

Thermo-fluidic significance of non Newtonian fluid with hybrid nanostructures

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ABSTRACT

The main purpose of this investigation is to analyze the flow and heat transfer behavior of a non Newtonian fluid (Oldroyd B fluid) with hybrid nanostructure subject to externally inclined magnetic field. Cattaneo Christov heat flux model is incorporated in the energy equation. Non linear thermal radiation and viscous dissipation mechanisms are considered. Shooting technique is implemented to accomplish the numerical solution. The outcomes of the numerical computations are discussed with the aid of relevant graphs. The performed study reveals that rise in magnetic field strength shows opposite behavior to fluid velocity and temperature. Thermal relaxation time factor controls the rate of heat transportation. Rise in curvature parameter accelerates the fluid flow while that of angle of inclination of magnetic field imparts the reverse effect.

1. Introduction

Indeed, non-Newtonian liquids are the material which characterizes a non linear rapprochement between the stress and strain rate [1–3]. Examples of such fluids include polymer solutions, concrete/china clay, foams and emulsions, sewage sludge, pharmaceutical chemicals, cosmetics, paints and biological fluids such as blood, synovial fluid, salvia etc. They are applied in turbid water bodies, photo chemical reactors, nuclear power plants, gas turbines, combustion, furnace design, fluidized bed heat exchangers, solar ponds, solar collectors, and different propulsion devices for aircraft, missiles, satellites and space vehicles. The Oldroyd-B liquid is a category of non Newtonian liquid that defines both the retardation and relaxation time phenomena [4,5]. In this connection, an analytical treatment for MHD mixed convection BLF of Oldroyd-B fluid was executed by Mustafa [6]. Further, the effects of Cattaneo-Christov double diffusion along with relaxation-retardation viscous dissipation (Zhang et al. [7]) and chemical reactions and stratifications (Alshomrani et al. [8]) were investigated. It is well observed from several research on the HT and viscous drag in area of dynamics that TC is higher in solids than liquids ([9,10]). The nfs are, therefore, developed from the insertion nano-structural particles into the

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Nomenclature

(u, v)	axial and radial velocity components ($m s^{-1}$)
hc	heat capacitance
nf	nanofluid
bf	base fluid
TC	thermal conductivity
HT	heat transfer
HTR	heat transfer rate
BLF	boundary layer flow
PDE	partial differential equation
NLTR	non linear thermal radiation
VBLT	velocity boundary layer thickness
TBLT	thermal boundary layer thickness
CCHF	Cattaneo Christov heat flux
$\rho_{hnf}, \rho_{nf}, \rho_f, \rho_{s_1}, \rho_{s_2}$	effective density of the hnf, nf, bf, $MnZnFe_2O_4Fe_3O_4$ ($kg m^{-3}$)
$(\rho C_p)_{hnf}, (\rho C_p)_{nf}, (\rho C_p)_f, (\rho C_p)_{s_1}, (\rho C_p)_{s_2}$	hc of the hnf, nf, bf, $MnZnFe_2O_4Fe_3O_4$ ($J kg^2 m^3 K^{-1}$)
$\mu_{hnf}, \mu_{nf}, \mu_f$	effective dynamic viscosity of the hnf, nf, bf ($kg m^{-1} s^{-1}$)
$k_{hnf}, k_{nf}, k_f, k_{s_1}, k_{s_2}$	tc of hnf, nf, $MnZnFe_2O_4Fe_3O_4$ ($W m^{-1} K^{-1}$)
$\nu_{hnf}, \nu_{nf}, \nu_f$	kinematic viscosity of hnf, nf, bf ($m^2 s^{-1}$)
Γ^*	momentum accommodation coefficient
λ_m	molecular mean free path
Kn	Knudsen number
σ^*	Stefan-Boltzmann constant
k^*	mean absorption coefficient
φ_1, φ_2	solid volume fraction for $MnZnFe_2O_4Fe_3O_4$
T	fluid temperature in the boundary layer (K)
T_∞	ambient fluid temperature (K)
M	Hartmann number
ε	curvature parameter
δ	stretching strength parameter
Γ_1	Deborah number for relaxation time
Γ_2	Deborah number for retardation time
τ_1	first order slip parameter
τ_2	second order slip parameter
Ω	angle of inclination
U_{slip}	slip velocity
λ_E	thermal relaxation time factor
Nr	radiation parameter
θ_r	temperature ratio parameter
Bi	Biot number
Re	Reynolds number
U_0	constant velocity
$Pr = \frac{\nu_f}{\alpha_f}$	Prandtl number
l	length
$\xi = \frac{x}{l}$	a non dimensional ratio parameter
Λ_E	thermal relaxation time
Ec	Eckert number
$Br = Pr.Ec = Br$	Brinkman number
θ_r	temperature ratio parameter
Suffixes	
hnf	nanofluid
nf	nanofluid
f	base fluid
s_1	nanoparticle $MnZnFe_2O_4$
s_2	nanoparticle Fe_3O_4

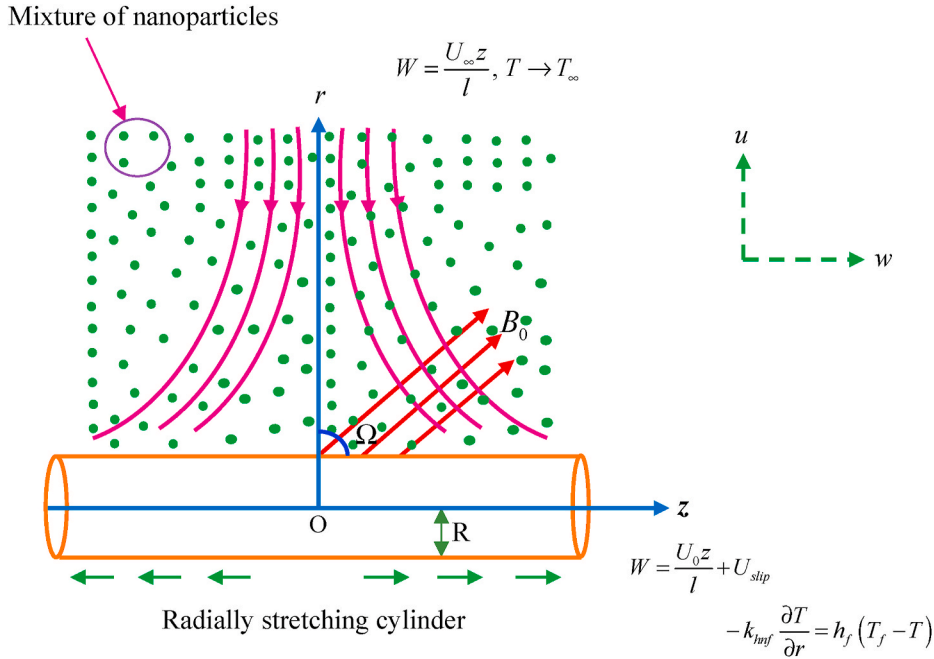


Fig. 1. Flow geometry of the problem.

conventional fluids. Such nanometer-sized particles are usually made up of metals, metal oxides, carbides, semiconductors, nanowires, nanotubes, nanorods and nanofibers ([11,12]). Such nfs have higher TC and stability when compared with the bfs [13–31].

In order to enhance the thermal properties of common nfs a novel type of fluid called ‘hybrid nanofluid’ comprising of two or more nps was developed. In hybrid nf, the nps have distinguished thermophysical and chemical characteristics, for instance, the capability to escalate the rate of heat transportation even at a small amount of nanoparticle volume fractions by hybridizing an appropriate combination of nps (Hemmat Esfe et al. [32]). It’s true that nfs have numerous engineering and industrial applications in fluid flows due to their better TC. However, hybrid nfs exhibit tremendous heat transfer enhancement. It is observed that hybrid nanoliquids are utilized for the in solar collectors of various shapes, very small channel heat sink, heat exchangers and others. Many investigations have been performed regarding the hybrid nanofluids characterization, models and their utilization in enhancing the HT capability significantly (Wang et al. [33], Sidik et al. [34], Leong et al. [35], Babu et al. [36], Ajili et al. [37], Dehkordi et al. [38], Asgari et al. [39], Karimipour et al. [40], Moradi et al. [41], Farzinpour et al. [42], Liu et al. [43], Ahmadi et al. [44], Zheng et al. [45], He et al. [46]). Oldroyd B fluid with Hybrid nanostructures has very significant role in medical applications.

An intensive survey makes clear that the flow and HT behavior of non Newtonian fluid (Oldroyd B fluid) with hybrid nanostructure subject to Cattaneo Christov heat flux over a stretched cylinder has not been studied yet. The novelty of the present investigation is to explore the impact of magnetic field on the flow of Oldroyd B fluid comprising $MnZnFe_2O_4$ as nps and $C_2H_2F_4$ as base particles. The hybrid nf in this article contains two ferrites, i.e., Magnetite ferrite (Fe_3O_4) and Manganese zinc ferrite ($MnZnFe_2O_4$) as constituents because their presence in the base fluid enables them efficient for HT in ferromagnetic fluid flows.

