon-shaped meson fields serve as the basis for the self-consistency resulting in the generation of the boundstate energies and related wave functions for single-particle states. The meson fields are used as sources for the scalar and vector densities that are derived from these wave functions, which then determine the mean-field potentials for the nucleons [10, 54, 62].

Different parameters for various RMF models used in this study are presented in Table 4.1 with the meson masses: m_s, m_v and m_ρ and κ given in units of MeV with the rest of the terms dimensionless.

Table 4.1: The model parameters for the different RMF models: QHDII, NL3, FSUG-old and FSUGold2, obtained from Refs. [34, 58, 62, 67]. The nucleon mass is fixed at $M = 939$ MeV.

Model	m_s	m_v	m_ρ	g_s^2	g_v^2	g_ρ^2	κ	λ	ζ	Λ
QHDII	520.0	783.0	770.0	109.6	190.4	65.2	0.0000	0.000 000	0.0000	0.000 000
NL3	508.2	782.5	763.0	104.4	165.6	79.6	3.8599	-0.015 905	0.0000	0.000 000
FSUGold	491.5	782.5	763.0	112.2	204.5	138.5	1.4203	0.023 762	0.0600	0.030 000
FSUGold2	497.5	782.5	763.0	108.1	183.8	80.5	3.0029	-0.000 533	0.0256	0.000 823

4.4 Relativistic Mean Field predictions

With the different parameter sets given in Table 4.1 for the various RMF models applied in this study, bulk properties of infinite nuclear matter at saturation point were calculated and compared with published values from literature. These properties: saturation density (ρ_0), binding energy per nucleon (BE/A), compression modulus (K), symmetry energy (J) and gradient of the symmetry energy (L).

4.4.1 Properties of saturated nuclear matter

In this subsection, the different bulk properties of saturated nuclear matter are defined and comparisons between the predictions and the data published in the literature are made. A FORTRAN code called Hartree.f from Piekarewicz [68] was modified to extract the RMF potentials, nucleon densities, boundstate proton wave functions and single-particle proton energies from various RMF models for different target nuclei (^{208}Pb , ^{40}Ca and ^{12}C).

4.4.1.1 Saturation density

The nucleus is made up of nucleons which are held together by a strong nuclear force, although this force is short ranged, it is attractive which is crucial for maintaining a stable nucleus. The interaction is between nearest neighbouring nucleons, and at a certain density, addition of more nucleons to the system will not increase the central density of the system [39]. At saturation, the nuclear system remains in its stable state if it is not disturbed. The binding energy of the system is minimal at this saturation point [59]. The radius of a spherical nucleus is given by: $R = r_0 A^{1/3}$, whereby R is the radius, $r_0 = 1.2$ fm and A represents the atomic mass of the nucleus, the constant proportionality (r_0) is related to saturation density by [69]:

$$\rho_0 = \left[\frac{4\pi}{3} r_0^3 \right]^{-1}, \quad (4.16)$$

The saturation density is given by [69]:

$$\rho_0 = \frac{2k_F^3}{3\pi^2}, \quad (4.17)$$

with the Fermi momentum (k_F) given by:

$$k_F = \left(\frac{3\pi^2 \rho_0}{2} \right)^{\frac{1}{3}}. \quad (4.18)$$

The saturation densities for QHDII, NL3 and FSUGold are estimated to be around 0.148 fm^{-3} from Refs. [34, 54, 57, 65] whereas for FSUGold2 it is estimated around 0.151 fm^{-3} from Refs. [57, 65]. The different parameter sets given in Table 4.1 for the different RMF models (QHDII, NL3, FSUGold and FSUGold2) are used to calculate the saturation density, and the results in Table 4.2 are similar to those published in Refs. [54, 57, 65].

4.4.1.2 Binding energy

The binding energy (BE/A) of a nuclear system is the energy required to separate the nucleus into its individual components. At saturation density, the nuclear system acquires stability and the binding energy reaches its minimum, therefore, a higher binding energy implies a more stable nucleus. Stable nuclear systems have a negative binding energy [59]. This value is estimated around -16.3 MeV from Refs. [34, 57, 65, 69, 70], and -15.75 MeV from Ref. [54, 71]. Fig. 4.1 shows the relationship between the binding energy and the saturation density for the different RMF models applied in this study. The results for the binding energies for these different RMF models are tabulated in Table 4.2 and are similar to results of the published values found in Refs. [59, 65, 69]. The binding energies extracted from the application of the RMF models investigated are shown in Fig. 4.1.

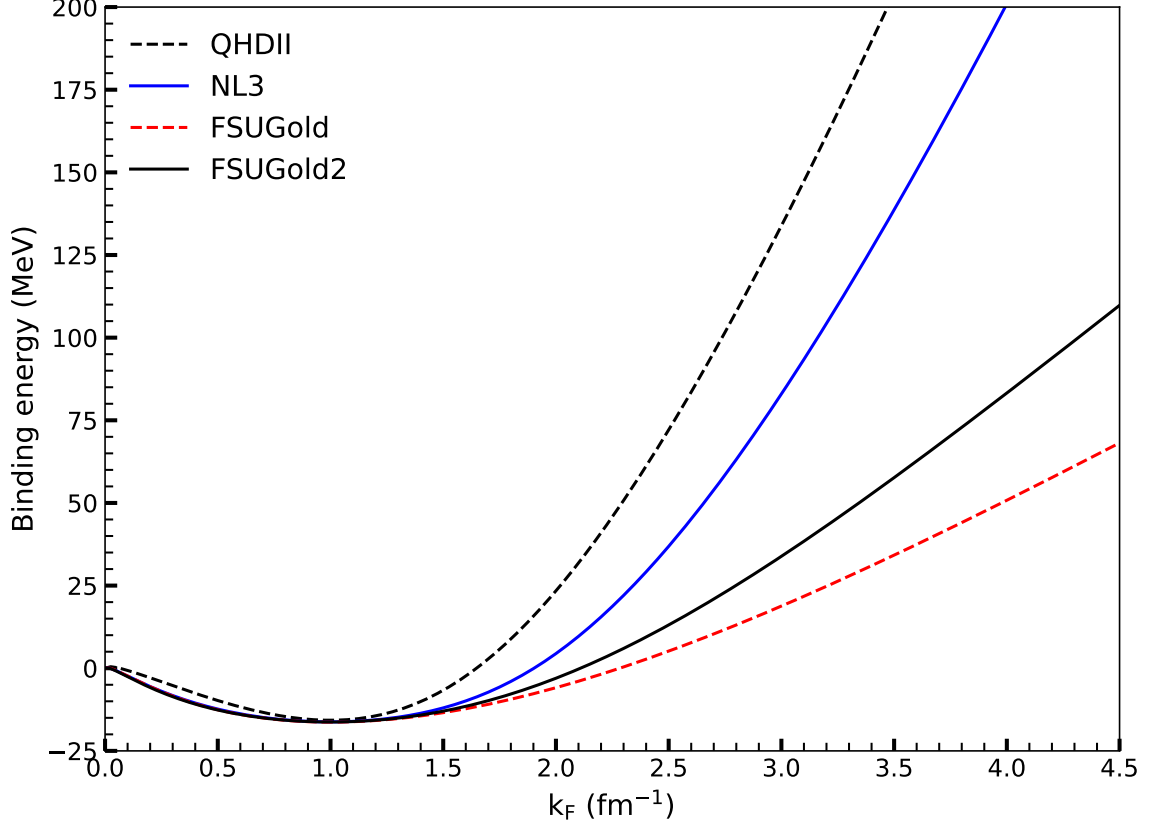


FIG. 4.1. The comparisons of the binding energies (MeV) of saturated nuclear matter as a function of the Fermi momentum (k_F) for different RMF models: QHDII, NL3, FSUGold and FSUGold2.

The binding energy curves for different RMF models are shown in Fig. 4.1. At saturation density ($\rho_0 \approx 0.148 \text{ fm}^{-3}$), the RMF models have approximately the same binding energies ($\approx 16 \text{ MeV}$) and deviations arise when moving away from the saturation point where there is no pressure. This behaviour is identical to observations made from Refs. [50, 54, 57, 65, 71].

4.4.1.3 Compression modulus

The compression modulus or incompressibility coefficient (K) defines how easily nuclear matter can be compressed. The curvature of the equation of state is characterized by the compression modulus at saturation, which influences the steepness of the equation of state. A rapid increase in pressure as a function of density is associated with a stiff equation of state, while a gradual increase in pressure as density increases is related to a soft equation of state [59, 69]. The compression modulus is expressed as [59]:

$$K = 9 \left[\rho^2 \frac{d^2}{d\rho^2} \left(\frac{\mathcal{E}}{\rho} \right) \right]_{\rho=\rho_0}. \quad (4.19)$$

where ρ represents the baryon density and $\mathcal{E}$ denotes the energy density. From Ref. [69], the value of K lies between 200 and 300 MeV with 234 MeV being the preferred value. QHDII estimation of the compression modulus value is 545 MeV from Refs. [54, 71], whereas for NL3, FSUGold and FSUGold2 yields the value of K to be 271.5 MeV, 230.0 MeV and 238.0 MeV, respectively, from Refs. [34, 57, 65]. These values are similar to the results in Table 4.2.

4.4.1.4 Symmetry energy

Stable nuclei having a low mass number ($A < 20$) have the same number of protons and neutrons, while stable nuclei in a region of high atomic mass numbers ($A > 100$), have more neutrons than protons [61]. As the number of protons increases, the Coulomb repulsive forces also increase, making the system less stable. The balance between protons and neutrons is explained by the symmetry energy (J). The contribution of the isospin asymmetry to the energy of the nucleus is represented by the symmetry energy coefficient (a_4) originating from the liquid-drop model of the nucleus and is given by [59, 72]:

$$a_4 = \frac{1}{2} \left(\frac{\partial^2 \mathcal{E}}{\partial t^2 \rho_b} \right)_{t=0}, \quad (4.20)$$

where t is given by:

$$t \equiv \frac{\rho_n - \rho_p}{\rho_b}. \quad (4.21)$$

and $\mathcal{E}$ is the energy density, $\rho_b = \rho_p + \rho_n$ is the baryon density, ρ_n and ρ_p represent the neutron and proton densities, respectively.

The values of the symmetry energy from Ref. [69] is 32.5 MeV, from Ref. [54] for the QHDII model, the symmetry energy is valued at 35 MeV. Refs. [34, 57, 65] yields the values for the symmetry energy for NL3, FSUGold, and FSUGold2 to be 37.3 MeV, 32.6 MeV and 37.6 MeV respectively. The values of the slope of the symmetry energy (L) for NL3, FSUGold and FSUGold2 are 118.2 MeV, 60.5 MeV and 112.8 MeV respectively from Refs. [57, 65]. The values for the symmetry energy in Table 4.2 are similar to the values from Refs. [34, 54, 57, 65].

4.4.1.5 Summary of results for properties of infinite nuclear matter

Table 4.2 displays the calculated bulk properties of infinite nuclear matter at saturation for the different RMF models investigated. Comparisons between the predictions and those found in the literature are in good agreement which means that the numerical code was applied correctly since published values were reproduced.

Table 4.2: Predicted properties of infinite nuclear matter at saturation using various RMF models (QHDII, NL3, FSUGold and FSUGold2) are presented. These include the saturation density (ρ_0), binding energy per nucleon (BE/A), incompressibility coefficient (K), the symmetry energy (J), and the gradient of symmetry energy (L).

Model	ρ_0 (fm ⁻³)	BE/A (MeV)	K (MeV)	J (MeV)	L (MeV)
QHDII	0.148	-15.73	546.59	34.98	70.06
NL3	0.148	-16.24	271.68	37.29	118.19
FSUGold	0.148	-16.30	230.02	32.59	60.52
FSUGold2	0.150	-16.27	237.88	37.59	112.72

As observed in Table 4.2, the values for the saturation density (ρ_0) for QHDII, NL3 and FSUGold yield the same values, 0.148 fm⁻³, while for FSUGold2 the saturation density is 0.150 fm⁻³. These values are similar to those found in the literature in Refs. [34, 54, 57, 65]. The same applies for the binding energy, the calculated values are within the values found in the published literature. The FSUGold model is observed to have a higher binding energy value (-16.30 MeV) and that QHDII has a lower value (-15.73 MeV), this is also captured from the literature

studied in Refs. [34, 54, 57, 65, 69–71].

QHDII has a higher compression modulus ($K = 546.59$ MeV) compared to the other RMF models [54]. The same behaviour is also shown in the literature. For the other RMF models, K is within the range of 230.02 MeV and 271.68 MeV. The symmetry energy (J) and its gradient (L) for NL3, FSUGold, and FSUGold2 are the same as those observed from Refs. [57, 65]. For QHDII, the estimated symmetry energy is also similar to the value in Ref. [69]. The bulk properties of saturated nuclear matter were reproduced and the predictions are within the same ranges as those found in the literature. The results for these properties of saturated nuclear matter gives us confidence that the numerical implementation of RMF is correct.

4.5 Properties of finite nuclear matter

Next, the different RMF models are applied to the nuclei of interest: ^{208}Pb , ^{40}Ca and ^{12}C to extract different nuclear properties, namely: the single-particle proton energies, RMF potentials, nucleon densities, and boundstate proton wave functions. These quantities carry meaningful information about the bound proton in the target nucleus which will be used to calculate the unpolarized triple differential cross sections and the polarization transfer observables for exclusive $(\vec{p}, 2\vec{p})$ knockout reactions.

4.5.1 Single-particle proton energies

Table 4.3 shows the single-particle proton energies for different nuclear states and target nuclei, namely: ^{12}C , ^{40}Ca and ^{208}Pb , as predicted by the various RMF models investigated in this study. Theoretical predictions are compared with experimental values to evaluate the reliability of the models and to see if the code has been implemented correctly. As observed in Table 4.3, the predictions from the different RMF models for the single-particle proton energies are $\approx \pm 4$ MeV from the experimental values from Refs. [2, 73, 74].

Table 4.3: Predictions of single-particle proton energies (in MeV) for different RMF models compared to the experimental data in Refs. [2, 73, 74] for corresponding nuclear states of target nuclei (^{208}Pb , ^{40}Ca and ^{12}C).

Target Nucleus	State (n_{lj})	Experiment	QHDII	NL3	FSUGold	FSUGold2
^{12}C	1p _{3/2}	-15.8	-12.8	-16.0	-13.7	-14.5
^{40}Ca	1d _{3/2}	-8.3	-6.6	-8.8	-9.3	-8.9
	2s _{1/2}	-11.6	-7.2	-9.7	-9.4	-9.8
^{208}Pb	3s _{1/2}	-8.0	-5.9	-8.0	-8.1	-8.1
	2d _{3/2}	-8.4	-7.0	-9.2	-9.2	-9.2
	1h _{11/2}	-9.4	-9.9	-10.2	-10.3	-10.1
	2d _{5/2}	-9.7	-9.2	-10.8	-10.9	-10.8
	1g _{7/2}	-11.5	-13.9	-15.0	-15.6	-15.2

The single-particle proton energies are tabulated in Table 4.3. For the 1p_{3/2} state of ^{12}C , the QHDII, FSUGold, and FSUGold2 models under-predict the experimental value for the single-particle proton energy, whereas the NL3 RMF model provides a reasonable prediction

since it is closer to the experimental data. The predictions for the $1d_{3/2}$ state in ^{40}Ca show reasonable predictions of the energies. QHDII under-predicts the experimental value and other RMF models predictions are closer to the data. For the $2s_{1/2}$ state in ^{40}Ca , all the RMF models under-predicts the single-particle proton energies. From all the RMF models, FSUGold2 predicts a value closer to the experimental data.

For the $3s_{1/2}$ state in ^{208}Pb , the experimental data is approximately the same as the predictions from the NL3, FSUGold and FSUGold2 RMF models, while the QHDII prediction is less than the experimental value. The NL3, FSUGold and FSUGold2 RMF models yield similar predictions for the $2d_{3/2}$, $1h_{11/2}$ and $2d_{5/2}$ states and over-predict the single-particle proton energies for those states whereas QHDII has lower values than the experimental values for both the $2d_{3/2}$ and $2d_{5/2}$ states. For the $1h_{11/2}$ state, all RMF models share similar values of the single-particle proton energies, and tend to have higher values as compared to the experiment. All RMF models over-predict the single-particle proton energies for the $1g_{7/2}$ state, with FSUGold having the higher value.

Fig. 4.2 shows the single-particle proton energies for the nuclear states of interest of ^{208}Pb extracted from the different RMF models.

