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Influence of the thermal radiation on the bioconvection of gyrotactic microorganism contains nanofluid

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Abstract— **Microorganisms play a vital role to understand the ecological system and so it is very important to understand the behavior of microorganism due to different parameters. In the present paper, we implement a mathematical model to understand the influence of the thermal radiation on the gyrotactic microorganism imbedded in water based nanofluid flow over a wedge. The Brownian motion and thermoforetic effects take place due to nanofluid. The governing equations based on the conservation of mass, momentum, energy, nanoparticle concentration and concentration of gyrotactic micro-organism are simplified to a set of coupled, non-linear ordinary differential equations using the similarity transformations. The transformed equations are then solved comfortably using a numerical method namely fourth order Runge-Kutta method. The numerical results accomplished in thepresent investigation are validated and are in good agreement with the prevously published results of some noteworthy researchers found in literature. The important outcomes of the present study is that the skin friction and local Nusselt number are enhanced during the suction while the local Sherwood number and local density of motile microorganism decrease due to suction. Thermal radiation reduces the heat transfer rate near the surface, while it increases the skin friction, concentration of the nanoparticle and concentration of the gyrotactic microorganism near the surface.**

Keywords— Bioconvection; Gyrotactic microorganism; thermal radiation; nanofluid; wedge.

Introduction

Microorganisms are denser than the cell fluid and so they easily move or able to move without any external propulsion. An overturning instability caused by the microorganisms swimming to the upper surface of a fluid which has a lower density to that of the microorganisms and it forms bioconvection. The first detailed observations of bioconvection were discussed by Wager [1] but it was taken up seriously after the work of Platt [2], who invented the term `bioconvection'. Pedley et al. [3] have developed a mathematical model on the swimming microorganisms which are contained in suspension of infinite depth. The work extended by Hill et al. [4] for a suspension of the finite depth.

A combination of a nanofluid and bioconvection is attractive due to its application on microfluidic devices and bio-chips. A mathematical model on bioconvection in suspension in a water based nanofluid has been developed by Kuznetsov and Avramenko [5]. Shaw et al. [6] have discussed the nature of bioconvection in a non-Darcy porous medium saturated with nanofluid in the presence of gyrotactic microorganism. Later, Shaw et al. [7] extended their work by considering the influence of magnetic field, Soret effect on bioconvection.
Radiation is quite significant for the microorganisms since at elevated levels, radiation can have a range of harmful effects on microorganisms. Radiation on microorganisms has been used in municipal wastewater sludge treatment mainly by UV and gamma ray [8], consider the effects on ecosystem [9]. In many laboratory and experiments, radiation is used to detect microorganisms [10]. El-Registan et al. [11] studied the effect and functions of microorganism cultures oxidative stress caused by irradiation. They showed that biological systems are substantially different in their sensitivity to radiation that caused a substantial but not completely lethal effect. Microorganism activation by UV and ionizing radiation has been shown by [12]. Relative resistance of microorganisms to different kinds of radiation has been discussed by several researchers [13, 14].

Bhattyacharya et al. [15] have studied the boundary layer flow of nanofluid over a stretching sheet. In the present study, we have developed a mathematical model on the stagnation point flow past a stretching / shrinking wedge saturated in a suspension nanofluid containing gyrotactic microorganisms and water as base fluid. The influence of the suction / injection, magnetic field and angle of the wedge on the bioconvection have been presented and discussed through graphs and tables.