2. Mathematical description and equations of the physical model

We consider a steady incompressible flow of non Newtonian fluid (Oldroyd B fluid) with hybrid nanostructure comprising $MnZnFe_2O_4$ and Fe_3O_4 as nanoparticles. Flow of Oldroyd B hybrid nanofluid is induced by stretching of a cylinder with velocity $U_w(x) = \frac{xU_0}{l}$. A constant magnetic field is applied externally at an angle Ω to the flow pattern. Cattaneo Christov heat flux model, non linear thermal radiation and viscous dissipation aspects are implemented in the thermal treatment process. We assume the cylindrical coordinates where the x-axis (axial direction) and r-axis (radial direction) are perpendicular to each other. A schematic diagram of the assumed problem is exhibited in Fig. 1.

The governing equations for the designed problem are ([4,5,7,40,47]):

$$\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z} = 0 \quad (1)$$

$$u \frac{\partial w}{\partial r} + w \frac{\partial w}{\partial z} + \lambda_1 \left[w^2 \frac{\partial^2 w}{\partial z^2} + u^2 \frac{\partial^2 w}{\partial r^2} + 2uw \frac{\partial^2 w}{\partial r \partial z} \right]$$

$$= \frac{\mu_{hnf}}{\rho_{hnf}} \left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r} \frac{\partial w}{\partial r} \right) + \frac{\mu_{hnf}}{\rho_{hnf}} \lambda_2 \left[\frac{u}{r^2} \frac{\partial w}{\partial r} - \frac{1}{r} \frac{\partial w}{\partial r} \frac{\partial w}{\partial z} - \frac{2}{r} \frac{\partial u}{\partial r} \frac{\partial w}{\partial r} + \frac{w}{r} \frac{\partial^2 w}{\partial r \partial z} \right. \\ \left. - \frac{\partial w}{\partial r} \frac{\partial^2 w}{\partial r \partial z} - 2 \frac{\partial w}{\partial r} \frac{\partial^2 u}{\partial r^2} + \frac{u}{r} \frac{\partial^2 w}{\partial r^2} - \frac{\partial w}{\partial z} \frac{\partial^2 w}{\partial r^2} \right. \\ \left. + w \frac{\partial^3 w}{\partial r^2 \partial z} + u \frac{\partial^3 w}{\partial r^3} \right] + W_e \frac{dW_e}{dz} + \sin^2 \Omega \frac{\sigma_{hnf}}{\rho_{hnf}} B_0^2 \left[(W_e - w) + \lambda_1 u \frac{\partial w}{\partial r} \right] \quad (2)$$

$$u \frac{\partial T}{\partial r} + w \frac{\partial T}{\partial z} + \Lambda_E \left(w^2 \frac{\partial^2 T}{\partial z^2} + u^2 \frac{\partial^2 T}{\partial r^2} + 2uw \frac{\partial^2 T}{\partial r \partial z} + w \frac{\partial w}{\partial z} \frac{\partial T}{\partial z} \right) \\ + w \frac{\partial u}{\partial z} \frac{\partial T}{\partial r} + u \frac{\partial w}{\partial r} \frac{\partial T}{\partial z} + u \frac{\partial u}{\partial r} \frac{\partial T}{\partial r} \\ = \alpha_{hnf} \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) + \frac{16\sigma^*}{3k^* (\rho C_p)_{hnf}} \left\{ T^3 \frac{\partial^2 T}{\partial r^2} + 3T^2 \left(\frac{\partial T}{\partial r} \right)^2 \right\} \\ + \sin^2 \Omega \frac{\sigma_{hnf}}{(\rho C_p)_{hnf}} B_0^2 (w - W_e)^2 + \frac{\mu_{hnf}}{(\rho C_p)_{hnf}} \left(\frac{\partial w}{\partial r} \right)^2 \quad (3)$$

With BCs [42]:

$$\left. \begin{aligned} w(z, r) = W(z) + U_{slip} = \frac{U_0 z}{l} + U_{slip}, \quad u = 0, \quad -k_{hnf} \frac{\partial T}{\partial r} = h_f (T_f - T) \text{ at } r = R \\ w \rightarrow W_e(z) = \frac{U_\infty z}{l}, \quad \frac{\partial u}{\partial r} = 0, \quad T \rightarrow T_\infty \quad \text{as } r \rightarrow \infty \end{aligned} \right\} \quad (4)$$

The second order slip is defined as

$$U_{slip} = \frac{2}{3} \left(\frac{3 - \Gamma^* l^3}{\Gamma^*} - \frac{3}{2} \frac{1 - l^2}{Kn} \right) \lambda_m \frac{\partial w}{\partial r} - \frac{1}{4} \left[l^4 + \frac{2}{Kn^2} (1 - l^2) \right] \lambda_m^2 \frac{\partial^2 w}{\partial r^2} \quad (5)$$

$$Or \quad U_{slip} = L_1 \frac{\partial w}{\partial r} + L_2 \frac{\partial^2 w}{\partial r^2} \quad (6)$$

$$\text{where } L_1 = \frac{2}{3} \left(\frac{3 - \Gamma^* l^3}{\Gamma^*} - \frac{3}{2} \frac{1 - l^2}{Kn} \right) \lambda_m \quad (7)$$

$$\text{and } L_2 = -\frac{1}{4} \left[l^4 + \frac{2}{Kn^2} (1 - l^2) \right] \lambda_m^2 \quad (8)$$

In (7) and (8) $l = \min \left[\frac{1}{Kn}, 1 \right]$, Γ^* is the momentum accommodation coefficient with $0 \leq \Gamma^* \leq 1$, λ_m is the molecular mean free path, $Kn = \frac{l}{l}$ is the Knudsen number.

Here, (u, w) are respectively the velocities of fluid along radial and axial directions, λ_1 and λ_2 are the relaxation and retardation times respectively, μ_{hnf} is the dynamic viscosity of hybrid nanofluid, ρ_{hnf} is the density of hybrid nanofluid, W_e is the free stream velocity, Ω is the angle of inclination of magnetic field, B_0 is the uniform strength of applied magnetic field, Λ_E is the thermal relaxation time factor, α_{hnf} is the thermal diffusivity of hybrid nanofluid, U_0 is the reference velocity, T is the temperature of the fluid within boundary layer, T_f is the temperature for convective heating, σ^* is the Stefan Boltzmann constant, k^* is the mean absorption coefficient, $(\rho C_p)_{hnf}$ is the heat capacity of hybrid nanofluid, σ_{hnf} is the electrical conductivity of hybrid nanofluid, U_{slip} is the slip velocity, l is the reference length, k_{hnf} is the thermal conductivity of hybrid nanofluid, h_f is the heat transfer coefficient, and U_∞ is the reference free stream velocity.

The physical properties of the hybrid nanofluid ($MnZnFe_2O_4 - Fe_3O_4 - C_2H_2F_4$) [43]:

$$\text{Dynamic viscosity } \mu_{hnf} = \mu_f (1 - \varphi_1)^{-2.5} (1 - \varphi_2)^{-2.5} \quad (9)$$

$$\text{Density } \rho_{hnf} = (1 - \varphi_2) [(1 - \varphi_1) \rho_f + \varphi_1 \rho_{s1}] + \varphi_2 \rho_{s2} \quad (10)$$

$$\text{Heat capacitance } (\rho C_p)_{hnf} = (1 - \varphi_2) [(1 - \varphi_1) (\rho C_p)_f + \varphi_1 (\rho C_p)_{s1}] + \varphi_2 (\rho C_p)_{s2} \quad (11)$$

$$\text{Thermal diffusivity } \alpha_{hnf} = \frac{k_{hnf}}{(\rho C_p)_{hnf}} \quad (12)$$

$$\text{Electrical conductivity } \frac{\sigma_{hmf}}{\sigma_{bf}} = \frac{\sigma_{s2} + 2\sigma_{bf} - 2\varphi_2(\sigma_{bf} - \sigma_{s2})}{\sigma_{s2} + 2\sigma_{bf} + \varphi_2(\sigma_{bf} - \sigma_{s2})} \quad (13)$$

$$\frac{\sigma_{bf}}{\sigma_f} = \frac{\sigma_{s1} + 2\sigma_f - 2\varphi_1(\sigma_f - \sigma_{s1})}{\sigma_{s1} + 2\sigma_f + \varphi_1(\sigma_f - \sigma_{s1})} \quad (14a)$$

$$\text{Thermal conductivity } \frac{k_{hmf}}{k_{bf}} = \frac{2k_{bf} + k_{s2} - 2\varphi_2(k_{bf} - k_{s2})}{2k_{bf} + k_{s2} + \varphi_2(k_{bf} - k_{s2})} \quad (14b)$$

$$\frac{k_{bf}}{k_f} = \frac{2k_f + k_{s1} - 2\varphi_1(k_f - k_{s1})}{2k_f + k_{s1} + \varphi_1(k_f - k_{s1})} \quad (15)$$

Where σ_{s1} and σ_{s2} are electrical conductivities of $MnZnFe_2O_4$ and Fe_3O_4 nps respectively, σ_f is the electrical conductivity of $C_2H_2F_4$, k_{s1} and k_{s2} are the thermal conductivities of $MnZnFe_2O_4$ and Fe_3O_4 nps respectively, k_f is the thermal conductivity of $C_2H_2F_4$, φ_1 and φ_2 are solid volume fractions of $MnZnFe_2O_4$ and Fe_3O_4 nps respectively.