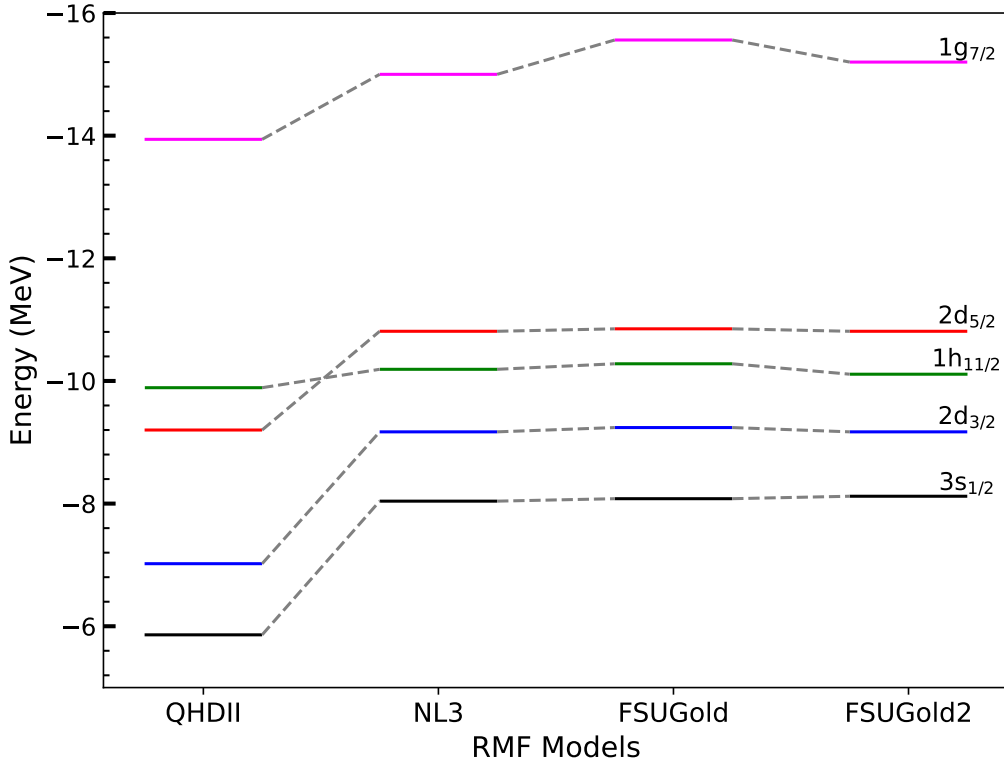


FIG. 4.2. Single-particle proton energies for different nuclear states of ^{208}Pb for various RMF models: QHDII, NL3, FSUGold and FSUGold2.

The proton energy levels of ^{208}Pb for the nuclear states of interest and various RMF models are shown in Fig. 4.2. As observed, QHDII has lower energy levels in comparison with the respective states for other RMF models. There is a swap between the $2d_{5/2}$ and $1h_{11/2}$ states for the QHDII results relative to the other RMF models (NL3, FSUGold, and FSUGold2). Another observation made from Fig. 4.2 is that, for the $1g_{7/2}$ state, FSUGold shows a higher single proton energy level. These results are similar to those found in the literature in Ref. [71]. The extracted single-particle proton energies from this study are similar to the published values in

Refs. [2, 73, 74]. This gives confidence that the numerical code has been implemented properly, since the energies were reproduced for the different nuclei and their respective single-particle states from the available experimental data for the unpolarized triple differential cross sections and the analyzing powers.

From the predictions given of the single-particle proton energies in Table 4.3, the results are similar for the RMF models applied in this study. The FSUGold2 will be treated as the reference point for other models (QHDII, NL3, and FSUGold) since it is better constrained to experimental data of the properties of saturated nuclear matter. These other RMF models (QHDII, NL3, and FSUGold) are presented to show the variations of the FSUGold2 model for the nuclei of interest and their respective single-particle states. The FSUGold2 model will be represented by the solid black line and the grey shaded band encloses the other RMF models: QHDII, NL3 and FSUGold, on Figs. 4.3 - 4.7.

4.5.2 Relativistic Mean Field potentials

The RMF potentials (MeV), namely, the scalar (σ), vector (ω) and the isovector (ρ) meson potentials obtained from different RMF models are depicted in Fig. 4.3 for different spherical nuclei, namely: ^{208}Pb , ^{40}Ca and ^{12}C .

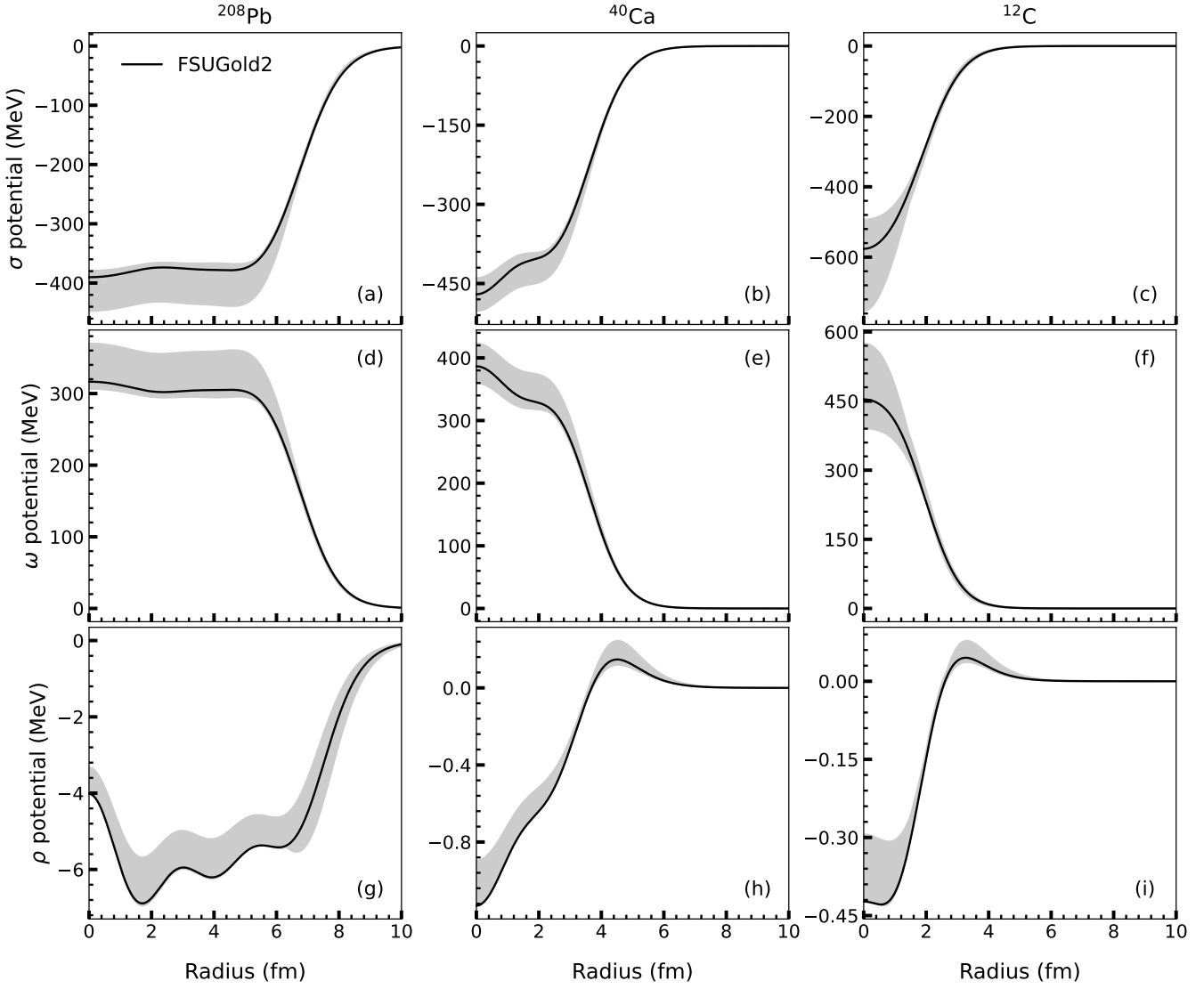


FIG. 4.3. RMF potentials (MeV) for scalar (σ), vector (ω) and isovector (ρ) mesons, of different RMF models for varying nuclei against the nuclear radius (fm). The FSUGold2 model is represented by a solid black line and other RMF models: QHDII, NL3 and FSUGold are enclosed in the grey shaded band.

From Fig. 4.3, the panels (a), (b) and (c) represents the scalar (σ) potentials for ^{208}Pb , ^{40}Ca and ^{12}C respectively. The scalar potential is negative, showing the attractive nature of the scalar field, and decreases towards the center of the nucleus. Panels (d), (e) and (f) depict the vector (ω) potential for ^{208}Pb , ^{40}Ca and ^{12}C . The vector potential is large at the center of the nucleus and decreases toward the surface of the nucleus. This vector field is positive, indicating its repulsive nature that counteracts the scalar attraction. The isovector (ρ) potentials are represented on panels (g), (h) and (i). The ρ potential is relatively weak compared to the σ and ω potentials. For a nucleus with neutron-proton asymmetry (^{208}Pb), the isovector potential becomes significant (> -2 MeV) and becomes insignificant for $Z = N$ (^{40}Ca and ^{12}C) where Z and N represent the proton and neutron numbers respectively. The Wood-Saxon-like potentials are depicted for scalar and vector potentials of ^{208}Pb .

4.5.3 Relativistic Mean Field densities

Fig. 4.4 illustrates the predictions of relativistic nucleon densities for target nuclei of interest (^{208}Pb , ^{40}Ca and ^{12}C) from different RMF models (QHDII, NL3, FSUGold and FSUGold2). The predictions include vector proton densities, vector neutron densities, scalar proton densities, and scalar neutron densities.

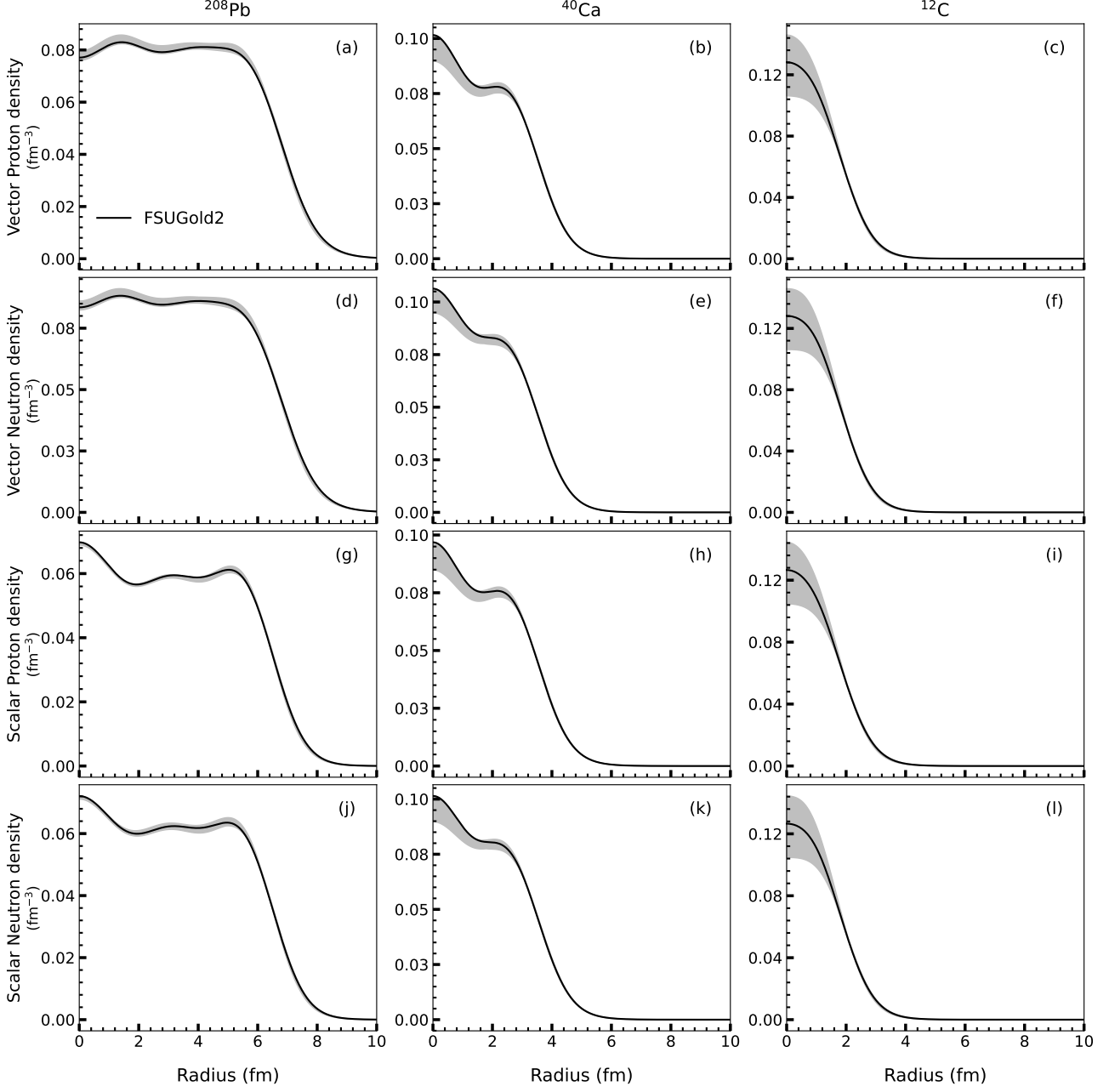


FIG. 4.4. RMF densities (fm^{-3}) for different target nuclei: ^{208}Pb , ^{40}Ca and ^{12}C , as functions of the nuclear radius (fm) for different RMF models (QHDII, NL3, FSUGold and FSUGold2). The RMF representations are the same as those used in Fig. 4.3 for the RMF potentials.

The different nucleon densities (fm^{-3}) for different RMF models plotted as functions of the nuclear radius (fm) for different target nuclei are shown in Fig. 4.4. The results for these densities are similar to those found in the literature from Refs. [44, 54, 64, 66, 71]. From Fig. 4.4, the panels (a), (b) and (c) show the vector proton densities for ^{208}Pb , ^{40}Ca and ^{12}C , respectively. Vector proton densities illustrate the distributions of protons within the nucleus in the vector

field which contribute to the repulsive nature of the meson (ω) field. Panels (d), (e), and (f) show the vector neutron densities also contributing to the vector field. The scalar proton densities are shown in panels (g), (h), and (i), while the scalar neutron densities are shown in panels (j), (k), and (l). These scalar densities illustrate the distributions of protons and neutrons to the scalar meson field (σ) which contributes to its attractive nature and effective mass of the nucleus.

From all the panels shown in Fig. 4.4, a strong central density is observed followed by exponentially decaying tails. This feature is also observed from literature from Refs. [44, 54, 64, 66, 71]. ^{208}Pb shows higher values for the scalar and vector neutron densities compared to the scalar and vector proton densities, this comes from the fact that ^{208}Pb is a neutron-rich nucleus. For ^{40}Ca and ^{12}C , the scalar and vector proton densities exhibit the same values as the scalar and vector neutron densities, since they each have equal numbers of protons and neutrons. These densities provide information about the spatial distribution of the protons and neutrons, which affect the binding energy and stability of the nucleus. Insignificant variations from the different applied RMF models are also observed for the investigated nuclei.

4.5.4 Relativistic proton boundstate wave functions

4.5.4.1 Single-particle states in ^{208}Pb

The relativistic proton boundstate wave functions were extracted using the RMF models (QHDII, NL3, FSUGold and FSUGold2) and the Figs. 4.5 - 4.7 display the structure of the different proton radial wave functions for different occupied nuclear states of interest for ^{208}Pb , ^{40}Ca and ^{12}C . Fig. 4.5 shows the proton radial wave functions of ^{208}Pb for states: $3s_{1/2}$, $2d_{3/2}$, $1h_{11/2}$, $2d_{5/2}$ and $1g_{7/2}$.