The transformations used [22]:

$$\left. \begin{aligned} w &= \frac{U_0 z}{l} f'(\eta), \quad u = -\frac{R}{r} \sqrt{\frac{v_f U_0}{l}} f(\eta), \\ \theta(\eta) &= \frac{T - T_\infty}{T_f - T_\infty}, \quad \eta = \frac{r^2 - R^2}{2R} \sqrt{\frac{U_0}{v_f l}} \end{aligned} \right\} \quad (16)$$

$$\text{In this case, } T = T_\infty[1 + (\theta_r - 1)\theta] \quad (17)$$

$$\text{where } \theta_r \left(= \frac{T_f}{T_\infty} > 1 \right)$$

Using (9)-(17) in (2), (3) and (4), we obtain

$$\begin{aligned} A \left[(1 + 2\varepsilon\eta)f''' + 2\varepsilon f'' + \Gamma_2 \left\{ \frac{1}{(1 + 2\varepsilon\eta)}(f'')^2 + 2(1 + 2\varepsilon\eta)(f'')^2 \right. \right. \\ \left. \left. - \Gamma_1[f^2 f''' + \varepsilon(1 + 2\varepsilon\eta)f^2 f'' - 2ff' f''] - f f'' - (f')^2 + \delta^2 \right. \right. \\ \left. \left. + \left(\frac{\sigma_{hmf}}{\sigma_f} \right) DM(\delta - 1 - \Gamma_1 f f'') \sin^2 \Omega = 0 \right. \right] \end{aligned} \quad (18)$$

$$\begin{aligned} E \left(\frac{k_{hmf}}{k_f} \right) [(1 + 2\varepsilon\eta)\theta'' + 2\varepsilon\theta'] + E \left(\frac{4}{3Nr} \right) \left[(1 + 2\varepsilon\eta)\theta'' + \varepsilon(\theta_r - 1)^3 \theta' \right] \\ - Pr f \theta' - Pr \lambda_E \left[f^2 \theta'' - f f' \theta' + \frac{2\varepsilon}{(1 + 2\varepsilon\eta)} f^2 \theta' \right] \\ + \left(\frac{\sigma_{hmf}}{\sigma_f} \right) EMB r \xi^2 (f' - \delta)^2 \sin^2 \Omega + FBr \xi^2 (1 + 2\varepsilon\eta)(f'')^2 = 0 \end{aligned} \quad (19)$$

$$\left. \begin{aligned} f'(\eta) &= 1 + \varepsilon \tau_1 f'' + \tau_2 [\varepsilon f'' + (1 + 2\varepsilon\eta)f''], \\ f(\eta) &= 0, \quad \frac{k_{hmf}}{k_f} \theta'(\eta) = -Bi[1 - \theta(\eta)]at\eta = 0 \\ f(\infty) &= 2\delta, f'(\infty) = \delta, \theta \rightarrow 0 \text{ as } \eta \rightarrow \infty \end{aligned} \right\} \quad (20)$$

$$\left. \begin{aligned} \varepsilon &= \frac{1}{R} \sqrt{\frac{l v_f}{U_0}}, \quad Nr = \frac{k^* k_f}{4\sigma^* T_\infty^3}, \quad \delta = \frac{U_\infty}{U_0}, \quad M = \frac{\sigma_f B_0^2 l}{\rho_f U_0}, \quad \theta_r = \frac{T_w}{T_\infty}, \quad Re = \frac{U_0 l}{v_f}, \\ Pr &= \frac{v_f}{\alpha_f}, \quad Ec = \frac{U_0^2}{T_\infty(\theta_r - 1)(C_p)_f}, \quad Br = Pr Ec, \quad \Gamma_1 = \frac{U_0 \lambda_1}{l}, \quad \Gamma_2 = \frac{U_0 \lambda_2}{l}, \\ \tau_1 &= \frac{L_1 U_0 r}{l v_f}, \quad \tau_2 = \frac{L_2 U_0}{l v_f}, \quad \lambda_E = \frac{\Lambda_e U_0}{l}, \quad \xi = \frac{x}{l}, \quad Bi = \frac{h_f}{k_f \left(\frac{r}{R} \sqrt{\frac{U_0}{v_f l}} \right)} \end{aligned} \right\} \quad (21)$$

Here, ε is the curvature parameter, Nr is the radiation parameter, δ is the stretching strength parameter, M is the Hartmann number,

θ_r is the temperature ratio parameter, Re is the Reynolds number, Pr is the Prandtl number, E_c is the Eckert number, Br is the Brinkman number, Γ_1 is the Deborah number for relaxation time, Γ_2 is the Deborah number for retardation time, τ_1 is the first order slip parameter, τ_2 is the second order slip parameter, λ_E is the thermal relaxation time factor, ξ is the non-dimensional ratio parameter, and Bi is the Biot number.

Here,

$$A = \frac{1}{(1 - \varphi_1)^{2.5} (1 - \varphi_2)^{2.5} \left\{ (1 - \varphi_2) \left[(1 - \varphi_1) + \varphi_1 \left(\frac{\rho_{s1}}{\rho_f} \right) \right] + \varphi_2 \left(\frac{\rho_{s2}}{\rho_f} \right) \right\}} \quad (22)$$

$$D = \frac{1}{(1 - \varphi_2) \left[(1 - \varphi_1) + \varphi_1 \left(\frac{\rho_{s1}}{\rho_f} \right) \right] + \varphi_2 \left(\frac{\rho_{s2}}{\rho_f} \right)} \quad (23)$$

$$E = \frac{1}{(1 - \varphi_2) \left[(1 - \varphi_1) + \varphi_1 \left(\frac{(\rho_{cp})_{s1}}{(\rho_{cp})_f} \right) \right] + \varphi_2 \left(\frac{(\rho_{cp})_{s2}}{(\rho_{cp})_f} \right)} \quad (24)$$

$$F = \frac{1}{(1 - \varphi_1)^{2.5} (1 - \varphi_2)^{2.5} \left\{ (1 - \varphi_2) \left[(1 - \varphi_1) + \varphi_1 \left(\frac{(\rho_{cp})_{s1}}{(\rho_{cp})_f} \right) \right] + \varphi_2 \left(\frac{(\rho_{cp})_{s2}}{(\rho_{cp})_f} \right) \right\}} \quad (25)$$

3. Numerical procedure

The system of highly coupled non linear and non dimensional equations (18) and (19) along with the BCs (20) are solved by using vp4c routing along with shooting technique. The present problem is a BVP which initially converted into IVP and then solve numerically using bvp4c routine. Initially, the higher order derivative terms are written in the form of remaining lower orders as

$$f^{iv} = \frac{A(1+2\epsilon\eta)f''' + 2\epsilon A f'' + A \left(\frac{1}{1+2\epsilon\eta} \right) \Gamma_2 (f'')^2 + 2A(1+2\epsilon\eta) \Gamma_2 (f'')^2 - 4\epsilon A \Gamma_2 f f''' + A(1+2\epsilon\eta) \Gamma_2 f' f'' - \Gamma_2 f^2 f''' - \epsilon(1+2\epsilon\eta) \Gamma_2 f^2 f'' + 2\Gamma_2 f f' f'' - f f'' - (f')^2 + \delta^2 + \left(\frac{\sigma_{hmf}}{\sigma_f} \right) DM(\delta - 1 - \Gamma_2 f f'') \sin^2 \Omega}{A(1+2\epsilon\eta) \Gamma_2 f} \quad (26)$$

$$\theta'' = \frac{\text{Pr} f \theta' - \text{Pr} \lambda_E f f' \theta' - 2DE\epsilon \theta' - \frac{4}{3Nr} \epsilon E(\theta_r - 1)^2 \theta' - \frac{4}{Nr} \epsilon(1+2\epsilon\eta)(\theta_r - 1)^2 (\theta')^2 - \left(\frac{\sigma_{hmf}}{\sigma_f} \right) EMB r \Lambda^2 (f' - \delta)^2 - FBr \Lambda^2 (1+2\epsilon\eta) (f'')^2}{E(1+2\epsilon\eta) \left(\frac{k_{hmf}}{k_f} \right) + \frac{4}{3Nr} E(1+2\epsilon\eta) - \text{Pr} \lambda_E f^2 - \text{Pr} \lambda_E \left(\frac{2\epsilon}{1+2\epsilon\eta} \right) f^2} \quad (27)$$

equations (26) and (27) are written in the form of matrix and for that introduce a new variable

$$y = [y_1, y_2, y_3, y_4, y_5, y_6]^T = [f, f', f'', f''', \theta, \theta']^T \quad (28)$$

and written in the matrix form as

The BCs at the initial stage ($\eta = 0$) are chosen as the initial condition and other parameters considered as an unknown constants and hence the initial guess is written as

$$y(0) = \left[0, 1 + \epsilon \tau_1 S_1 + \tau_2 \{ \epsilon S_1 + (1 + 2\epsilon\eta) S_2 \}, S_1, S_2, S_3, -\frac{Bi}{\left(\frac{k_{hmf}}{k_f} \right)} (1 - S_3) \right] \quad (30)$$

where S_1, S_2, S_3 are unknowns.

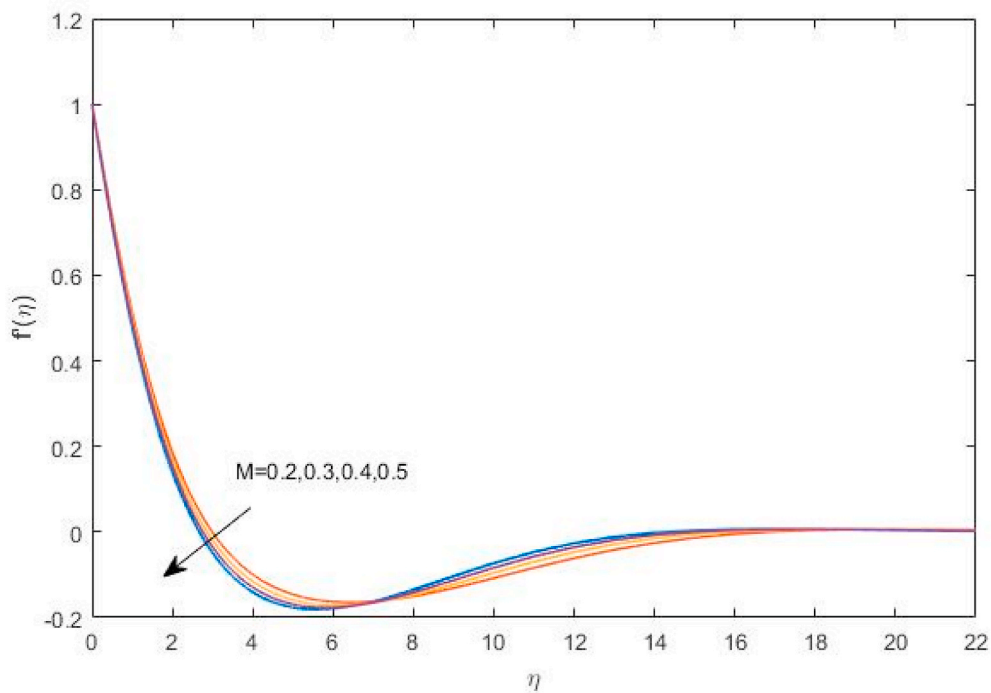
Solve the matrix equation (29) using the MATLAB's ODE solver with using the initial condition (30). Shooting method is employed to obtain the initial guess based on the linear interpolation of the boundary value specified on the initial mesh of equally spaced point. Continue the process until arrive at a convergence and we bound the convergence criteria as $\text{error}|E_i| < \text{tolerance} = 10^{-10}$ where errors are defined as

$$\begin{aligned} E_1 &= y_4(\infty, S_1) - f''(\infty) \\ E_2 &= y_5(\infty, S_2) - \theta'(\infty) \end{aligned} \quad (31)$$

Here, it is assumed the maximum value of η (η_∞) as 25. However, the system is converged before that value of η .

Table 1
Thermophysical properties of nanoparticles and water.

Properties	Fe_3O_4	$MnZnFe_2O_4$	$C_2H_2F_4$
$\rho(\text{kg}/\text{m}^3)$	5180	4700	1199.7
$c_p(\text{J}/\text{kgK})$	670	1050	1432
$k(\text{W}/\text{mK})$	9.7	3.9	0.0803



a. Behavior of M on $f'(\eta)$

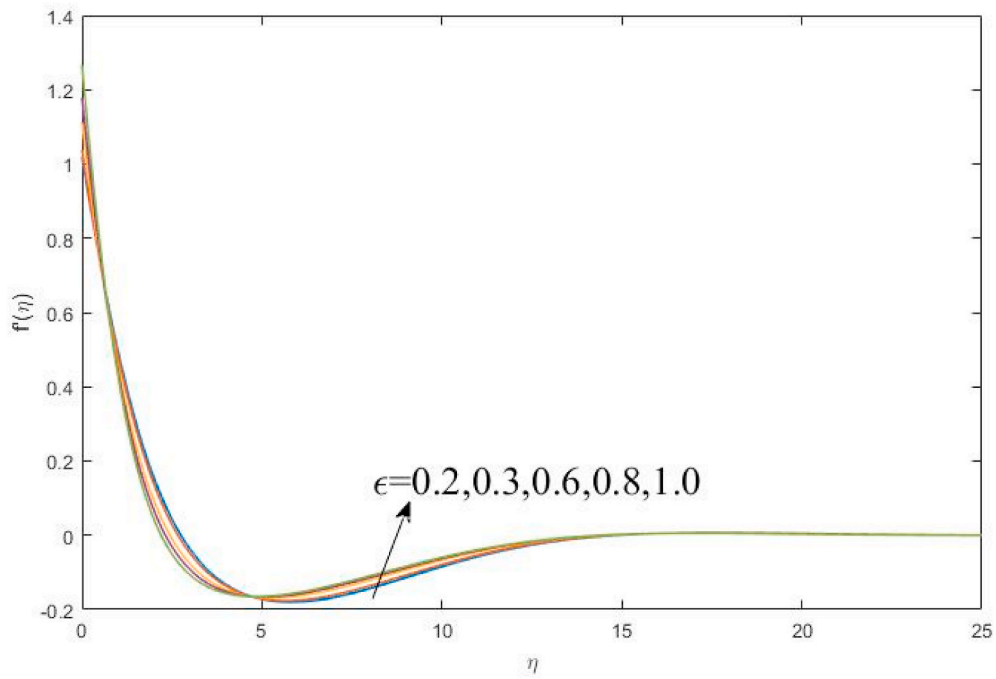
Fig. 2a. Behavior of $\text{Mon.}f'(\eta)$

$$\begin{pmatrix} y_1' \\ y_2' \\ y_3' \\ y_4' \\ y_5' \\ y_6' \end{pmatrix} = \begin{pmatrix} y_2 \\ y_3 \\ y_4 \\ \frac{A(1+2\varepsilon\eta)y_4 + 2\varepsilon Ay_3 + A\left(\frac{1}{1+2\varepsilon\eta}\right)\Gamma_2(y_3)^2 + 2A(1+2\varepsilon\eta)\Gamma_2(y_3)^2}{A(1+2\varepsilon\eta)\Gamma_2 y_1} \\ \frac{-4\varepsilon A\Gamma_2 y_1 y_4 + A(1+2\varepsilon\eta)\Gamma_2 y_2 y_4 - \Gamma_1(y_1)^2 y_4 - \varepsilon(1+2\varepsilon\eta)\Gamma_1(y_1)^2 y_3 + 2\Gamma_1 y_1 y_2 y_3}{y_5} \\ \frac{-y_1 y_3 - (y_2)^2 + \delta^2 + \left(\frac{\sigma_{hmf}}{\sigma_f}\right)DM(\delta - 1 - \Gamma_1 y_1 y_3)\sin^2 \Omega}{y_6} \\ \frac{\text{Pr } y_1 y_6 - \text{Pr } \lambda_E y_1 y_2 y_6 - 2DE\varepsilon y_6 - \frac{4}{3Nr}\varepsilon E(\theta_r - 1)^3 y_6 - \frac{4}{Nr}\varepsilon(1+2\varepsilon\eta)(\theta_r - 1)^2 (y_6)^2}{E(1+2\varepsilon\eta)\left(\frac{k_{hmf}}{k_f}\right) + \frac{4}{3Nr}E(1+2\varepsilon\eta) - \text{Pr } \lambda_E(y_1)^2 - \text{Pr } \lambda_E\left(\frac{2\varepsilon}{1+2\varepsilon\eta}\right)(y_1)^2} \\ -\left(\frac{\sigma_{hmf}}{\sigma_f}\right)EMBr\Lambda^2(y_2 - \delta)^2 - FBr\Lambda^2(1+2\varepsilon\eta)(y_3)^2 \end{pmatrix}, \quad (29)$$