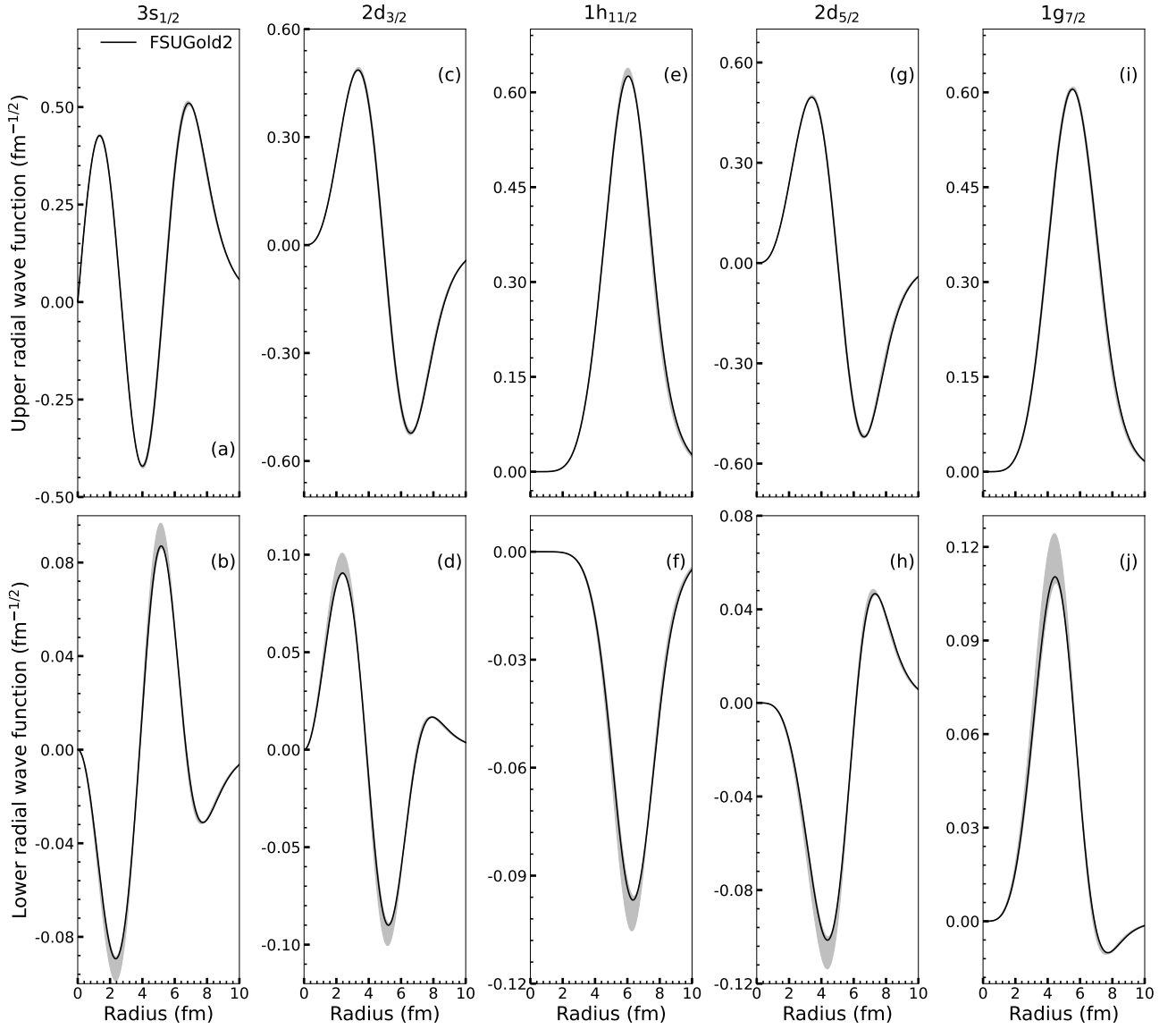


FIG. 4.5. ^{208}Pb boundstate proton radial wave functions for the different nuclear states: $3s_{1/2}$, $2d_{3/2}$, $1h_{11/2}$, $2d_{5/2}$ and $1g_{7/2}$ as functions of the nuclear radius (fm) for the upper and lower proton radial wave functions. The upper panels represent the upper proton radial wave functions while the lower panels show the lower proton radial wave functions for various RMF models. The solid black line represents the FSUGold2 model and the other models: QHDII, NL3 and FSUGold, are enclosed within the grey shaded band.

Fig. 4.5 represents the proton boundstate wave functions of ^{208}Pb for different nuclear states. The upper row shows the upper radial proton boundstate wave functions whereas the lower row illustrates the lower proton boundstate wave functions. Panels (a) and (b) display the wave functions for the $3s_{1/2}$ state. The $2d_{3/2}$ state wave functions are shown by panels (c) and (d), and panels (e) and (f) illustrate the $1h_{11/2}$ nuclear state. Panels (g) and (h) show the $2d_{5/2}$ and the $1g_{7/2}$ state represented in panels (i) and (j). In Fig. 4.5, the upper radial proton boundstate wave functions display a dominant nature for all nuclear states compared to the lower proton boundstate wave functions. The lower radial wave functions have small contributions arising from relativistic effects on the lower component of the Dirac spinor. The RMF models show insignificant variations between the predictions of the different proton boundstate wave functions for the respective nuclear states in ^{208}Pb .

4.5.4.2 Single-particle states in ^{40}Ca

The different relativistic proton boundstate wave functions extracted from different RMF models for the nuclear states in ^{40}Ca are shown in Fig. 4.6.

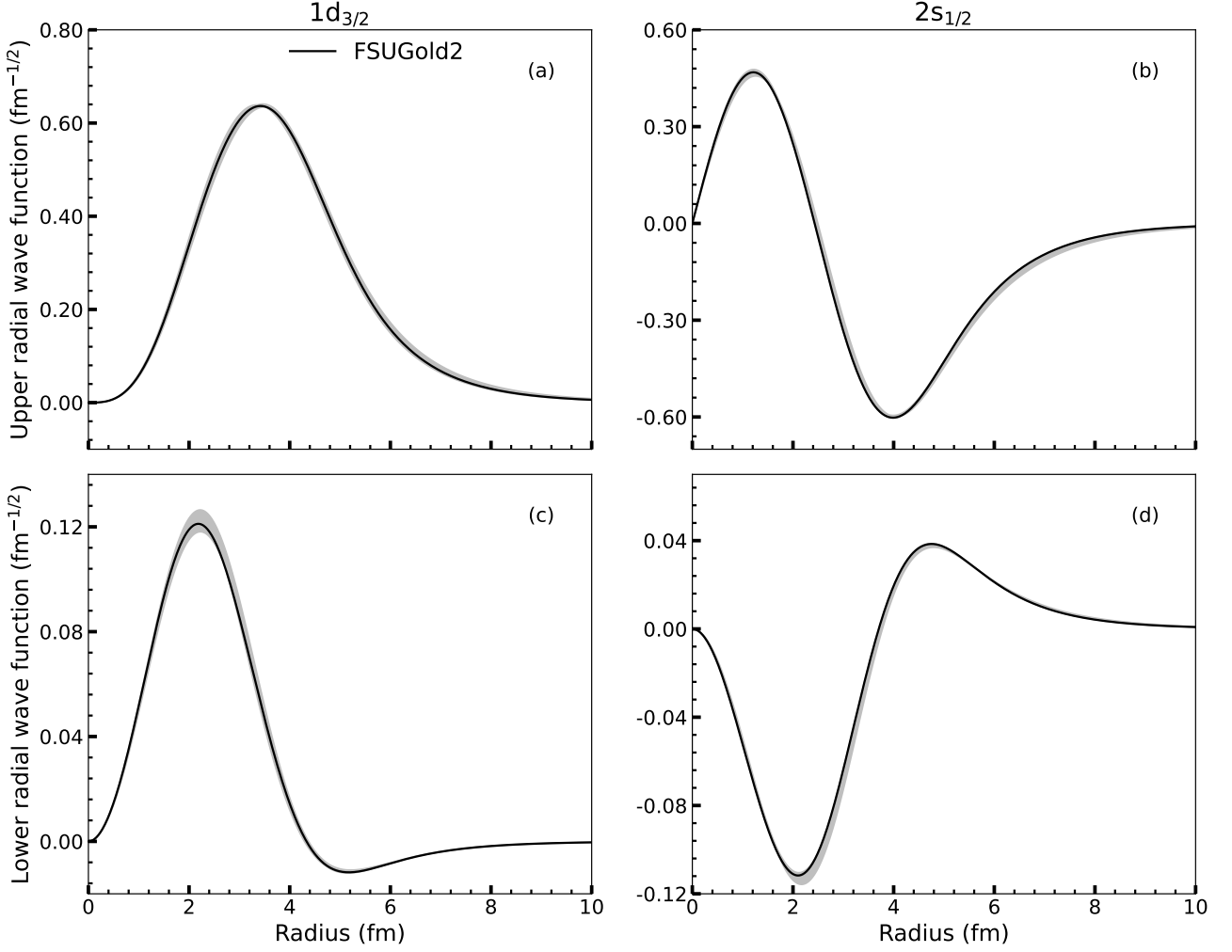


FIG. 4.6. ^{40}Ca upper and lower boundstate proton radial wave functions for the $1d_{3/2}$ and $2s_{1/2}$ states as functions of the nuclear radius (fm). The RMF representations are the same as for ^{208}Pb in Fig. 4.5.

The upper proton radial wave functions for the nuclear states of ^{40}Ca ($1d_{3/2}$ and $2s_{1/2}$) are shown in panels (a) and (b), whereas the lower radial boundstate wave functions are represented in panels (c) and (d). The major contributions arise from the upper radial wave functions for both nuclear states as compared to the lower wave functions. From Fig. 4.6, the different RMF models show insignificant variations. The $1d_{3/2}$ boundstate wave functions have a similar structure to that found in Ref. [47].

4.5.4.3 Single-particle state in ^{12}C

Fig. 4.7 shows the proton boundstate radial wave functions extracted from different RMF models for the $1p_{3/2}$ state in ^{12}C .

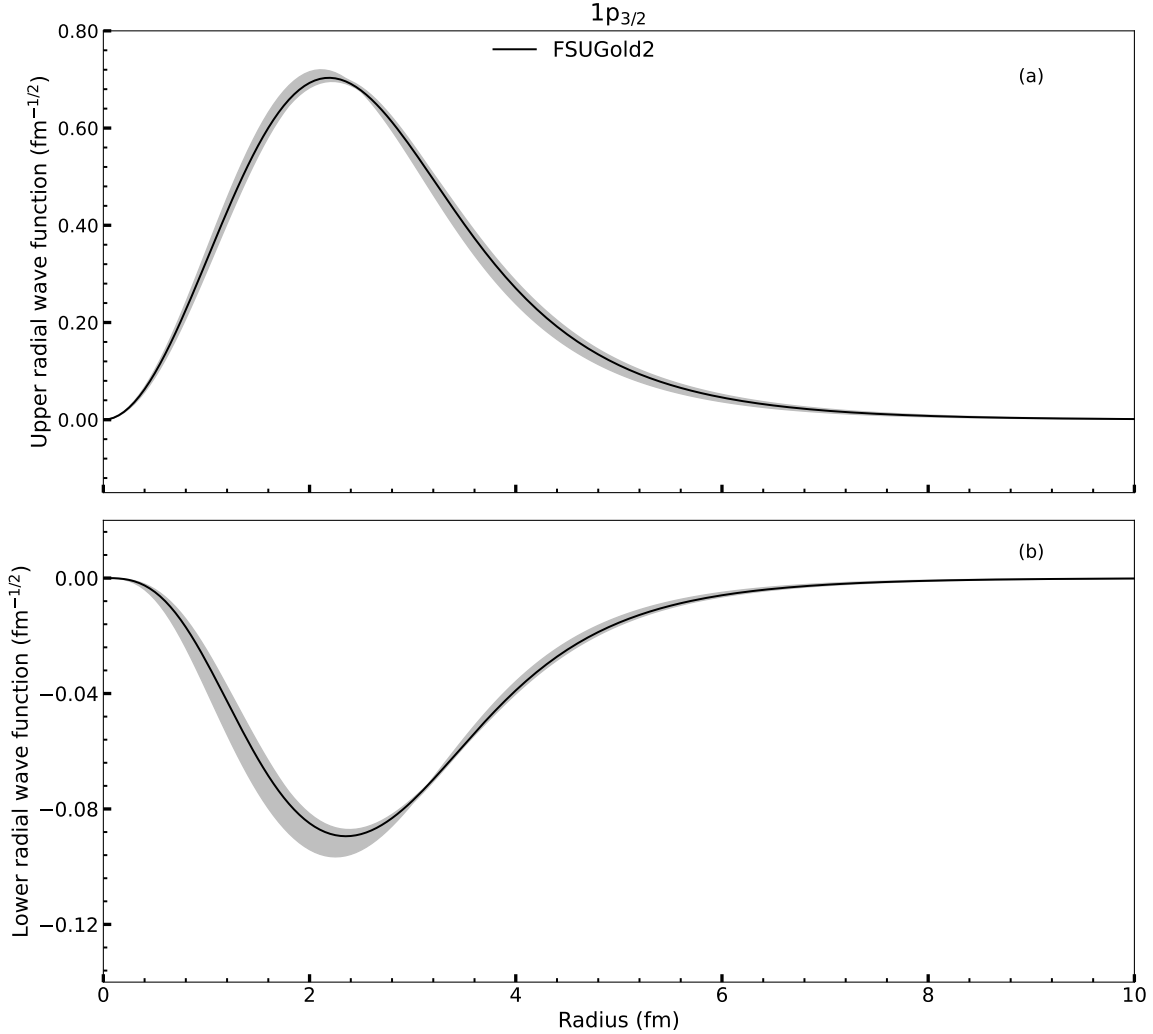


FIG. 4.7. ^{12}C upper and lower radial wave functions for the $1p_{3/2}$ state in ^{12}C as a function of the nuclear radius (fm). The RMF representations are the same as in Fig. 4.5 for ^{208}Pb .

The upper and lower proton radial boundstate wave functions of $1p_{3/2}$ in ^{12}C are shown in panels (a) and (b), respectively. Similar to the boundstate wave functions of ^{208}Pb and ^{40}Ca , the major contribution arises from the upper radial wave functions compared to the lower radial wave functions. Insignificant variations are observed for the different RMF models studied in this work.

The proton radial wave functions for the target nuclei of interest and nuclear states are represented from Figs. 4.5 - 4.7. As observed from the figures, there are insignificant variations between the boundstate proton wave functions obtained using different RMF models. The choice for the nuclear states shown was motivated by the availability of the experimental data which will be compared with the relativistic plane wave predictions for the unpolarized triple differential cross sections and the analyzing powers of the target nuclei of interest. These boundstate wave functions are essential in the computation of the observables since they carry information about the bound proton before being ejected from the target nucleus.

4.6 Summary

The relativistic boundstate proton radial wave functions are extracted for different RMF models (QHDII, NL3, FSUGold and FSUGold2). Different RMF models were used to calculate the properties of finite nuclei matter for different target nuclei (^{208}Pb , ^{40}Ca and ^{12}C), namely the relativistic nuclear densities, potentials, boundstate proton radial wave functions and single-particle proton energies. Insignificant variations between the RMF models are observed for all nuclei of interest. Since the relativistic boundstate proton wave functions were extracted, all the ingredients required to evaluate the relativistic transition amplitude in Eq. (3.3) shown in Section 3.2 are determined. This brings the study a step closer to the computation of predictions of the relativistic plane wave polarization transfer observables and the unpolarized triple differential cross section. Chapter 5 presents the results and the discussion for the $(\vec{p}, 2\vec{p})$ complete set of polarization transfer observables and the unpolarized triple differential cross sections for ^{208}Pb , ^{40}Ca and ^{12}C for selected kinematical conditions.

Chapter 5

Results and Discussion

5.1 Introduction

In this Chapter, the objectives stated in Chapter 1 are addressed by presenting the results of this study. The main objective was to develop and apply a theoretical nuclear model based on RPWIA for the predictions of the unpolarized triple differential cross sections [σ in units of $\mu\text{b sr}^{-2} \text{ MeV}^{-1}$] and the complete set of polarization transfer observables (D_{ij}) for $(\vec{p}, 2\vec{p})$ knockout reactions from stable spherical nuclei. The unpolarized triple differential cross section provides information about the boundstate wave functions expressed by Eq. (3.14) extracted from various RMF models: QHDII, NL3, FSUGold, and FSUGold2. The sensitivity of the different polarization transfer observables to these RMF models is studied, and the best-calibrated RMF models (FSUGold and FSUGold2) are investigated to determine if they can yield better predictions compared to the QHDII and NL3 models.

A systematic study from a heavy target nucleus, ^{208}Pb , through ^{40}Ca to a lighter target nucleus, ^{12}C , is conducted for different single-particle states, and the regions where the RPWIA model predicts the experimental data are determined. The choice of applied kinematics is based on the availability of the $(\vec{p}, 2\vec{p})$ knockout reaction experimental data of the unpolarized triple differential cross section and the analyzing power (A_y) at coplanar scattering angles, $\theta_{a'} = 32.5^\circ$ and $\theta_b = -50.0^\circ$ and incident proton energies of 392 MeV at RCNP (Japan) from Ref. [2]. At 392 MeV, the RIA which assumes that the interaction between the incident proton and the target proton in the nucleus resembles the interaction of protons in free space is valid since the incident proton energies are much greater than the binding energies of the single-particle states of interest.

A FORTRAN computer code called Relpw.f which calculates the unpolarized triple differential cross sections and the complete set of polarization transfer observables given by Eqs. (3.43) and (3.44) was developed, tested and implemented. The code synopsis is given in the Appendix 6.3. The calculations were carried out for the stable spherical nuclei (^{208}Pb , ^{40}Ca and ^{12}C) for different single-particle states given in Table 5.1. Currently, the only available data are that of the unpolarized triple differential cross sections and the analyzing power (polarization transfer observable) for nuclei of interest. There are no data for the other polarization transfer observables, and this study serves to motivate measurements of these observables (D_{ij}).

The kinematic regions where the RPWIA model predictions quantitatively describe the data are identified. Furthermore, within those kinematic regions, the predictions of the RPWIA model are compared to the predictions of the RDWIA model found in Refs. [14, 22, 27, 47] as a way of investigating how the RPWIA model performs against the RDWIA model for those kinematic regions. For the RDWIA model, a FORTRAN code, Reldw.f, from Hillhouse in Refs. [14, 23, 27], which also adopts the IA1 parameterization of the NN interaction was used. Within the RDWIA model, the global optical potentials, Energy-Dependent Mass-Independent (EDADI) parameter set is employed [5, 14, 25, 75]. Comparisons between the RPWIA and RDWIA predictions for the complete set of polarization transfer observables are made to determine whether similar predictions can be achieved through an application of a model which neglects the distortion effects on the incident and outgoing protons from the nucleus.

The predictions of the complete set of polarization transfer observables over the entire range relative to unpolarized triple differential cross sections are provided. The focus is on the kinetic energy of the outgoing proton [$T_{a'}$ (MeV)] in a certain range where the predictions of the RPWIA model are relatively close to the experimental data. It is worth noting that the results presented in this Chapter are for the proton knockout from the nuclear states where the experimental data are available.

5.2 Knockout reactions in various nuclei

The RPWIA model was applied to predict the unpolarized triple differential cross sections and the complete set of polarization transfer observables for stable spherical nuclei. The results are presented for incident proton kinetic energies of 392 MeV and coplanar scattering angles ($\theta_{a'} = 32.5^\circ$ and $\theta_b = -50.0^\circ$) and for various RMF models. The same kinematics are applied in all nuclei and their respective nuclear states given in Table 5.1. The RPWIA predictions of the unpolarized triple differential cross sections and the analyzing powers (A_y) are compared with the experimental data from Ref. [2]. Furthermore, these predictions (RPWIA) will also be compared to the RDWIA predictions of Hillhouse *et al.* [14, 27, 47] for the complete set of polarization transfer observables for regions where the RPWIA predictions are closer to data.

These regions are identified to be around the maximum peak of the unpolarized triple differential cross sections, therefore, the complete set of polarization transfer observables will be presented within the ranges of ± 10 MeV from the peaks of these cross sections, for each state with the kinetic energies of the outgoing protons, $T_{a'}$ (MeV), given in Table 5.2. For all results (Figs. 5.1 - 5.5), RMF model presentations are such that the solid black line represents the RPWIA predictions from applying the FSUGold2 model, and the grey shaded area represents the QHDII, NL3 and FSUGold models. The RDWIA predictions are represented by the dashed line and the experimental data are from Ref. [2]. The sensitivity and the effects of these RMF models on the polarization transfer observables are studied.

The RPWIA predictions for the single-particle states: $3s_{1/2}$, $2d_{3/2}$, $1h_{11/2}$, $2d_{5/2}$ and $1g_{7/2}$ in ^{208}Pb , are shown in Figs. 5.1 - 5.5. For ^{40}Ca , the results for the $1d_{3/2}$ and $2s_{1/2}$ states are presented in Figs. 5.6 - 5.7, while for ^{12}C , they are shown in Fig. 5.8 for the $1p_{3/2}$ state. The discussion and analysis of the results are covered in Section 5.6. Table 5.1 presents the nuclei of interest and their respective states.

Table 5.1: The spherical stable nuclei of interest and their single-particle states.

Nucleus	Single-particle states
^{12}C	$1p_{3/2}$
^{40}Ca	$1d_{3/2}, 2s_{1/2}$
^{208}Pb	$3s_{1/2}, 2d_{3/2}, 1h_{11/2}, 2d_{5/2}, 1g_{7/2}$.

5.3 Knockout from single-particle states in ^{208}Pb

In this section, the results for the states in ^{208}Pb are presented in Figs. 5.1 - 5.5. The results of the $3s_{1/2}$ state are shown in Fig. 5.1, with the $2d_{3/2}$ state shown in Fig. 5.2, $1h_{11/2}$ state in Fig. 5.3, followed by the $2d_{5/2}$ and $1g_{7/2}$ states presented in Figs. 5.4 and 5.5, respectively.

5.3.1 $3s_{1/2}$ state

The RPWIA predictions for the unpolarized triple differential cross section and polarization transfer observables for the $3s_{1/2}$ state of ^{208}Pb are shown in Fig. 5.1.

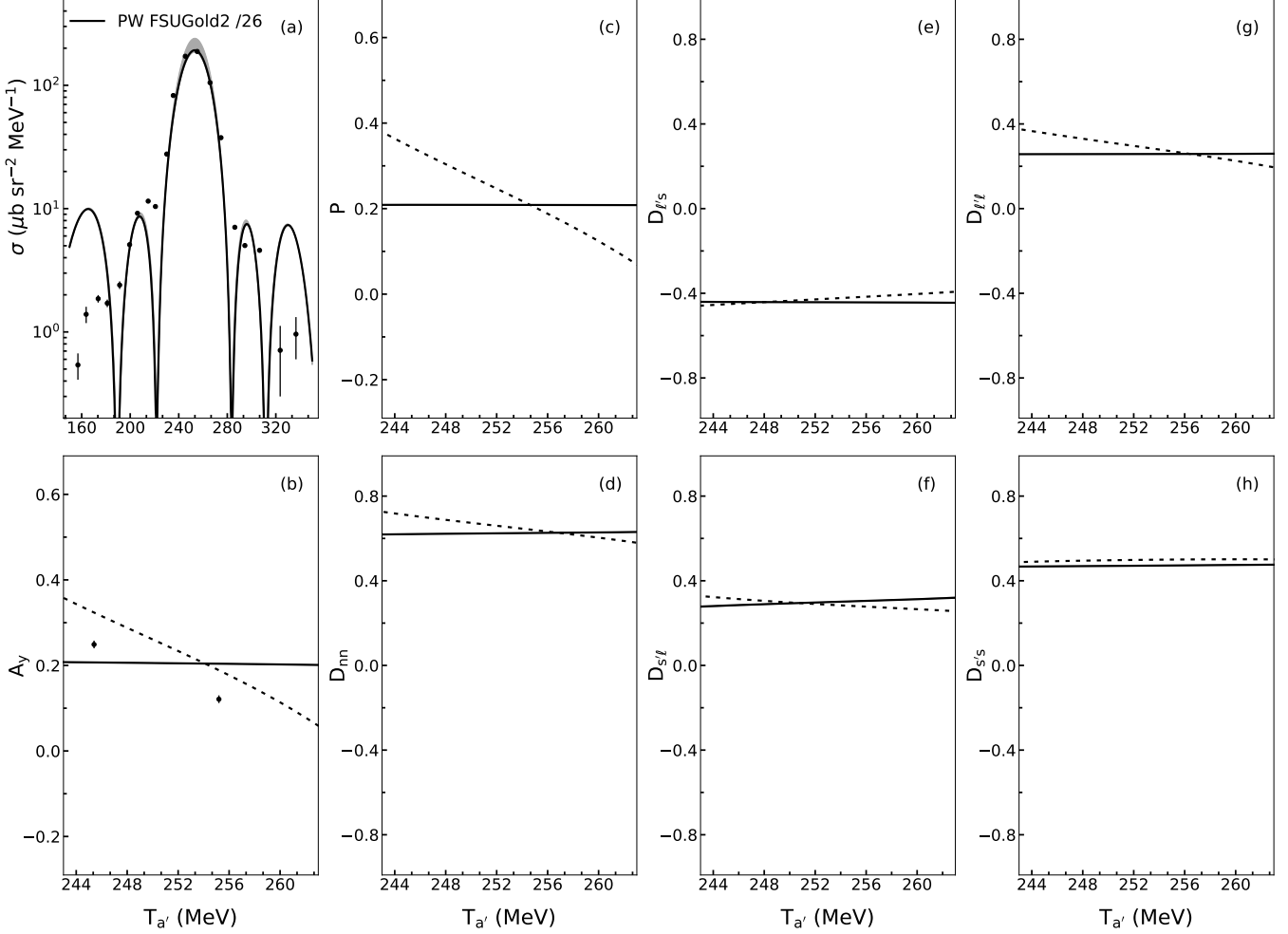


FIG. 5.1. $3s_{1/2}$ state for ^{208}Pb unpolarized triple differential cross σ ($\mu\text{b sr}^{-2} \text{MeV}^{-1}$), and polarization transfer observables (D_{ij}) versus the kinetic energy of the outgoing proton [$T_{a'}$ (MeV)] for various RMF models: QHDII, NL3, FSUGold and FSUGold2. The solid black line represents the RPWIA results for the FSUGold2 RMF model for the $3s_{1/2}$ boundstate wave functions. The shaded band shows the variations in the RPWIA predictions of the RMF models (QHDII, NL3, FSUGold). The dashed line represents the RDWIA predictions using FSUGold2 for the boundstate wave functions. The coplanar scattering angle pairs are: $\theta_{a'} = 32.5^\circ$ and $\theta_b = -50.0^\circ$ and the incident proton kinetic energy is 392 MeV. The data are from Ref. [2].

5.3.2 $2d_{3/2}$ state

The RPWIA unpolarized triple differential cross section and the complete set of polarization transfer observables of ^{208}Pb for the $2d_{3/2}$ state are shown in Fig. 5.2.

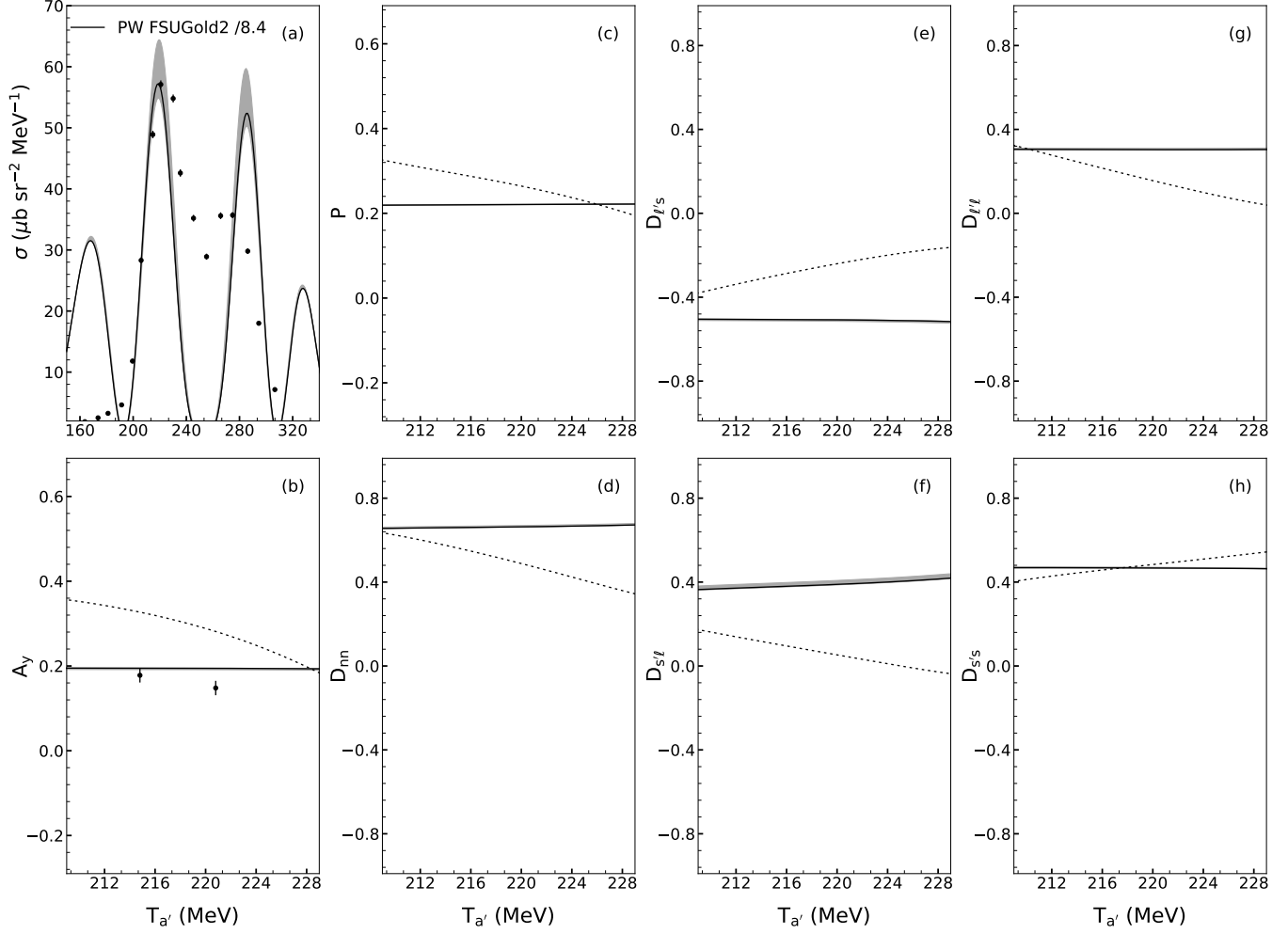


FIG. 5.2. The same as for Fig. 5.1, except for the $2d_{3/2}$ state in ^{208}Pb .

5.3.3 $1h_{11/2}$ state

The RPWIA unpolarized triple differential cross section and the complete set of polarization transfer observables of ^{208}Pb for the $1h_{11/2}$ state are shown in Fig. 5.3.

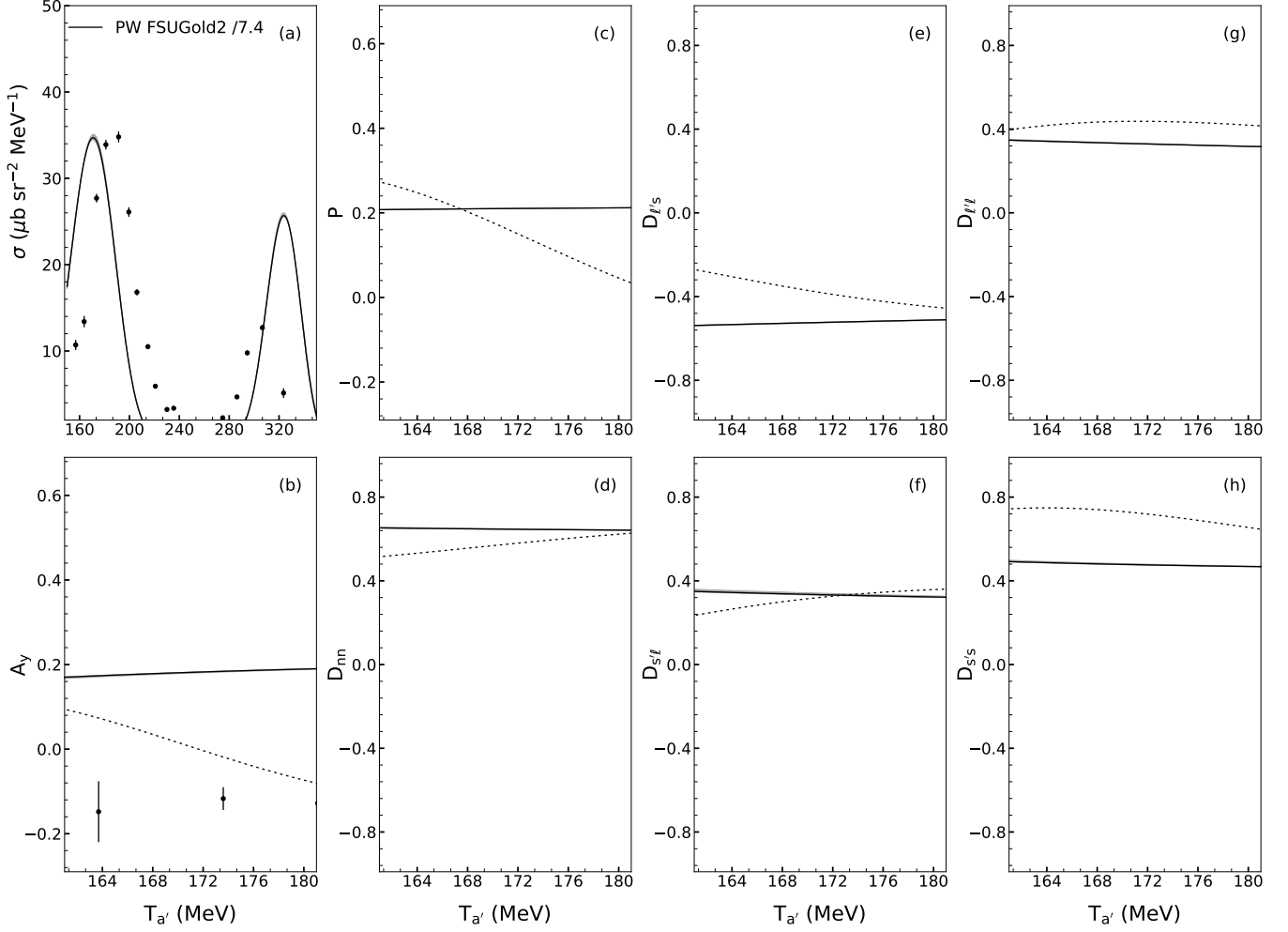


FIG. 5.3. The same for Fig. 5.1, except for the $1h_{11/2}$ state in ^{208}Pb .

5.3.4 $2d_{5/2}$ state

The RPWIA unpolarized triple differential cross section and the complete set of polarization transfer observables of ^{208}Pb for the $2d_{5/2}$ state are shown in Fig. 5.4.