4. Results and discussion

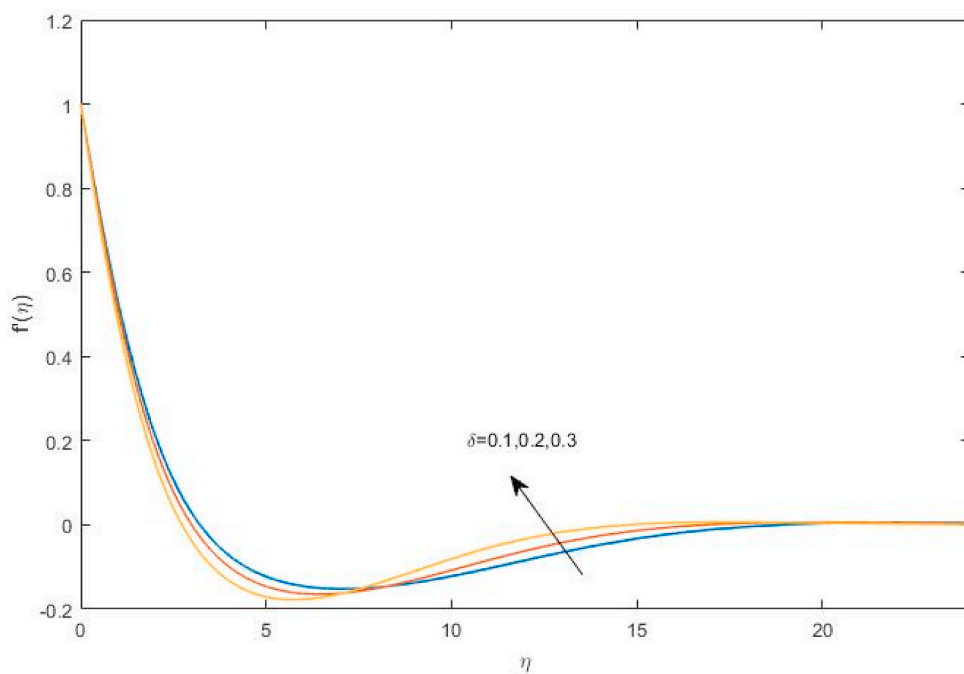
This section comprises of the graphical illustration of significant inferential parameters on velocity and temperature fields. The bvp4c routing has been executed for the elucidations of non dimensionalized ODEs. The values of the embedded parameters such as Hartmann number M , curvature parameter ε , stretching strength parameter δ , Deborah number for relaxation time Γ_1 , Deborah number for retardation time Γ_2 , first order slip parameter τ_1 , second order slip parameter τ_2 , angle of inclination Ω , thermal relaxation time factor γ_E , radiation parameter Nr , temperature ratio parameter θ_r and Biot number Bi used in the numerical computations are $M = 0.5$, $\varepsilon = 0.5$, $\delta = 0.2$, $\Gamma_1 = \Gamma_2 = 0.2$, $\tau_1 = \tau_2 = 0.2$, $\Omega = \frac{\pi}{6}$, $\gamma_E = 0.2$, $Nr = Bi = 0.2$, $\theta_r = 1.1$. Additionally, Table 1 is constructed for enlisting the thermophysical properties of hybrid nps and bf.

Physical aspects of velocity flow of Oldroyd B fluid with hybrid nanoparticle suspensions in response to varied pertinent parameters such as M , ε , δ , Γ_1 , Γ_2 , τ_1 , τ_2 , Ω have been highlighted in Fig. 2a-5b. Fig. 2a delineates a general trend that rise in M ($M = 0.1, 0.2, 0.3$) implicating intensification of magnetic field strength contributes to a decelerated non Newtonian flow $f'(\eta)$ within appreciable layers of BLR (up to $\eta \approx 7$). In this flow environment, Lorentz force due to the interaction of the conducting fluid and applied transverse magnetic field contributes resistance to the fluid movement and therefore causing velocity decrement. However, beyond $\eta \approx 7$ the flow pattern characterizes the opposite behavior. Augmented ε ($\varepsilon = 0.2, 0.3, 0.6, 0.8, 1.0$) is the core impetus that gives rise to growth of $f'(\eta)$ in the region (beyond $\eta = 5$). Accordingly, VBLT upsurges. However, within $\eta = 5$ (adjacent to the solid boundary of the stretched surface of the horizontal cylinder), reverse flow pattern is accomplished. This implicates that ε regulates the flow of Oldroyd B hybrid nf involving $MnZnFe_2O_4$ and Fe_3O_4 as nps and $C_2H_2F_4$ as bf. The enhanced curvature parameter indicates the reduction in the radius of the cylinder. Such reduction belittles the solid boundary and nanofluid contact area. The latest situation peters out the resistance to the motion of nps. Consequently, the velocity of the Oldroyd B hybrid nanofluid ameliorates. The impact of rise in δ ($\delta = 0.1, 0.2, 0.3$) on $f'(\eta)$ is portrayed in Fig. 3a. That logically means where the free stream value is less than the stretching value, $f'(\eta)$ and the related boundary layer thickness grows in the flow zone at few distance from the stretched surface of horizontal cylinder while a paradigm shift in the flow pattern (i.e., opposite flow pattern) develops adjacent to the stretched boundary. It is seen that $f'(\eta)$ ebbs gradually with incremented Γ_1 ($\Gamma_1 = 0.5, 0.8, 1.2$) (because Γ_1 corresponds to relaxation time λ_1) and finally disappears towards the ambient fluid as outlined in Fig. 3b. Physically for higher values of Γ_1 acts as core thrust making the viscous force upsurges which create more resistance to fluid movement and therefore the fluid velocity whittles down. On the contrary, progressive values of Γ_2 ($\Gamma_2 = 0.2, 0.3, 0.5$) spectacle descending trend in the region $5 \leq \eta \leq 15$ (because Γ_2 corresponds to retardation time λ_2). Beyond $\eta = 15$, Γ_2 has no influence on $f'(\eta)$ (See Fig. 4a). Higher values of Γ_2 lead to less viscous effect and therefore yield rise in velocity. Hike in first order slip (augmenting τ_1) reduces $f'(\eta)$. But one thing is that the portrayal is visualized in the region little distance away from the stretched surface of the cylinder (Fig. 4b). However, the opposite behavior of $f'(\eta)$ is scrutinized in response to varied τ_2 ($\tau_2 = 0.0, 1.0, 1.5, 2.0, 3.0$).