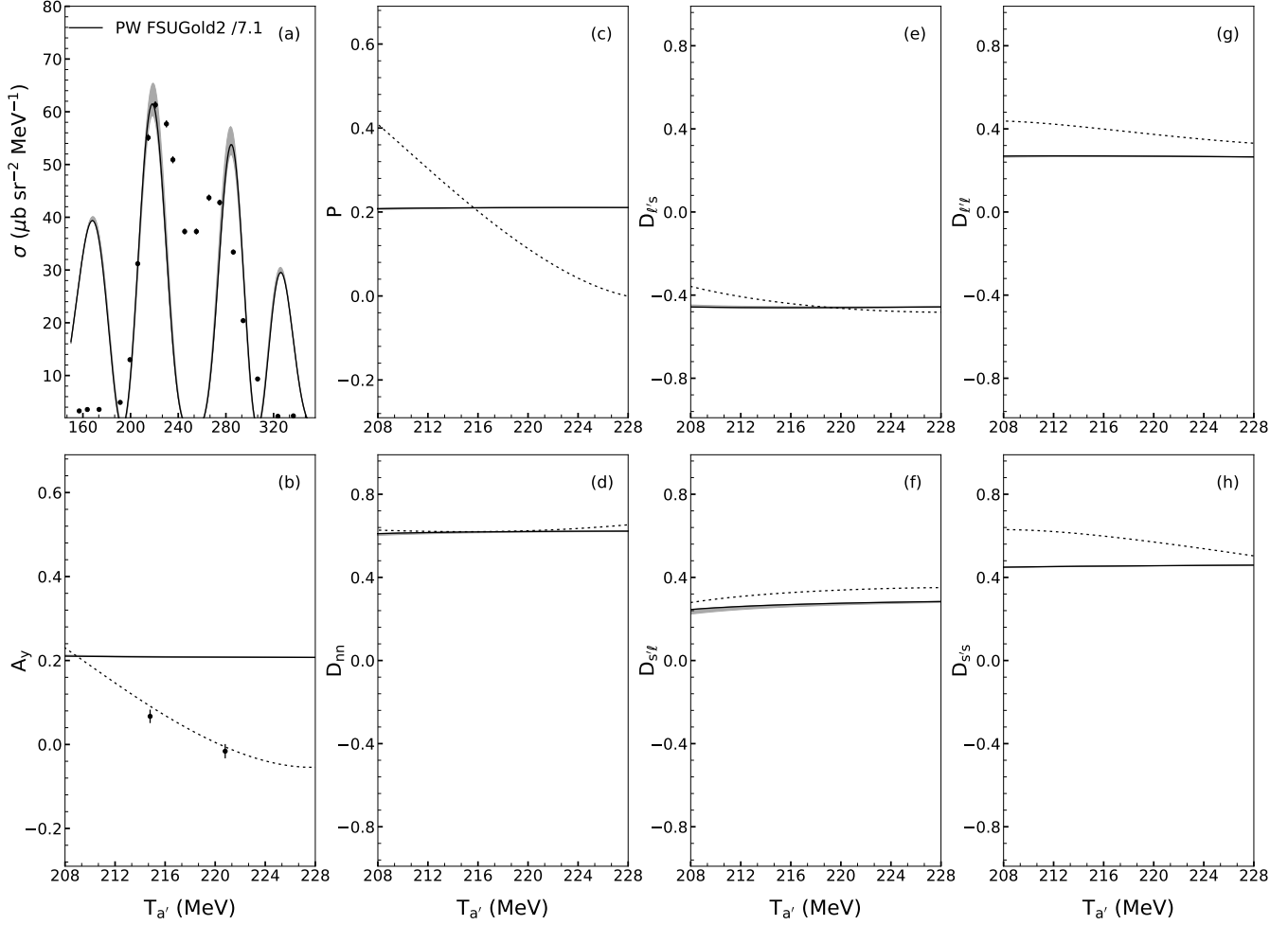


FIG. 5.4. The same as for Fig. 5.1, except for the $2d_{5/2}$ state in ^{208}Pb .

5.3.5 $1g_{7/2}$ state

The RPWIA unpolarized triple differential cross section and the complete set of polarization transfer observables of ^{208}Pb for the $1g_{7/2}$ state are shown in Fig. 5.5.

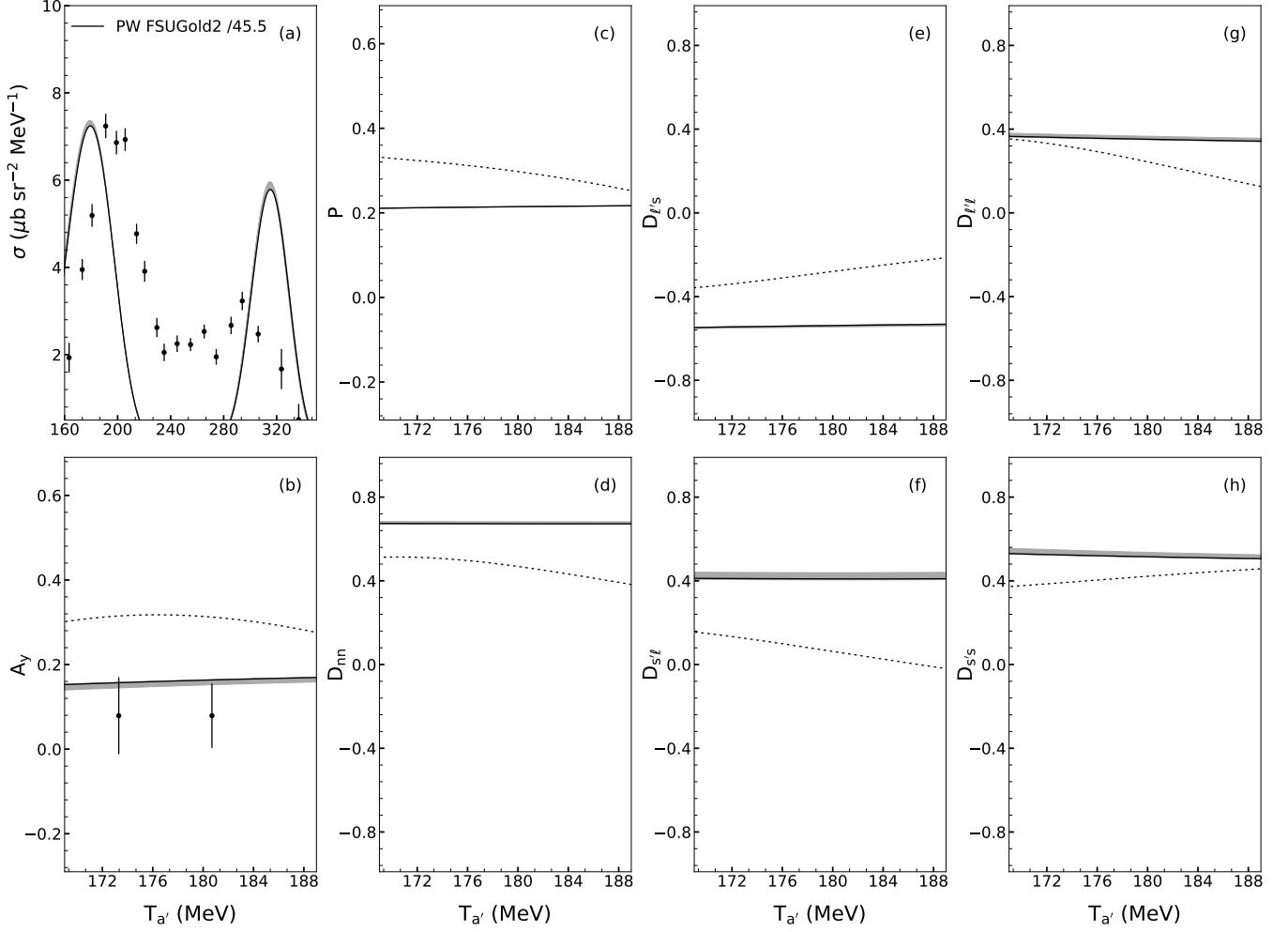


FIG. 5.5. The same as for Fig. 5.1, except for the $1g_{7/2}$ state in ^{208}Pb .

5.4 Knockout from single-particle states in ^{40}Ca

Another nucleus of interest investigated in this study is ^{40}Ca . The same kinematic conditions are applied as those given in section 5.2.

5.4.1 $1d_{3/2}$ state

Fig. 5.6 shows the RPWIA unpolarized triple differential cross section and the complete set of polarization transfer observables for the $1d_{3/2}$ state in ^{40}Ca .

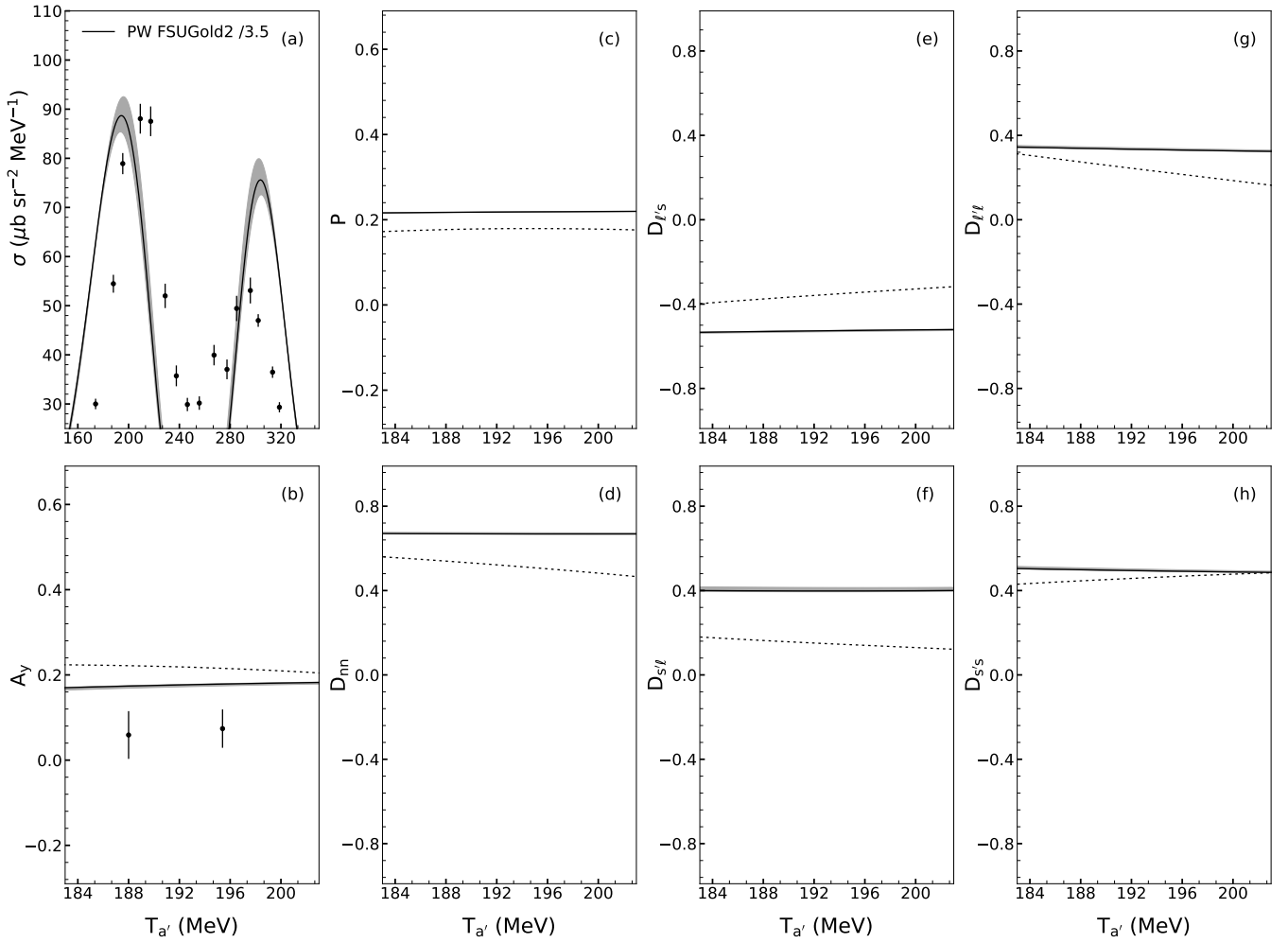


FIG. 5.6. The same as Fig. 5.1, except for the $1d_{3/2}$ state of ^{40}Ca .

5.4.2 $2s_{1/2}$ state

The RPWIA unpolarized triple differential cross section and the polarization transfer observables for ^{40}Ca for the $2s_{1/2}$ state are shown in Fig. 5.7.

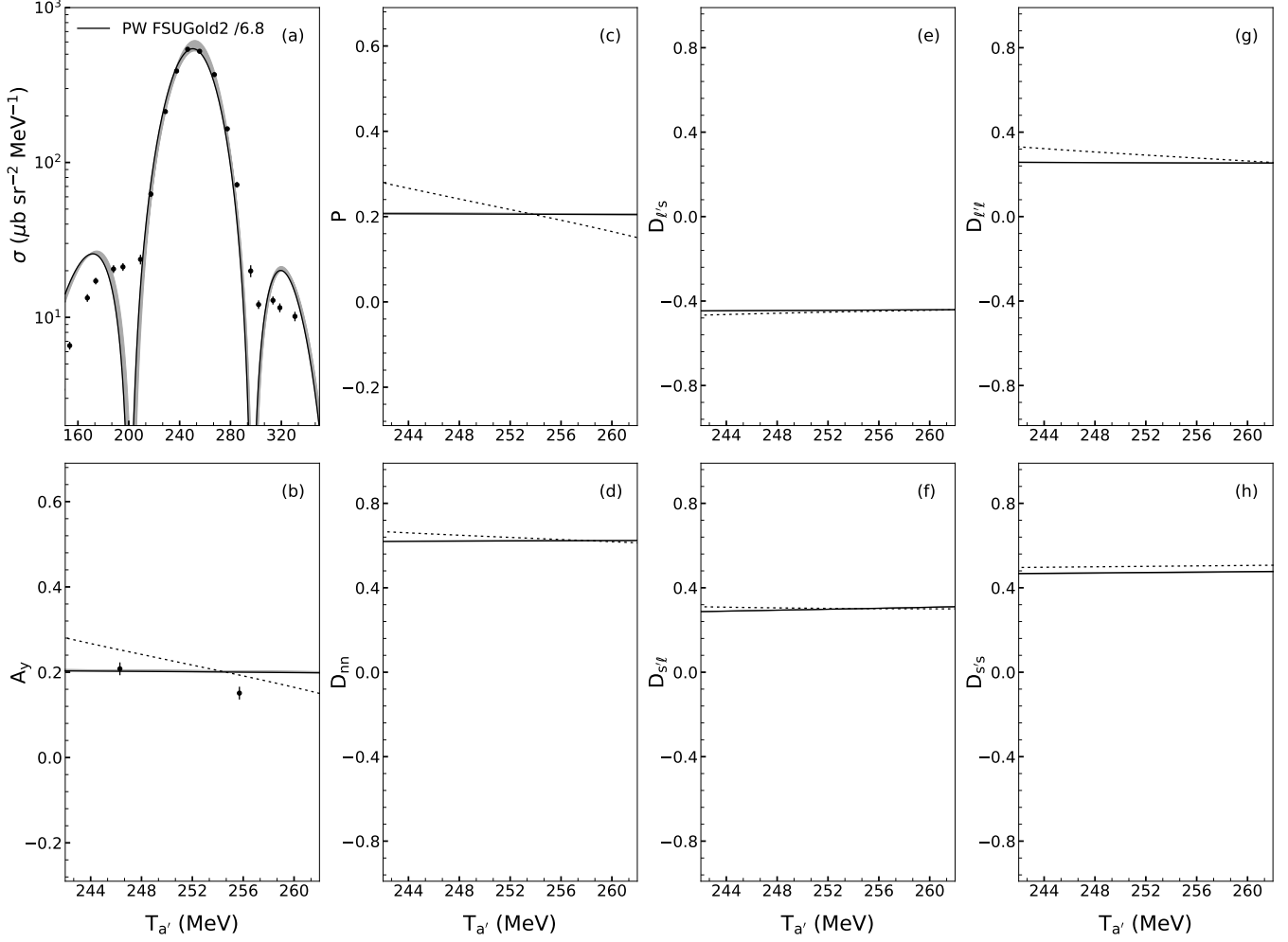


FIG. 5.7. The same as Fig. 5.1, except for the $2s_{1/2}$ state of ^{40}Ca .

5.5 Knockout from single-particle state in ^{12}C

In this section, the RPWIA predictions for the single-particle state, $1p_{1/2}$ for ^{12}C are presented in Fig. 5.5.

5.5.1 $1p_{1/2}$ state

The RPWIA unpolarized triple differential cross sections and polarization transfer observables of the $1p_{1/2}$ state in ^{12}C are shown in Fig. 5.8.

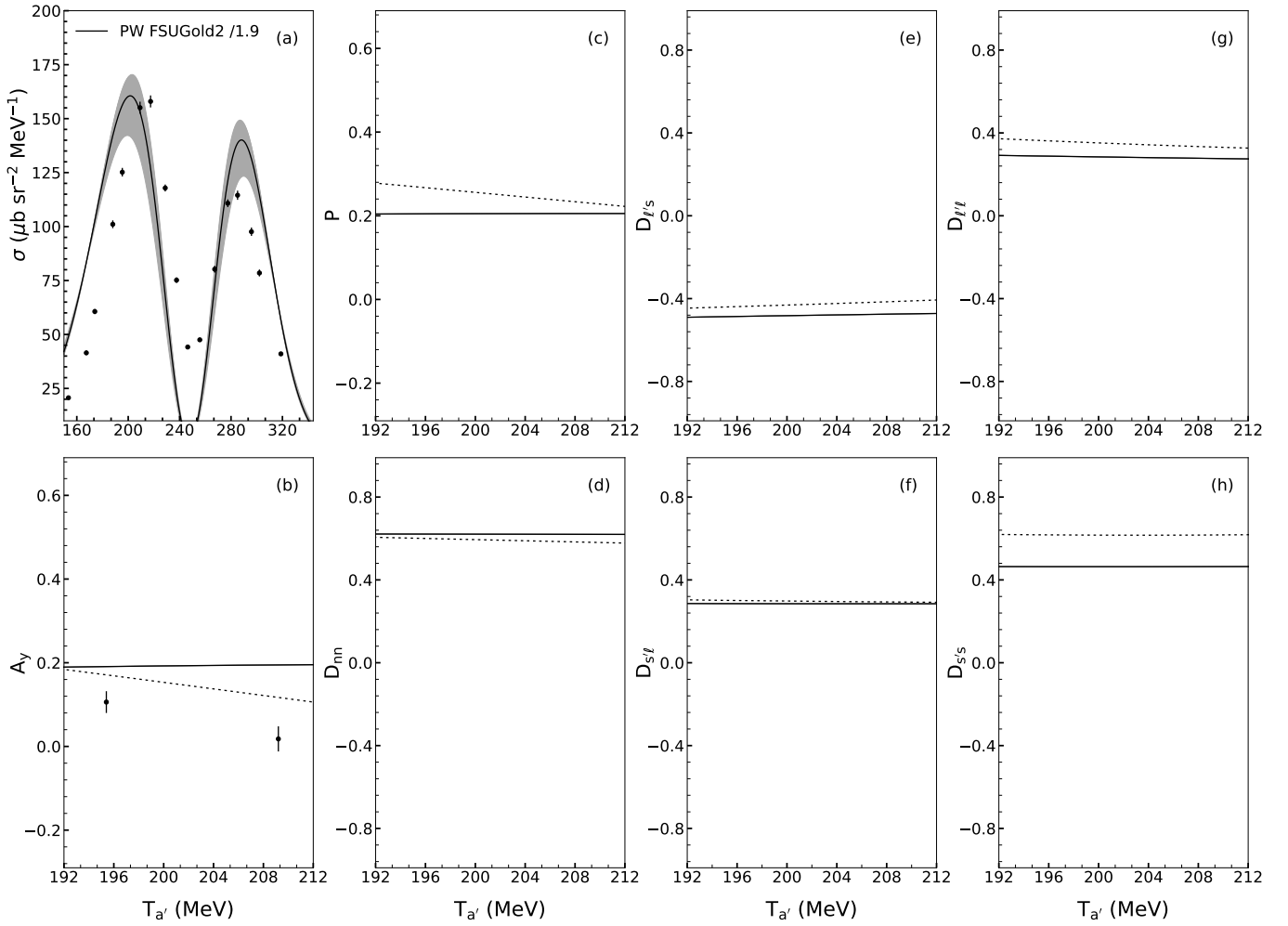


FIG. 5.8. The same as Fig. 5.1, except for the $1p_{3/2}$ state in ^{12}C .

5.6 Discussion of results

In this section, the results of Figs. 5.1 - 5.8, are discussed, and some of the key questions of this study are addressed. The kinematic regions where the RPWIA predictions are compared with the data and the RDWIA predictions are identified and presented in Table 5.2. The RPWIA calculations are compared with the experimental data of the unpolarized triple differential cross sections and the analyzing powers to determine the predictive power of the RPWIA model. Comparisons between the RPWIA (neglecting distortion effects) and RDWIA model predictions, which incorporate optical potentials, are performed. Another question to be addressed is how sensitive are the complete set of polarization transfer observables and the unpolarized triple differential cross sections to different RMF models (QHDII, NL3, FSUGold, and FSUGold2). The predictions of the RPWIA model for the $2d_{3/2}$ and $2d_{5/2}$ states of ^{208}Pb are also compared as they have the same principal quantum number ($n = 2$) and the angular quantum number ($l = 3$), but different values of j .

5.6.1 Kinematic regions for polarization transfer observables

Table 5.2 presents the kinetic energies of the outgoing protons ($T_{a'}$) corresponding to the highest peaks of unpolarized triple differential cross sections (σ_{peak}) closer to the data.

Table 5.2: The peak of the unpolarized triple differential cross section, σ_{peak} , and the kinetic energy of the outgoing proton, $T_{a'}$ (MeV), of the different target nuclei and respective nuclear states (n_{lj}) for the RPWIA model.

Nucleus	State (n_{lj})	$T_{a'}$ (MeV)	± 10 MeV	σ_{peak} ($\mu\text{b sr}^{-2} \text{ MeV}^{-1}$)	$\sigma_{\text{pw}}/\sigma_{\text{exp}}$
^{12}C	$1p_{3/2}$	202.0	$192.0 \leq T_{a'} \leq 212.0$	305.0	1.9
^{40}Ca	$1d_{3/2}$	194.0	$184.0 \leq T_{a'} \leq 204.0$	310.4	3.5
	$2s_{1/2}$	250.0	$240.0 \leq T_{a'} \leq 260.0$	3689.5	6.8
^{208}Pb	$3s_{1/2}$	253.0	$243.0 \leq T_{a'} \leq 263.0$	5003.0	26.6
	$2d_{3/2}$	219.0	$209.0 \leq T_{a'} \leq 229.0$	480.6	8.4
	$1h_{11/2}$	170.0	$160.0 \leq T_{a'} \leq 180.0$	258.8	7.4
	$2d_{5/2}$	218.0	$208.0 \leq T_{a'} \leq 228.0$	436.7	7.1
	$1g_{7/2}$	179.0	$169.0 \leq T_{a'} \leq 189.0$	329.6	45.5

From Table 5.2, the different quantities of interest being the kinetic energy of the outgoing proton [$T_{a'}$ (MeV)], the kinematic regions where the complete set of polarization transfer observables are plotted and compared with the RDWIA predictions (± 10 MeV from the peak of the unpolarized triple differential cross section closer to the experimental data) are shown. The peak values of the unpolarized triple differential cross section [σ_{peak} ($\mu\text{b sr}^{-2} \text{ MeV}^{-1}$)], and the scaling factors ($\sigma_{\text{pw}}/\sigma_{\text{exp}}$) which fit the theoretical predictions of the cross sections with experimental data are also presented. Although the RPWIA predictions describe the data, the RPWIA model does not reproduce the absolute values of the experimental unpolarized triple differential cross sections. The $3s_{1/2}$ state of ^{208}Pb has the highest value of the σ_{peak} (5003.0 $\mu\text{b sr}^{-2} \text{ MeV}^{-1}$) compared to all other states in ^{208}Pb which may suggest a higher probability of knockout of the bound proton. In terms of scaling factors ($\sigma_{\text{pw}}/\sigma_{\text{exp}}$), the $1g_{7/2}$ state has the

highest value of 45.5 with the ^{12}C having the least $\sigma_{\text{pw}}/\sigma_{\text{exp}}$ ratio in comparison with all the nuclei (^{208}Pb and ^{40}Ca).

5.6.2 Comparison of the RPWIA calculations to data

The predictive power of the RPWIA model is investigated by comparing the predictions with the experimental data of the unpolarized triple differential cross sections and the analyzing powers from RCNP [2].

5.6.2.1 ^{208}Pb analyses

From Figs. 5.1 - 5.5, the results of RPWIA model predictions for ^{208}Pb for proton knockout of the following single-particle states: $3s_{1/2}$, $2d_{3/2}$, $1h_{11/2}$, $2d_{5/2}$ and $1g_{7/2}$ are shown. For all these states, the RPWIA model qualitatively describes the shape of the experimental data of the unpolarized triple differential cross section with the specific scaling factors ($\sigma_{\text{pw}}/\sigma_{\text{exp}}$) given in Table 5.2 used to fit the RPWIA predictions to data. The analyzing powers and other polarization transfer observables are plotted within a certain energy range ± 10 MeV from the peak of the unpolarized triple differential cross section, where the RPWIA predictions are closer to the experimental data of the cross section. The RPWIA predictions of the analyzing powers for the $3s_{1/2}$ and $2d_{3/2}$ states quantitatively describe the data whereas, for the $1h_{11/2}$, $2d_{5/2}$ and $1g_{7/2}$, the RPWIA fails to describe the data within the selected energy ranges.

5.6.2.2 ^{40}Ca analyses

The RPWIA predictions for ^{40}Ca for the single-particle states $1d_{3/2}$ and $2s_{1/2}$, are shown in Figs. 5.6 - 5.7. For these states, the RPWIA unpolarized triple differential cross sections quantitatively describe the data for both the $1d_{3/2}$ and $2s_{1/2}$ states and this is also observed from the literature from Ref. [2]. The scaling factors applied to fit the RPWIA unpolarized triple differential cross sections closer to the data of Ref. [2] are shown in Table 5.2. The RPWIA predictions quantitatively describe the analyzing power data for the $2s_{1/2}$ state, but fail for the $1d_{3/2}$ state.

5.6.2.3 ^{12}C analyses

Fig. 5.8 presents the RPWIA predictions for the $1p_{3/2}$ state in ^{12}C for the RPWIA predictions of the unpolarized triple differential cross section and the analyzing powers compared to data from Ref. [2]. The RPWIA predictions of the unpolarized triple differential cross sections describe the experimental data and follow a trend similar to the results from Ref. [2]. The scaling factor applied to fit the predictions of the RPWIA model to data is 1.9, presented in Table 5.2. For the analyzing powers, the RPWIA predictions do not describe the data in the selected energy ranges for the kinetic energy of the outgoing proton ($T_{a'}$ MeV).

5.6.3 Comparison of RPWIA to RDWIA calculations

Within selected energy ranges of the kinetic energy of the outgoing proton [$T_{a'}$ (MeV)], the complete set of polarization transfer observables of the RPWIA model predictions are compared to the RDWIA model of Hillhouse *et al.* from Refs. [14, 27].

5.6.3.1 ^{208}Pb analyses

The RPWIA predictions of the polarization transfer observables are compared to the RDWIA predictions for all states of interest in ^{208}Pb . For the $3s_{1/2}$ state, the RDWIA predictions in-

intersect with the RPWIA predictions for the polarization transfer observables: A_y , P , D_{nn} , $D_{\ell's}$, $D_{s'\ell}$, $D_{\ell'\ell}$. For these observables, it implies that a RPWIA model which ignores distortions can be used to yield the same polarization transfer observables at specific kinetic energies of the outgoing protons [$T_{a'}$ (MeV)].

The RPWIA and RDWIA predictions for the complete set of polarization transfer observables for the $2d_{3/2}$ state intersect for observables: A_y , P , $D_{\ell'\ell}$ and $D_{s's}$. There are no intersection points for the predictions of D_{nn} , $D_{\ell's}$ and $D_{s'\ell}$.

The RPWIA predictions for the complete set of polarization transfer observables for the $1h_{11/2}$ state are shown in Fig. 5.3. For this state, the RPWIA and RDWIA predictions intersect for the observables P and $D_{s'\ell}$. There are no intersection points for the observables: A_y , D_{nn} , $D_{\ell's}$, $D_{\ell'\ell}$ and $D_{s's}$. This implies that for the selected regions, the RPWIA model does not yield similar results compared to the RDWIA model. Therefore, experimental data are required to properly gauge both models.

Fig. 5.4 shows the RPWIA predictions of the complete set of polarization transfer observables compared to the RDWIA predictions. Within the selected energy ranges for the kinetic energy of the outgoing proton, RPWIA and RDWIA have intersection points only for A_y , P , D_{nn} and $D_{\ell's}$. For other observables: $D_{s'\ell}$, $D_{\ell'\ell}$ and $D_{s's}$, the RPWIA and RDWIA do not intersect for the selected kinematic regions.

The RPWIA predictions of the polarization transfer observables of the $1g_{7/2}$ state in ^{208}Pb compared to the RDWIA predictions are shown in Fig. 5.5. For this particular state, the only observable which shows the intersection between the RPWIA and RDWIA predictions is $D_{\ell'\ell}$.

5.6.3.2 ^{40}Ca analyses

Figs. 5.6 - 5.7 show the RPWIA and RDWIA predictions of the complete set of polarization transfer observables for the single-particle states: $1d_{3/2}$ and $2s_{1/2}$, in ^{40}Ca . These observables are plotted as functions of the kinetic energy of the outgoing proton [$T_{a'}$ (MeV)]. For the $1d_{3/2}$ state in Fig. 5.6, only $D_{s's}$ shows intersection points between the predictions of the RPWIA and RDWIA models.

The predictions of RPWIA and RDWIA of the polarization transfer observables for the $2s_{1/2}$ state are presented in Fig. 5.7. For A_y , P , D_{nn} , $D_{\ell's}$, $D_{s'\ell}$, $D_{\ell'\ell}$, the RPWIA and RDWIA predictions intersect at certain kinetic energies of the outgoing proton. The RPWIA and RDWIA predictions for $D_{\ell's}$, $D_{s'\ell}$ show similar features. Only the $D_{s's}$ observable does not show the intersection between the RPWIA and RDWIA predictions.

5.6.3.3 ^{12}C analyses

The RPWIA predictions of the complete set of the polarization transfer observables for a lighter target nucleus, ^{12}C , and the state, $1p_{3/2}$ compared to the RDWIA predictions are shown in Fig. 5.8. For the analyzing power, A_y and $D_{s'\ell}$, the RPWIA and RDWIA predictions share intersection points for the selected energy ranges. The RPWIA and RDWIA predictions for $D_{s'\ell}$ share similar features. For the other observables: P , D_{nn} , $D_{\ell's}$, $D_{\ell'\ell}$ and $D_{s's}$, there are no intersection points.

5.6.4 Sensitivity of observables to RMF models

In this section, the sensitivity of the complete set of polarization transfer observables to the different RMF models (QHDII, NL3, FSUGold and FSUGold2) is investigated. The RMF models are used to extract the relativistic boundstate wave functions given by Eq. 3.14 of the different single-particle states. These models are applied to various nuclei (^{208}Pb , ^{40}Ca and ^{12}C) and their respective single-particle states. The results are shown from Figs. 5.1 - 5.8 with the solid black line representing the predictions from the FSUGold2 model and the other RMF models: QHDII, NL3, FSUGold, are enclosed within a grey shaded band.

For all nuclei, the complete set of polarization transfer observables are insensitive to the different RMF models. However, the unpolarized triple differential cross sections yield varying results and scaling factors ($\sigma_{\text{pw}}/\sigma_{\text{exp}}$) across different RMF models considered, while maintaining the same structure as the experimental data. The choice of the RMF models considered in this study have a limited impact on the observables since they show little to insignificant variations between the RPWIA predictions for the selected kinetic energies of the outgoing proton [T_a (MeV)].

5.6.4.1 ^{208}Pb analyses

For states in a heavier target nucleus, ^{208}Pb , the $3s_{1/2}$, $1h_{11/2}$ and $1g_{7/2}$ states show insignificant variations between the different RMF models, whereas for the $2d_{3/2}$ and $2d_{5/2}$ states, there are small variations around the maximum peak of the unpolarized triple differential cross section where the RPWIA predictions are closer to the experimental data.

5.6.4.2 ^{40}Ca analyses

As observed from the $1d_{3/2}$ state in Fig. 5.6, there are little variations between the RMF models applied for the unpolarized triple differential cross section. The shape of the RPWIA cross section is similar to that in Ref. [2]. For the $2s_{1/2}$ state of ^{40}Ca , there are insignificant variations in the RPWIA predictions of the unpolarized triple differential cross section from the application of various RMF models. The shape of the cross section is symmetric, and this is also observed in Ref. [2].

5.6.4.3 ^{12}C analyses

For a lighter target nucleus, ^{12}C , the focus was on the $1p_{3/2}$ single-particle state. The results of the RPWIA predictions for the unpolarized triple differential cross section from using different RMF models are shown in Fig. 5.8. There are some small variations between the different RMF models applied, however, the shape of the unpolarized triple differential cross section is similar to that observed in Ref. [2].

5.6.5 Comparisons of the observables from the 2d states in ^{208}Pb

The RPWIA predictions of the observables from the $2d_{5/2}$ and $2d_{3/2}$ states in ^{208}Pb are compared to each other since they share the same principal quantum number (n) and the angular momentum quantum number (l), but different values of the total angular quantum numbers (j). The comparisons are presented in Fig. 5.9.