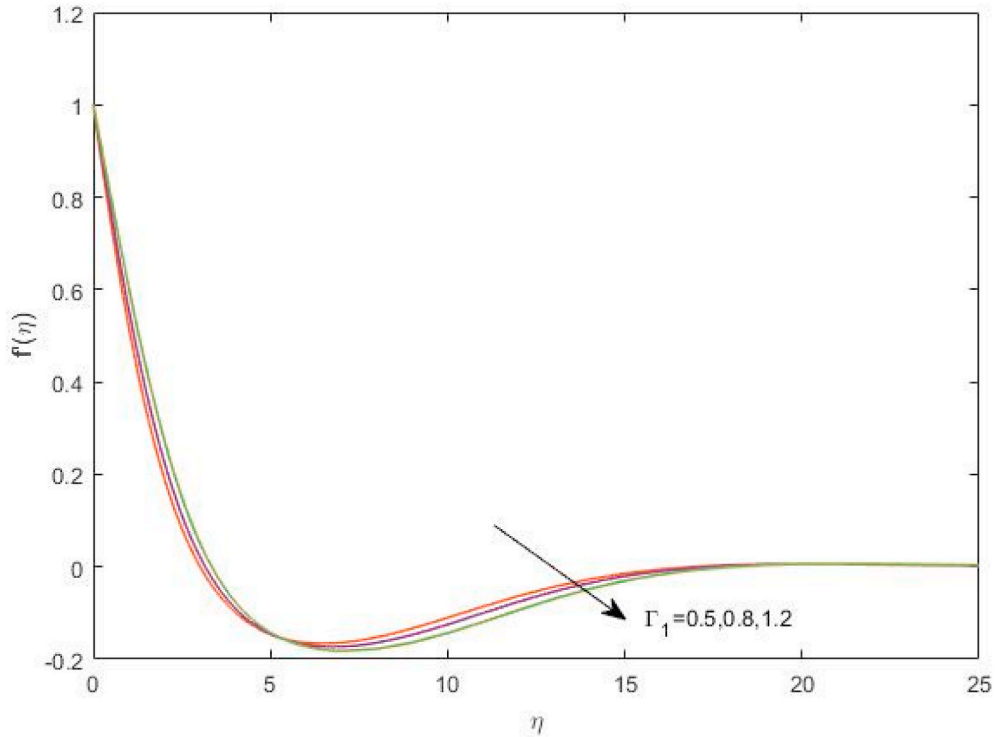
b. Behavior of ϵ on $f'(\eta)$

Fig. 2b. Behavior of ϵ on $f'(\eta)$



a. Behavior of δ on $f'(\eta)$

Fig. 3a. Behavior of δ on $f'(\eta)$



b. Behavior of Γ_1 on $f'(\eta)$

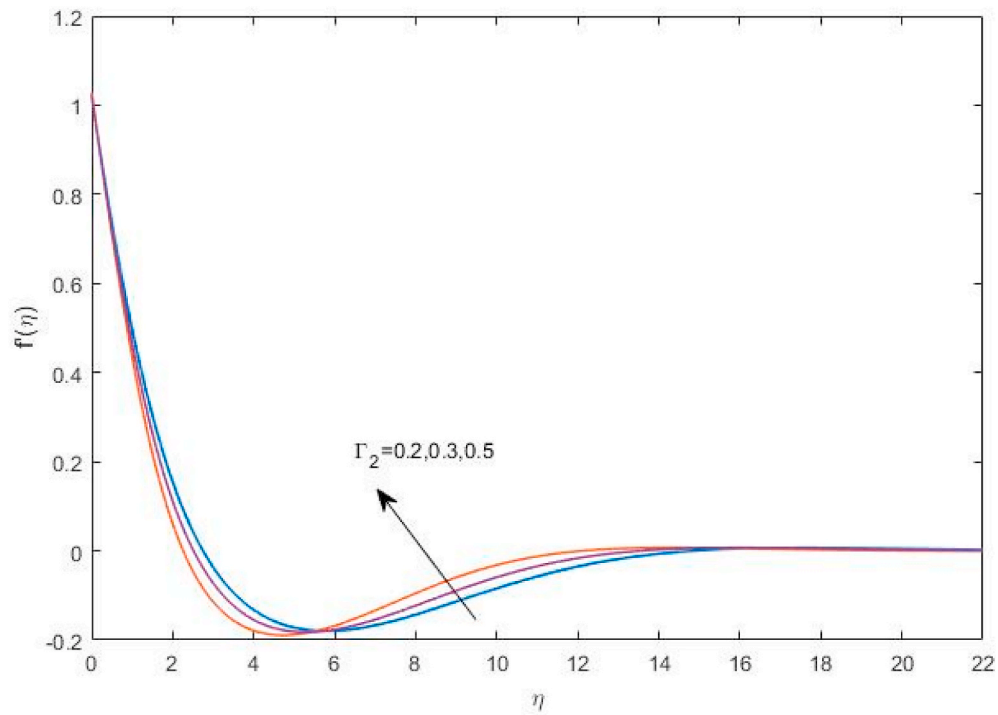
Fig. 3b. Behavior of Γ_1 on $f'(\eta)$

It is interesting to observe that $f'(\eta)$ was less when the flow was subject to second order slip (Fig. 5a). Most importantly, variation of $f'(\eta)$ with varied angle of inclination Ω of applied magnetic field is depicted in Fig. 5b. It is seen that as Ω ($\Omega = \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}$) enhances, $f'(\eta)$ whittles down within few layers of boundary layer (where the gravity effect upsurges for greater angle of inclination) and opposite flow pattern is the outcome in the beyond. Such behavior is due to the non linear nature of Oldroyd B fluid with hybrid nanostructure (see Fig. 6).

At the onset of thermal behavior discussion, Fig. 6a scrutinizes that the fluid temperature $\theta(\eta)$ and thermal thickness boost significantly due to rise in M ($M = 0, 1, 2, 3, 4, 5$). This typically happens when the retarding Lorentz force yields the availability of kinetic energy which in turn rises fluid temperature. Therefore, heat transfer amount declines for Oldroyd B hybrid nf. However, magnetic field strength has no impact on $\theta(\eta)$ in the region far away from the hot solid surface of the stretched cylinder. Analogous $\theta(\eta)$ profiles are obtained for Oldroyd B fluid comprising $MnZnFe_2O_4$ np only. In this case, $\theta(\eta)$ is less compared to $MnZnFe_2O_4 - Fe_3O_4 - C_2H_2F_4$ hybrid nf. Further, $\theta(\eta)$ feature is noticed to be analogous for Fe_3O_4 np, however, with less magnitude compared to Oldroyd B fluid $MnZnFe_2O_4$ np and Oldroyd B fluid with $MnZnFe_2O_4 - Fe_3O_4 - C_2H_2F_4$ hybrid nf (See Fig. 7a). Heightening δ ($\delta = 0.1, 0.2, 0.3$) upgrades temperature curve $\theta(\eta)$ and the related thermal thickness. The variation of $\theta(\eta)$ is marginal in increasing range of θ_r ($\theta_r = 1.1, 1.3$) subject to CCHF. This is the situation when T_∞ is less than T_w . In other words, the temperature on the surface of the stretched cylinder dominates that in the ambient fluid. Influence of λ_E on $\theta(\eta)$ is demonstrated in Fig. 8b. In this context, it is understood that uplifting the values of λ_E ($\lambda_E = 1.0, 1.5, 2.0$) i.e., with more and more CCHF into the flow domain, $\theta(\eta)$ and thermal thickness decline significantly (see Fig. 8) (see Fig. 7).

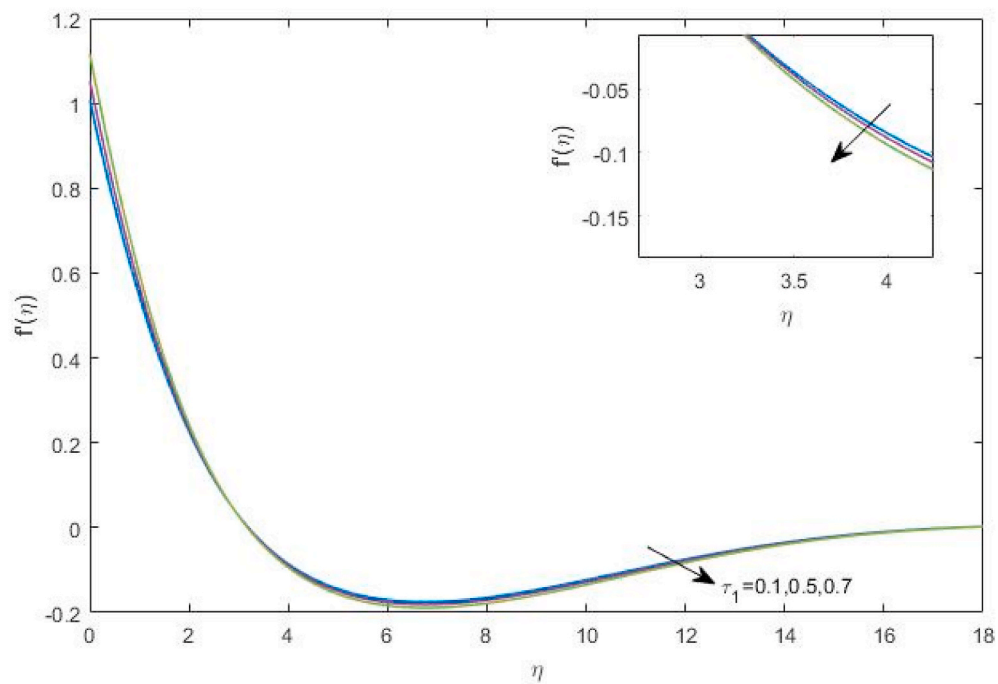
5. Major facts

The present study aims at the investigation of flow and heat transfer behavior of a non Newtonian fluid (Oldroyd B fluid) with hybrid nanostructure in the presence of externally inclined magnetic field. Cattaneo Christov heat flux model, non linear thermal



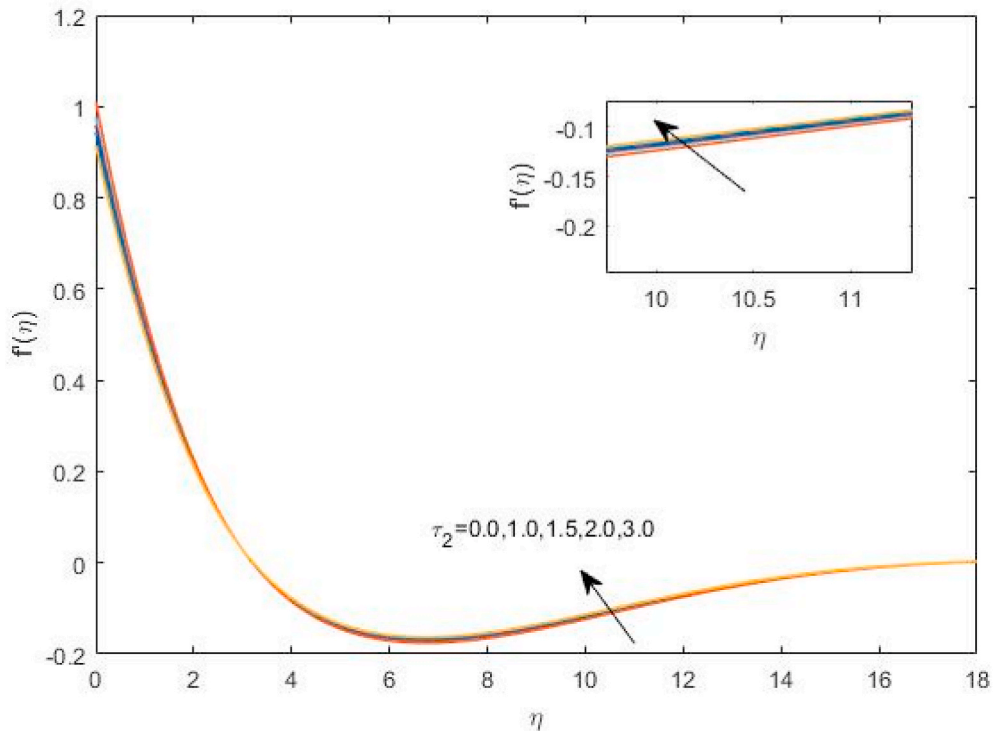
a. Behavior of Γ_2 on $f'(\eta)$

Fig. 4a. Behavior of Γ_2 on $f'(\eta)$



b. Behavior of τ_1 on $f'(\eta)$

Fig. 4b. Behavior of τ_1 on $f'(\eta)$



a. Behavior of τ_2 on $f'(\eta)$

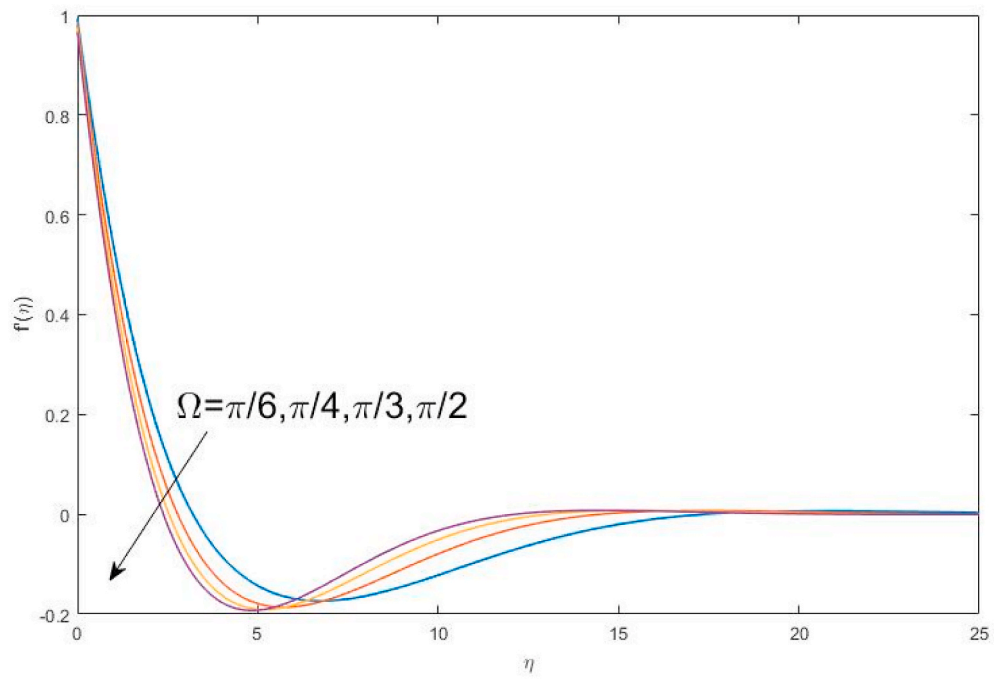
Fig. 5a. Behavior of τ_2 on $f'(\eta)$

radiation and viscous dissipation mechanisms are considered. Shooting technique is implemented to obtain the numerical solution. The study of flow and heat transfer of Oldroyd B fluid with hybrid nanostructure comprising $MnZnFe_2O_4$ and Fe_3O_4 as nanoparticles past a stretched cylinder is the novelty of the study. The relevant facts of this research are summarized as follows:

- > Elevated values of M results in diminution of $f'(\eta)$ and the related layer thickness.
- > Velocity field exhibits opposite behavior for Γ_1 and Γ_2 .
- > Rise in ϵ and δ favors to the escalation of $f'(\eta)$ and the related hydrodynamic layer thickness.
- > Greater Ω yields lower value of $f'(\eta)$. This implicates that $f'(\eta)$ has an inverse relation to Ω .
- > Augmented Me elevates $\theta(\eta)$ whatever the nature of Oldroyd B fluid may be. The temperature exhibits the same ascending trend for $MnZnFe_2O_4$ and Fe_3O_4 nanoparticles.
- > Heightening δ and θ_r uplifts $\theta(\eta)$. However, marginal variation of $\theta(\eta)$ due to rise in θ_r is marked.
- > Hiked λ_E whittles down $\theta(\eta)$ successfully leading to controlled HTR.

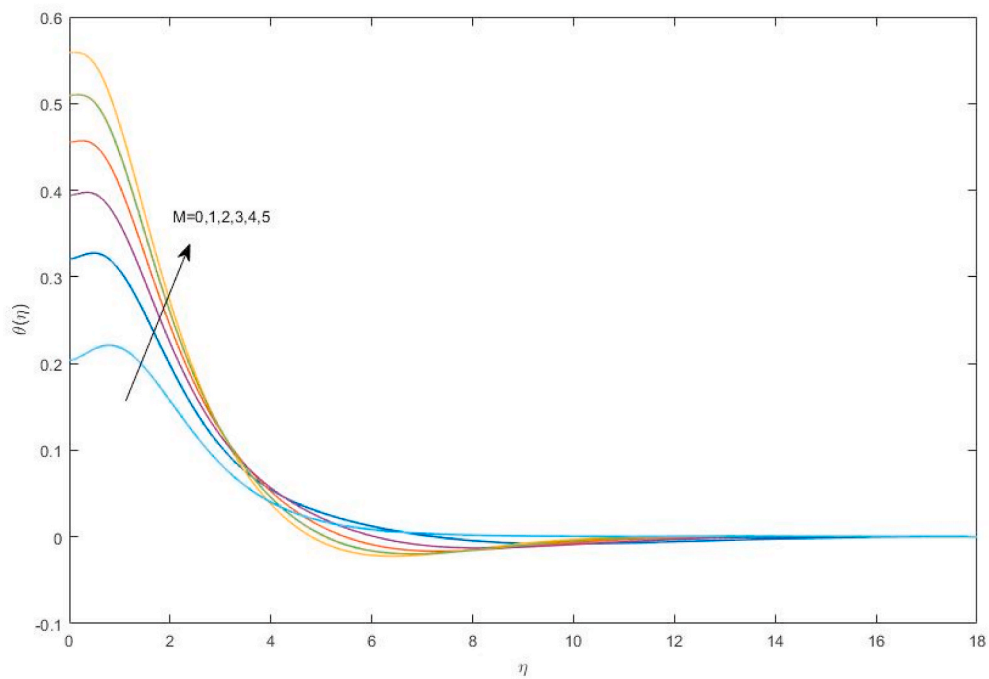
Author statement

M K Nayak: Conceptualization, Data curation, Formal analysis, Writing – original draft. V S Pandey: Conceptualization, Data curation, Formal analysis, Writing – original draft. S Shaw: Investigation, Methodology, Writing – original draft. O D Makinde: Investigation, Methodology, Writing – original draft. K M Ramada: Investigation, Methodology, Writing – original draft. Mouna Ben Henda: Investigation, Methodology, Writing – original draft. Iskander Tlili: Conceptualization, Data curation, Formal analysis, Writing – original draft.