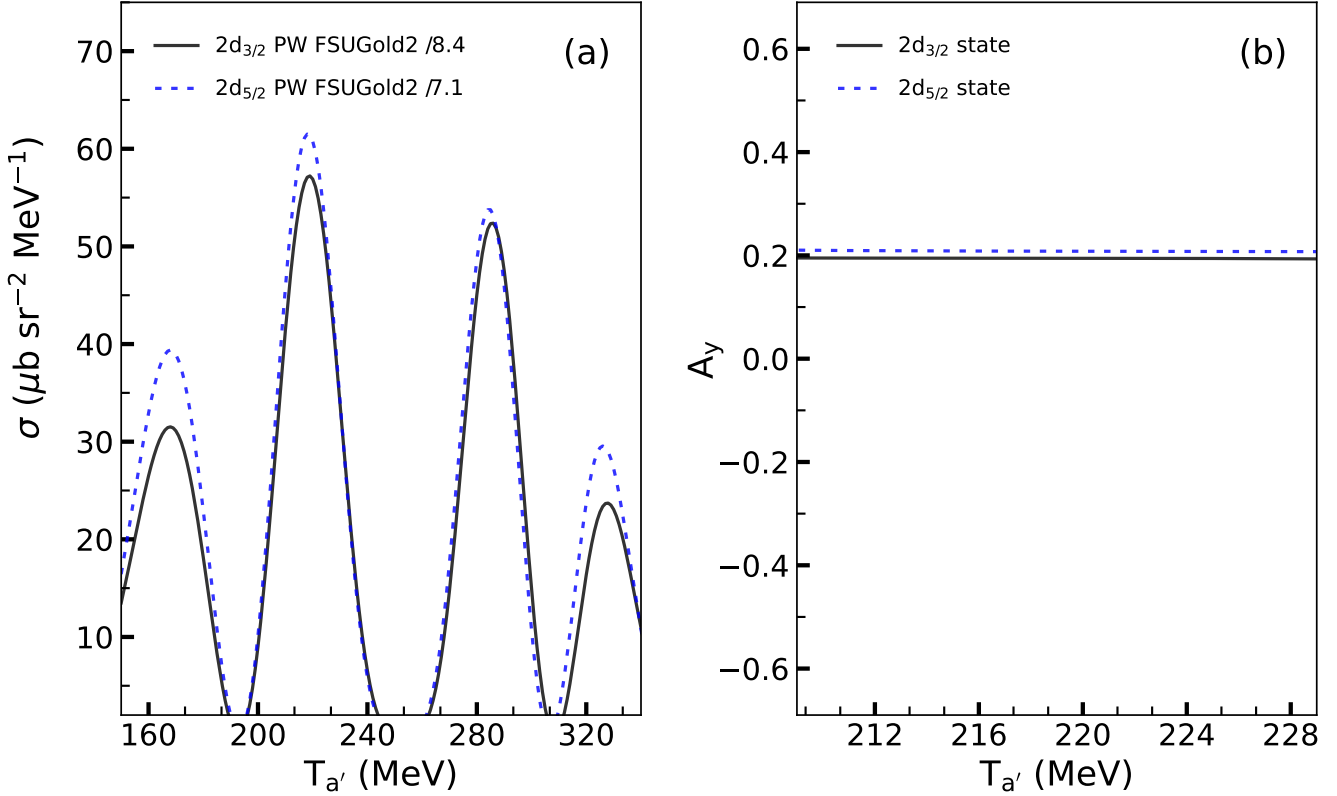


FIG. 5.9. Comparisons between the $2d_{3/2}$ and $2d_{5/2}$ states of ^{208}Pb unpolarized triple differential cross sections and the analyzing powers for the FSUGold2 RMF model.

The unpolarized triple differential cross sections shown in panel (a) and analyzing powers in panel (b), of the $2d_{5/2}$ and $2d_{3/2}$ states are shown in Fig. 5.9. The $2d_{3/2}$ state is represented by the black solid line, whereas the $2d_{5/2}$ state is represented by the blue dashed line. These states have different scaling factors used to fit the experimental data of the cross section. The comparisons are made only for the FSUGold2 RMF model, since it was established that the RMF models investigated in this study have a limited impact on the overall predictions. The unpolarized triple differential cross sections for the $2d_{5/2}$ and $2d_{3/2}$ states in ^{208}Pb are similar to each other and within the regions of interest, the analyzing powers are nearly identical.

Although the predictions of the RPWIA model for unpolarized triple differential cross sections and that of the analyzing powers describe the experimental data for most states investigated in this study, this model does not reproduce absolute values of the cross sections. This means that different scaling factors are applied to fit the predictions to the experimental data from RCNP. The RPWIA model does not yield reliable spectroscopic factors which are related to the state occupation probabilities, and these spectroscopic factors are reliably extracted through the use of the RDWIA model which incorporates optical potentials. The complete set of polarization transfer observables are presented within a limited range of the energy of the outgoing proton

(around ± 10 MeV from the peak of the cross section), and beyond this range, the RPWIA predictions do not yield desirable results of these observables.

5.7 Summary

In this Chapter, the theoretical predictions of the RPWIA model of the unpolarized triple differential cross sections and the complete set of polarization transfer observables for stable spherical nuclei (^{208}Pb , ^{40}Ca , and ^{12}C) were provided and compared with experimental data from RCNP and the RDWIA model. Using the RPWIA, the effects of different RMF models: QHDII, NL3, FSUGold, and FSUGold2 on the unpolarized triple differential cross sections and polarization transfer observables were analysed. The results showed small variations among the different RMF models, with RPWIA and RDWIA predictions intersecting for certain nuclear states for the investigated nuclei. The predictions of the unpolarized triple differential cross sections were in agreement with the data of Ref. [2]. The summary and conclusions that address the objectives stated in Chapter 1 are presented in Chapter 6.

Chapter 6

Summary and Conclusions

In this Chapter, the summary and conclusions that align with the objectives of this study which were stated in Chapter 1 are presented. The future outlook is provided in section 6.3 which may help drive future experimental work for the measurements of the complete set of polarization transfer observables to further gauge theoretical models.

6.1 Summary

This study was based on the development and application of the RPWIA model for exclusive polarized proton-induced proton-knockout reactions $(\vec{p}, 2\vec{p})$ from stable spherical nuclei, namely: ^{208}Pb , ^{40}Ca and ^{12}C , for predictions of unpolarized triple differential cross sections and the complete set of polarization transfer observables using direct kinematics. These predictions were compared with the experimental data from RCNP Japan in Ref. [2]. The $(\vec{p}, 2\vec{p})$ knockout reactions were applied to ^{208}Pb for the nuclear states: $3s_{1/2}$, $2d_{3/2}$, $1h_{11/2}$, $2d_{5/2}$ and $1g_{7/2}$. With the same kinematical conditions applied for the knockout from ^{208}Pb , the predictions were performed for ^{40}Ca nuclear states: $1d_{3/2}$, and $2s_{1/2}$ and for a lighter target nucleus, ^{12}C , $1p_{3/2}$ state. The choice of these states was determined by the availability of experimental data from Ref. [2].

Within the RPWIA model, the relativistic transition amplitude given by Eq. (3.3) in Chapter 3, which contains all information regarding the $(\vec{p}, 2\vec{p})$ scattering observables was related to the T -matrix which was used to compute the unpolarized triple differential cross section given by Eqs. (3.43) and (3.44) from Chapter 3. The incident and the two outgoing protons were treated as relativistic Dirac plane waves, the NN interaction was parameterized in terms of five Fermi invariant (SVPAT) scattering amplitudes known as the IA1 representation discussed in Section 3.3 [38, 76]. The radial wave functions of the bound proton were extracted through the application of the RMF theory from Chapter 4. The RMF models which were considered in this study are: QHDII, NL3, FSUGold and FSUGold2. Different polarization transfer observables, namely: D_{n0} , D_{0n} , D_{nn} , $D_{s'\ell}$, $D_{s's}$, $D_{\ell's}$ and $D_{\ell'\ell}$, were predicted with both the RPWIA model and compared to the corresponding RDWIA predictions from Refs. [14, 27].

As observed in Figs. 5.1 - 5.8, there is a noticeable trend in terms of the predictions of the RPWIA model of the complete set of polarization transfer observables extracted with the different RMF models. These models show insignificant variations as they yield similar results for each quantity predicted for the respective nuclear states of all nuclei (^{208}Pb , ^{40}Ca and ^{12}C). The RPWIA predictions of these observables were compared with the RDWIA predictions within a certain region, where the RPWIA predictions are closer to the experimental data and at the peak of the unpolarized triple differential cross section, to visualize how far off the RPWIA model performs compared to the well-established RDWIA model. For all nuclei and their re-

spective states, the peak of the unpolarized triple differential cross sections and energies of interest were identified, and the RPWIA predictions quantitatively described the experimental data after applying scaling factors.

Another observation in Fig. 5.9, for the nuclear states, $2d_{3/2}$ and $2d_{5/2}$ in ^{208}Pb , is that the unpolarized triple differential cross sections share similar features as in Ref. [2]. The RPWIA predictions of the unpolarized triple differential cross sections quantitatively describe the experimental data for both states ($2d_{3/2}$ and $2d_{5/2}$). Within the energy range of interest, the analyzing powers of these states are identical to each other. Various RMF models considered in this study show insignificant variations in the RPWIA predictions of the polarization transfer observables.

For the $1h_{11/2}$ and $1g_{7/2}$ states in ^{208}Pb , the unpolarized triple differential cross section predictions are shifted to the left from the experimental data, with similar structures, and this feature is also observed from Ref. [2]. The predictions of the RPWIA model for the analyzing power (A_y) for the $1h_{11/2}$ state does not describe the experimental data well in the selected energy ranges (± 10 MeV from σ_{peak}) since there are large error bars. On the polarization transfer observables: P , D_{nn} and $D_{s'\ell}$, it was observed that both RPWIA and RDWIA predictions intersect at certain energies, which helps to give confidence in the RPWIA model, while for $1g_{7/2}$, there are no intersection points between the two relativistic plane and distorted waves predictions. The RMF models still show insignificant variations between the predicted observables.

The predictions of the RPWIA and RDWIA models for the $1d_{3/2}$ state in ^{40}Ca are shown in Fig. 5.6. For the unpolarized triple differential cross section, the RPWIA predictions are shifted to the left of the experimental data, but they exhibit a similar structure from literature [2]. The $2s_{1/2}$ state of ^{40}Ca shown in Fig. 5.7, yields similar results for the unpolarized triple differential cross section as in Ref. [2]. The RPWIA predictions for both the unpolarized triple differential cross section and the analyzing power quantitatively describe the experimental data. There are no variations between the RMF models used in terms of the polarization transfer observables for both nuclear states.

For a lighter target nucleus, ^{12}C , the unpolarized triple differential cross section and the polarization transfer observables for the $1p_{3/2}$ state are shown in Fig. 5.8. The predictions of the RPWIA model of the unpolarized triple differential cross section describe the experimental data from Ref. [2]. There are insignificant variations between the different RMF model predictions for the various polarization transfer observables. For the selected energy ranges (± 10 MeV from σ_{peak}), the RPWIA predictions for the analyzing power (A_y) are not aligned in a satisfactory way. Since the predictions of the RPWIA model are compared to the predictions of the well-established RDWIA model, the polarization transfer observable, $D_{s'\ell}$, yields similar results even for the different RMF models. The RPWIA and RDWIA predictions for the observables: D_{nn} , $D_{\ell's}$, $D_{\ell'\ell}$ and $D_{s's}$, exhibit similar structures.

6.2 Conclusions

In conclusion, the RPWIA model provided a consistent description of the unpolarized triple differential cross sections and the analyzing power for most single-particle states of interest across different target nuclei, namely ^{208}Pb , ^{40}Ca , and ^{12}C . The RPWIA model gives the shape of the unpolarized triple differential cross sections for all nuclei of interest. It quantitatively describes the experimental data from RCNP at incident proton energies of 392 MeV. The RPWIA model also describes the analyzing power data within a certain kinematical region (± 10 MeV from the peak of the unpolarized triple differential cross section), which implies that the model is adequate to predict the analyzing power data as well as the unpolarized triple differential cross sections for nuclei of interest. Within the selected kinematic regions, the RPWIA model was used to predict the complete set of polarization transfer observables and comparisons with the predictions of the RDWIA model were made. For most of the observables, both the RPWIA model and the RDWIA model gave similar predictions, which means that the RPWIA is sufficient to predict these observables within the selected energy ranges. Although the RPWIA model performs well for unpolarized triple differential cross sections and analyzing powers, more experimental data for D_{ij} 's are needed to gauge the model. Different RMF models (QHDII, NL3, FSUGold, and FSUGold2) were used in the predictions of the unpolarized triple differential cross sections and the complete set of polarization transfer observables. For the unpolarized triple differential cross sections, the RMF models show little or insignificant variations in the predictions while maintaining the overall structure similar to the literature [2]. There are no variations between the predictions of the various RMF models for the complete set of polarization transfer observables. As shown by the results, the RPWIA model has a potential to be used to predict the observables of interest and can be used as a baseline for future experimental measurements.

6.3 Future work

For future work, the RPWIA formalism will be applied for the inverse kinematics for proton knockout reactions from unstable neutron-rich nuclei to evaluate the predictive power for the model against experiments using inverse kinematics. A more correct representation of the NN interaction referred to as IA2 parameterization needs to be considered.

Appendix

Introduction

The kinematics of the $(\vec{p}, 2\vec{p})$ knockout schematic shown in Fig. 3.1 in Chapter 3 are presented within both the laboratory and the center-of-mass frames. The natural units are adopted whereby $\hbar = c = 1$. The $\hbar$ and c represents the Planck's constant and the speed of light respectively.

Kinematics in the laboratory system

Within the laboratory system, the kinematical factor F_{kin} is given by:

$$F_{kin} = \frac{E_a E_{a'} E_b}{(2\pi)^5} \frac{k_a k_b}{k_a} \left[1 + \frac{E_b}{E_C} \left(1 - \frac{k_a}{k_b} \cos \theta_b + \frac{k_{a'}}{k_b} \cos(\theta_{a'} + \theta_b) \right) \right]^{-1}, \quad (\text{A.1})$$

whereby E_a represents the total energy of the incident proton, $E_{a'}$ and E_b represents the total energy of the scattered protons (a' and b). E_C represents the energy of the residual nucleus and the momentum of incident proton a , and scattered protons a' and b are denoted as k_a , $k_{a'}$ and k_b respectively. The angles θ_b and $\theta_{a'}$ represents the angles of the scattered protons at coplanar scattering angles.

$$\mathcal{E}_i = \sqrt{k_i^2 + m_i^2} = E_i + m_i, \quad (\text{A.2})$$

$\mathcal{E}_i$ and m_i denotes the total relativistic kinetic energy and the mass of the protons and the momentum (k_i) of the proton expressed as:

$$k_i^2 = E_i^2 + 2E_i m_i, \quad (\text{A.3})$$

Considering conservation of energy, the following expressions can be used:

$$\mathcal{E}_a + M_A = \mathcal{E}_{a'} + \sqrt{k_{b'}^2 + m_{b'}^2} + \sqrt{k_C^2 + M_C^2} + E_x, \quad (\text{A.4})$$

where the subscripts, a, A, a', b' and C represents the projectile proton, the target nucleus, a' and b' denotes the outgoing scattered protons and the residual nucleus respectively. E_x is the excitation energy. Through conservation of momentum:

$$\vec{k}_a = \vec{k}_{a'} + \vec{k}_{b'} + \vec{k}_C, \quad (\text{A.5})$$

with the expression

$$\begin{aligned} \vec{k}_C^2 &= (\vec{k}_a - \vec{k}_{a'} - \vec{k}_{b'})^2, \\ &= k_a^2 + k_{a'}^2 + k_{b'}^2 - 2k_a k_{a'} \cos \theta_{a'} - 2k_a k_{b'} \cos \theta_{b'} + 2k_{a'} k_{b'} \cos(\theta_{a'} + \theta_{b'}), \end{aligned} \quad (\text{A.6})$$

The residual nucleus momentum $\vec{k}_C$, can be eliminated by substituting:

$$\begin{aligned} \mathcal{E}_a + M_A &= \mathcal{E}_{a'} + \sqrt{k_{b'}^2 + m_{b'}^2} \\ &+ \left[k_a^2 + k_{a'}^2 + k_{b'}^2 - 2k_a k_{a'} \cos \theta_{a'} - 2k_a k_{b'} \cos \theta_{b'} + 2k_{a'} k_{b'} \cos(\theta_{a'} + \theta_{b'}) + M_C^2 \right]^{1/2} \\ &+ E_x, \end{aligned} \quad (\text{A.7})$$

Squaring the terms yields:

$$\left[\mathcal{E}_a + M_A - \mathcal{E}_{a'} - E_x - \sqrt{k_{b'}^2 + m_{b'}^2} \right]^2 = k_a^2 + k_{a'}^2 + k_{b'}^2 - 2k_a k_{a'} \cos \theta_{a'} - 2k_a k_{b'} \cos \theta_{b'} + 2k_{a'} k_{b'} \cos(\theta_{a'} + \theta_{b'}) + M_C^2, \quad (\text{A.8})$$

which can then be written as:

$$\begin{aligned} & 2(\mathcal{E}_a + M_A - \mathcal{E}_{a'} - E_x) \sqrt{k_{b'}^2 + m_{b'}^2} \\ &= (\mathcal{E}_a + M_A - \mathcal{E}_{a'} - E_x)^2 + m_{b'}^2 - M_C^2 - \left[k_a^2 + k_{a'}^2 - 2k_a k_{a'} \cos \theta_{a'} \right. \\ & \quad \left. - 2k_a k_{b'} \cos \theta_{b'} + 2k_{a'} k_{b'} \cos(\theta_{a'} + \theta_{b'}) \right], \\ &= 2 \left[k_a \cos \theta_{b'} - k_{a'} \cos(\theta_{a'} - \theta_{b'}) \right] k_{b'} + (\mathcal{E}_a + M_A - \mathcal{E}_{a'} - E_x)^2 \\ & \quad + m_{b'}^2 - M_C^2 + m_{b'}^2 - m_c^2 - \left[k_a^2 + k_{a'}^2 - 2k_a k_{a'} \cos \theta_{a'} \right], \\ &= C_1 k_{b'} + C_2, \end{aligned} \quad (\text{A.9})$$

whereby C_1 and C_2 acquires the form:

$$C_1 = 2 \left[k_a \cos \theta_{b'} - k_{a'} \cos(\theta_{a'} + \theta_{b'}) \right], \quad (\text{A.10})$$

$$C_2 = (\mathcal{E}_a + M_A - \mathcal{E}_{a'} - E_x)^2 + m_{b'}^2 - M_C^2 - \left[k_a^2 + k_{a'}^2 - 2k_a k_{a'} \cos \theta_{a'} \right], \quad (\text{A.11})$$

squaring Eq. (A.9) then yields:

$$4 \left[\mathcal{E}_a + M_A - \mathcal{E}_{a'} - E_x \right]^2 \left[k_{b'}^2 + m_{b'}^2 \right] = C_1^2 k_{b'}^2 + 2C_1 C_2 k_{b'} + C_2^2, \quad (\text{A.12})$$

or

$$C_1^2 - 4 \left[\mathcal{E}_a + M_A - \mathcal{E}_{a'} - E_x \right]^2 k_{b'}^2 + 2C_1 C_2 k_{b'} + C_2^2 - 4(m_{b'})^2 \left[\mathcal{E}_a + M_A - \mathcal{E}_{a'} - E_x \right]^2 = 0, \quad (\text{A.13})$$

which can be expressed as a quadratic function:

$$A k_{b'}^2 + 2C_1 C_2 k_{b'} + B = 0, \quad (\text{A.14})$$

where A and B are:

$$A = C_1^2 - 4 \left[\mathcal{E}_a + M_A - \mathcal{E}_{a'} - E_x \right]^2, \quad (\text{A.15})$$

$$B = C_2^2 - 4(m_{b'})^2 \left[\mathcal{E}_a + M_A - \mathcal{E}_{a'} - E_x \right]^2, \quad (\text{A.16})$$

The momentum $k_{b'}$ is determined through solving the quadratic equation (A.14) resulting in:

$$k_{b'} = \frac{-C_1 C_2 \pm \sqrt{C_1^2 C_2^2 - AB}}{A}, \quad (\text{A.17})$$

The kinetic energy in both the entrance and exit channels remains fixed and the momentum of particles a (k_a) and a' ($k_{a'}$) in these channels can be derived through:

$$k_a = \sqrt{(E_a + m_{a'})^2 - m_{a'}^2}, \quad (\text{A.18})$$

$$k_{a'} = \sqrt{(E_{a'} + m_{a'})^2 - m_{a'}^2}, \quad (\text{A.19})$$

The total relativistic energy, kinetic energy, and momentum of the residual nucleus C , are described by:

$$\mathcal{E}_C = \mathcal{E}_a + M_A - \mathcal{E}_{a'} - \mathcal{E}_{b'} - E_x, \quad (\text{A.20})$$

$$E_C = \mathcal{E}_C - M_C, \quad (\text{A.21})$$

$$k_C = \sqrt{\mathcal{E}_C - M_C^2}, \quad (\text{A.22})$$

Momentum conservation follows that, the direction of the incoming beam is chosen as the z-direction such that:

$$(k_a)_z = (k_{a'})_z + (k_{b'})_z + (k_C)_z, \quad (\text{A.23})$$

$$k_a = k_{a'} \cos \theta_{a'} + k_{b'} \cos \theta_{b'} + k_C \cos \theta_C, \quad (\text{A.24})$$

with $\cos \theta_C$:

$$\cos \theta_C = \frac{k_a - k_{a'} \cos \theta_{a'} - k_{b'} \cos \theta_{b'}}{k_C}, \quad (\text{A.25})$$

whereas for the y-direction:

$$(k_a)_y = (k_{a'})_y + (k_{b'})_y + (k_C)_y, \quad (\text{A.26})$$

$$0 = k_{a'} \sin \theta_{a'} - k_{b'} \sin \theta_{b'} - k_C \sin \theta_C, \quad (\text{A.27})$$

and $\sin \theta_C$ given as:

$$\sin \theta_C = \frac{k_{a'} \sin \theta_{a'} - k_{b'} \sin \theta_{b'}}{k_C}. \quad (\text{A.28})$$

Kinematics for the nucleon-nucleon system

In this section, the kinematics for the NN interaction are presented. The effective laboratory kinetic energy and the NN center-of-mass (cm) scattering angle will be determined. These quantities are used to calculate the NN t-matrix for a type of reaction, $A(a, a', b')C$, where the target nucleus is represented by A , the projectile proton beam (a), a' and b' the two outgoing protons and lastly the residual nucleus, C , one applies the laws of conservation to determine the quantities within the cm frame. Following the conservation of four momentum:

$$P_a + P_A = P_{a'} + P_{b'} + P_C, \quad (\text{A.29})$$

with P_i representing the four momenta of particle $i \in a, A, a', b', C$ within the laboratory system. Let:

$$P_B = P_A - P_C, \quad (\text{A.30})$$

Then Eq. (A.29) can be expressed as:

$$P_a - P_B = P_{a'} - P_{b'}, \quad (\text{A.31})$$

The Mandelstam variables, s and t (Lorentz invariant quantities) are written in the laboratory frame as:

$$s = (P_a + P_B)^2 = (P_{a'} + P_{b'})^2, \quad (\text{A.32})$$

$$t = (P_a - P_{a'})^2 = (P_B - P_{b'})^2, \quad (\text{A.33})$$

The effect of the nuclear medium is negligible due to impulse approximation which assumes the dominance of the reaction being that of the projectile and struck proton in the target nucleus. Initial energy prescription uses:

$$\begin{aligned} s_i &= (P_a + P_B)^2, \\ &= (\mathcal{E}_a + \mathcal{E}_B)^2 - (k_a + k_B)^2, \\ &= (\mathcal{E}_a + m_B)^2 - k_a^2, \\ &= m_a^2 + m_B^2 + 2m_B \mathcal{E}_a, \\ &= 2E_a m_B + (m_a + m_B)^2, \end{aligned} \quad (\text{A.34})$$

whereby an assumption is made that, m_B is equivalent to the free proton mass m_p and the effective kinetic energy of the proton in the laboratory frame is determined as:

$$E_a = \frac{s_i - 4m_p^2}{2m_p}, \quad (\text{A.35})$$

For the final energy prescription:

$$s_f = (P_{a'} + P_{b'})^2, \quad (\text{A.36})$$

Then the effective kinetic energy is given by:

$$E_a = \frac{s_f - 4m_p^2}{2m_p}, \quad (\text{A.37})$$

The Mandelstam variable, t , is used in the calculation of the NN center-of-mass scattering angle and from Eq. (A.33), in the laboratory system:

$$\begin{aligned} t &= (\mathcal{E}_a - \mathcal{E}_{a'})^2 - (\vec{k}_a - \vec{k}_{a'})^2, \\ &= \mathcal{E}_a^2 - k_a^2 + \mathcal{E}_{a'}^2 - k_{a'}^2 - 2\mathcal{E}_a \mathcal{E}_{a'} + 2\vec{k}_a \cdot \vec{k}_{a'}, \\ &= 2m_p^2 - 2\mathcal{E}_a \mathcal{E}_{a'} + 2k_a k_{a'} \cos \theta_{a'}, \end{aligned} \quad (\text{A.38})$$

Within the rest frame of the incoming proton and the struck proton, an assumption is made that, the projectile strikes a free proton, therefore, the center-of-mass of momenta of the projectile and the left scattered proton satisfies:

$$k_a^{cm} = k_{a'}^{cm} = k^{cm}, \quad (\text{A.39})$$

Hence following that:

$$\begin{aligned} t &= (P_a^{cm} - P_{a'}^{cm})^2, \\ &= (\mathcal{E}_a^{cm} - \mathcal{E}_{a'}^{cm})^2 - (\vec{k}_a^{cm} - \vec{k}_{a'}^{cm})^2, \\ &= m_a^2 + m_{a'}^2 - 2\mathcal{E}_a^{cm} \mathcal{E}_{a'}^{cm} + 2k_{cm}^2 \cos \theta_{a'}^{cm}, \\ &= 2m_p^2 - \mathcal{E}_a^{cm} \mathcal{E}_{a'}^{cm} + 2k_{cm}^2 \cos \theta_{a'}^{cm}, \end{aligned} \quad (\text{A.40})$$

using the condition $\theta_{a'}^{cm} = \theta^{cm}$, then from Eq. (A.40) one acquires:

$$\cos \theta^{cm} = \frac{t - 2m_p - 2\mathcal{E}_a^{cm}\mathcal{E}_{a'}^{cm}}{2(k^{cm})^2}, \quad (\text{A.41})$$

Due to the assumption in Eq. (A.39):

$$\mathcal{E}_a^{cm} = \mathcal{E}_{a'}^{cm} = E_{NN}^{cm} + m_p, \quad (\text{A.42})$$

E_{NN}^{cm} represents the center-of-mass kinetic energy for both the a and a' , therefore Eq. (A.41) takes the form:

$$\cos \theta^{cm} = \frac{t + 2(E_{NN}^{cm})^2 + 4m_p E_{NN}^{cm}}{2(k^{cm})^2}, \quad (\text{A.43})$$

Setting another condition $\mathcal{E}^{cm} = \mathcal{E}_a^{cm} = \mathcal{E}_{a'}^{cm}$, the overall relativistic center-of-mass energy for the protons, a and a' , is described by the equation:

$$(\mathcal{E}^{cm})^2 = (E_{NN}^{cm} + m_p)^2 = (k^{cm})^2 + m_p^2, \quad (\text{A.44})$$

which then follows:

$$2(k^{cm})^2 = 2(E_{NN}^{cm})^2 + 4m_p E_{NN}^{cm}, \quad (\text{A.45})$$

Thus Eq. (A.43) becomes:

$$\begin{aligned} \cos \theta^{cm} &= \frac{t + 2(k^{cm})^2}{2(k^{cm})^2}, \\ &= \frac{t}{2(k^{cm})^2} + 1, \end{aligned} \quad (\text{A.46})$$

The NN scattering angle in the center-of-mass frame is expressed as:

$$\theta^{cm} = \cos^{-1} \left[\frac{t}{2(k^{cm})^2} + 1 \right], \quad (\text{A.47})$$

k^{cm} is defined in the center-of-mass frame whereby $\vec{k}_{a'}^{cm} + \vec{k}_{b'}^{cm} = 0$ and within the laboratory frame, the Mandelstam variable, s_f , becomes:

$$\begin{aligned} s_f &= (P_{a'} + P_{b'})^2, \\ &= (\mathcal{E}_{a'} + \mathcal{E}_{b'})^2 - (\vec{k}_{a'} + \vec{k}_{b'})^2, \\ &= (\mathcal{E}_{a'} + \mathcal{E}_{b'})^2 - k_{a'}^2 - k_{b'}^2 + k_{a'} k_{b'} \cos \theta(\theta_{a'} + \theta_{b'}), \end{aligned} \quad (\text{A.48})$$

Due to the invariant nature of s_f under Lorentz transformation, for both a and a' , it can be defined in the center-of-mass system as:

$$\begin{aligned} s_f &= (\mathcal{E}_a^{cm} + \mathcal{E}_{a'}^{cm})^2 - (\vec{k}_a^{cm} + \vec{k}_{a'}^{cm})^2, \\ &= 4\mathcal{E}_{Tot}^{cm}, \end{aligned} \quad (\text{A.49})$$

The total energy of the protons, a and a' within their rest system, represented as $\mathcal{E}_{Tot}^{cm}$, is linked to the center-of-mass momentum, k^{cm} , through:

$$(k^{cm})^2 + m_p^2 = \mathcal{E}_{Tot}^{cm}, \quad (\text{A.50})$$

From Eq. (A.49 - A.50), the center-of-mass momentum is given by:

$$(k^{cm})^2 = \frac{s_f}{4} - m_p^2. \quad (\text{A.51})$$

Code documentation

Fig. A.2 shows the synopsis of the programming code used to calculate the unpolarized triple differential cross sections and the complete set of the polarization transfer observables on nuclei.

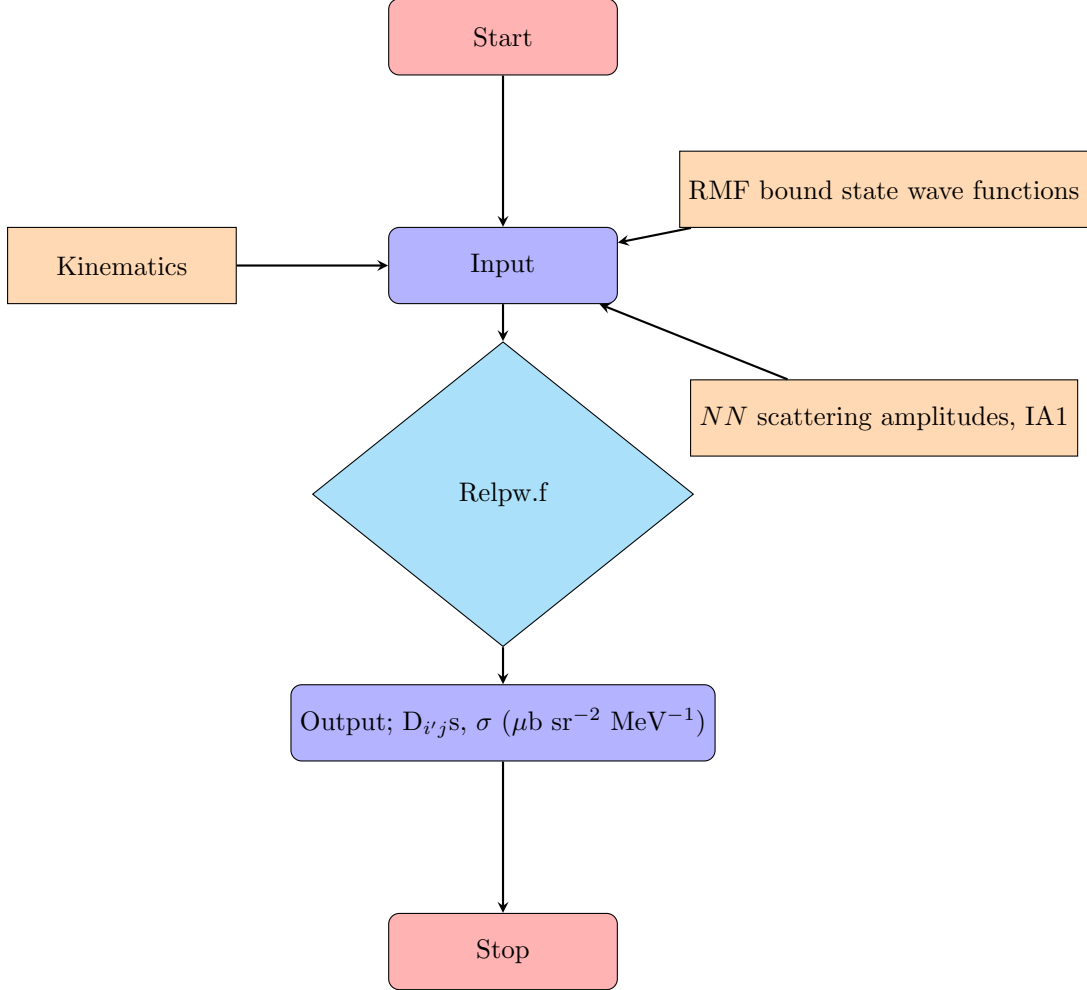


FIG. A.2. The code documentation used to calculate the unpolarized triple differential cross sections and the complete set of polarization transfer observables for stable nuclei: ^{208}Pb , ^{40}Ca and ^{12}C .

The kinematics are evaluated from the Appendix section 6.3. The atomic masses (A), proton numbers (Z) and neutron numbers (N) of respective nuclei: (^{208}Pb , ^{40}Ca and ^{12}C) are given. The energy of the incident polarized proton beam ($E_{a'}$) and the scattering angles for the two outgoing protons ($\theta_{a'}$ and $\theta_{b'}$) in the exit channels are also given. Different RMF models were used to extract the relativistic boundstate proton radial wave functions from Eq. (3.22). The NN amplitudes are evaluated from Eq. (3.37) following the IA1 parameterization discussed in section 3.3. All these quantities served as inputs into the main code, Relpw.f, which was used in the calculations of the unpolarized triple differential cross sections and the complete set of the polarization transfer observables given by Eqs. (3.43) and (3.44) respectively. The relativistic transition amplitude in Eq. (3.29) was related to the 2×2 T -matrix given by Eq. (3.46), and used to calculate the observables using Eqs. (3.43) and (3.44).

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