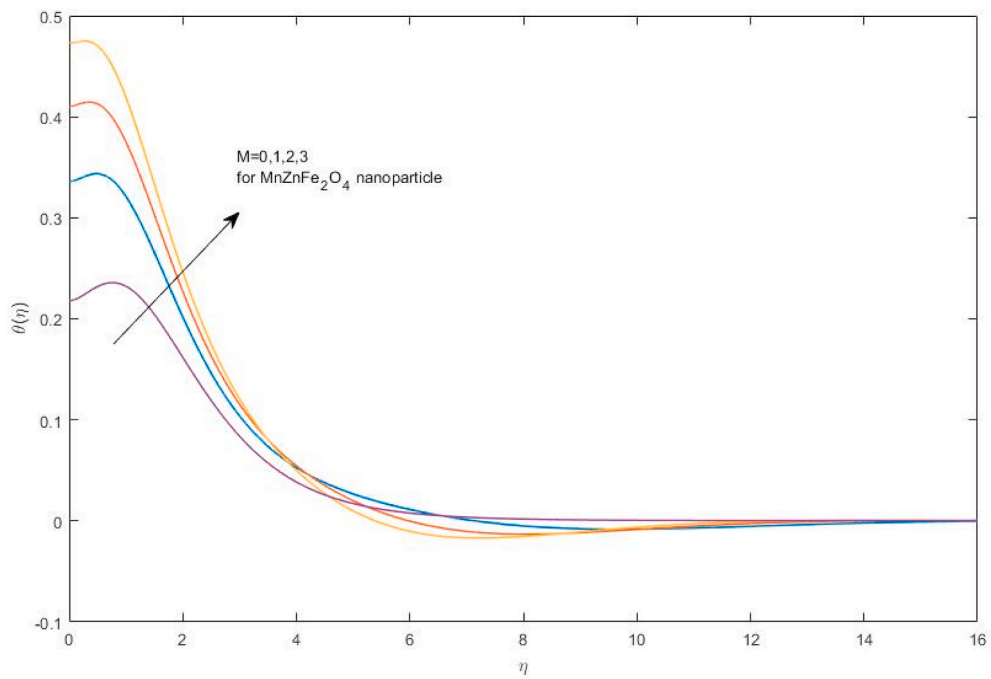
b. Behavior of Ω on $f'(\eta)$

Fig. 5b. Behavior of Ω on $f'(\eta)$



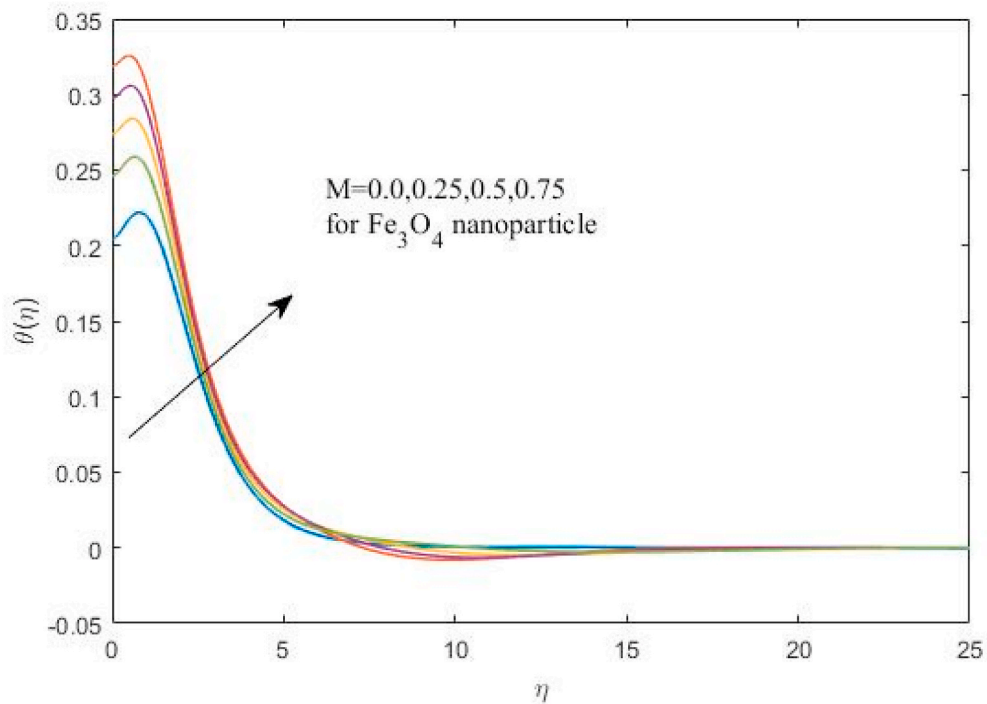
a. Behavior of M on $\theta(\eta)$

Fig. 6a. Behavior of M on $\theta(\eta)$



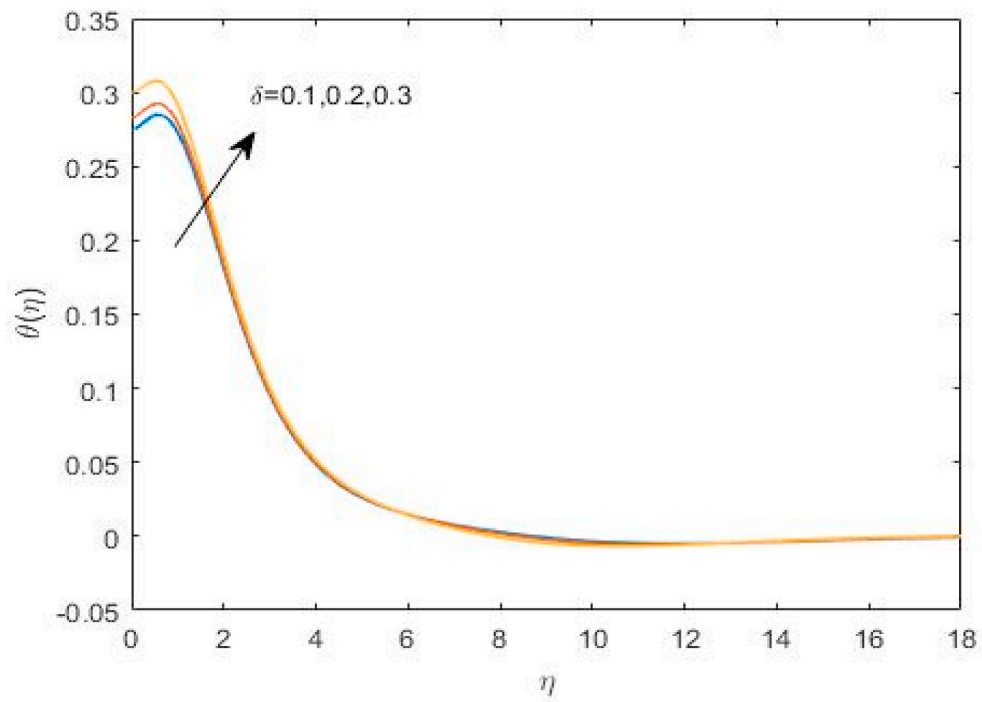
b. Behavior of M on $\theta(\eta)$ $MnZnFe_2O_4$ nanoparticle

Fig. 6b. Behavior of M on $\theta(\eta)$ $MnZnFe_2O_4$ nanoparticle.



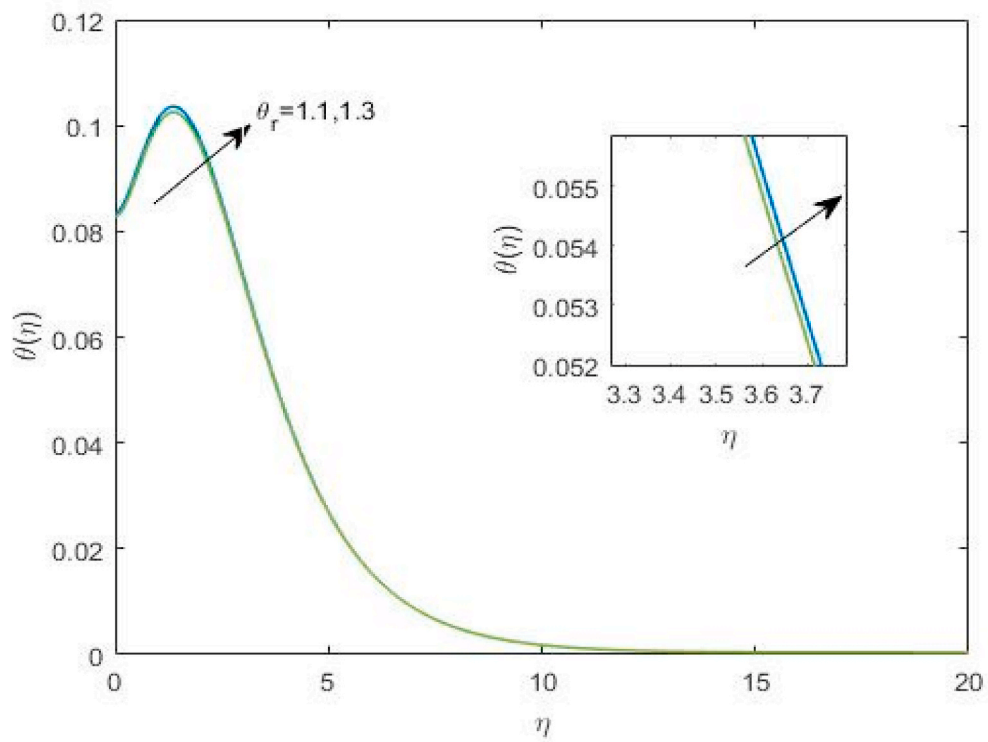
a Behavior of M on $\theta(\eta)$ Fe_3O_4 nanoparticle

Fig. 7a. Behavior of M on $\theta(\eta)$ Fe_3O_4 nanoparticle.



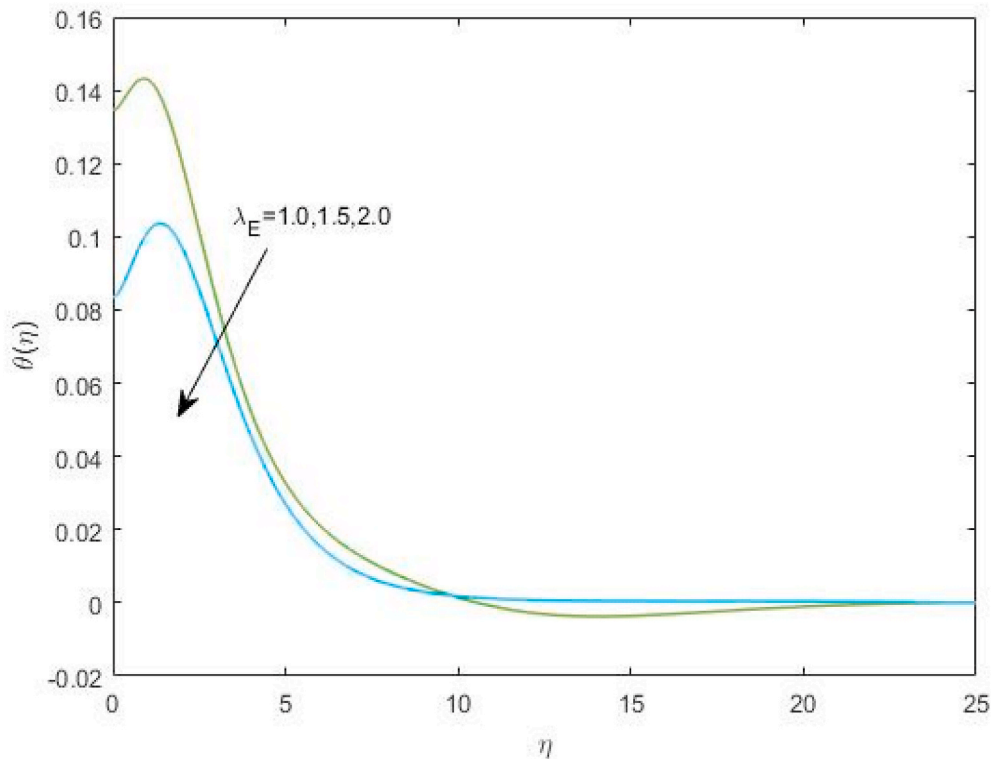
b. Behavior of δ on $\theta(\eta)$

Fig. 7b. Behavior of δ on $\theta(\eta)$



a. Behavior of θ_r on $\theta(\eta)$

Fig. 8a. Behavior of θ_r on $\theta(\eta)$



b. Behavior of λ_E on $\theta(\eta)$

Fig. 8b. Behavior of λ_E on $\theta(\eta)$

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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