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Mathematical Formulations

We considered the Cartesian coordinate system (x, y) to define the axis of the wedge surface. x and y –axis are defined along the wedge surface and normal to the wedge surface, respectively. The stretching / shrinking velocity of the surface is linear and defined as $u_w(x) = U_w x$, where U_w is a constant, for stretching surface U_w is non-zero positive while for shrinking sheet it is non-zero negative. It is static when $U_w = 0$. Similarly, we have considered the ambient fluid moves linearly following the relation $u_e(x) = U_\infty x$, where U_∞ is a constant. Utilizing the Oberbeck-Boussinesq approximation, the continuity and momentum equation of the boundary layer flow can be written as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{2.1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = u_e\frac{du_e}{dx} + \nu\frac{\partial^2 u}{\partial y^2} + g\beta_0(T - T_\infty)\sin\frac{\Omega}{2}, \tag{2.2}$$

where the velocity components of the nanofluid defined by u and v along the x-axis and y-axis, respectively, viscosity of the fluid by μ, acceleration due to gravity by g, temperature by T, volumetric thermal expansion coefficient of the base fluid, the ambient temperature and the wedge angle defined by β_0, T_∞ and Ω, respectively.

The thermal energy equation is written as

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha_m\frac{\partial^2 T}{\partial y^2} + \tau D_B\frac{\partial T}{\partial \bar{y}}\frac{\partial C}{\partial \bar{y}} + \frac{\tau D_T}{T_\infty}\left(\frac{\partial \bar{T}}{\partial \bar{y}}\right)^2 + \frac{16\sigma T_\infty^3\sigma}{3k}\frac{\partial^2 T}{\partial y^2}, \tag{2.3}$$

where, the Brownian and thermophoretic diffusion coefficient are defined as D_B and D_T, respectively, α_m is the effective thermal diffusivity of the porous medium and $\tau = (\rho c)_p / (\rho c)_f$, where $(\rho c)_p$ and $(\rho c)_f$ are the volumetric heat capacities for nanoparticle material and fluid, respectively.

The solute concentration equation is written as

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D_B\frac{\partial^2 C}{\partial y^2} + \frac{D_T}{T_\infty}\frac{\partial^2 T}{\partial y^2}, \tag{2.4}$$

While, the equation for the concentration of gyrotactic microorganism is defined as

$$u\frac{\partial n}{\partial x} + v\frac{\partial n}{\partial y} + \frac{\partial}{\partial y}\left(n\frac{bW_c}{\Delta C}\frac{\partial C}{\partial y}\right) = D_n\frac{\partial^2 n}{\partial y^2}, \tag{2.5}$$

where $\Delta C = C_w - C_\infty$ represents the characteristic nanoparticle volume fraction respectively, and C_w and C_∞ are related to the nanoparticle concentration at the wall and the ambient concentration of nanoparticle. Other parameters b and W_c are the chemotaxis constant and the maximum cell swimming speed, respectively. The product bW_c is assumed to be constant.

The corresponding boundary conditions are

$$\begin{aligned} &v = v_0, u = u_w(x) = U_w x, T = T_w, C = C_w, \\ &\qquad n = n_w \text{ at } y = 0, \\ &u = u_e(x) = U_\infty x, T = T_\infty, C = C_\infty, \\ &\qquad n = n_\infty \text{ as } y \to y_\infty \end{aligned} \tag{2.6}$$

where n_w is the density of motile microorganism at the plate surface. The ambient values of the density of motile microorganism are denoted by n_∞. We introducing the stream function ψ such that $u = \partial\psi/\partial y$ and $v = -\partial\psi/\partial x$ and applying the following similarity transformations

$$\begin{aligned} &\psi = \sqrt{U_\infty \nu}\, x f(\eta), \eta = \sqrt{\frac{U_\infty}{\nu}}\, y, \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \\ &\phi(\eta) = \frac{C - C_\infty}{\Delta C}, \chi(\eta) = \frac{n - n_\infty}{\Delta n}, \end{aligned} \tag{2.7}$$

where $\Delta n = n_w - n_\infty$. Governing equations after using the above mentioned similarity transformation is written as

$$f''' + ff'' + 1 - f'^2 + M\theta\sin\left(\frac{\Omega}{2}\right) = 0, \tag{2.8}$$

$$(1+\gamma)\theta'' + Pr\left(f\theta' + Nb\theta'\phi' + Nt\theta'^2\right) = 0, \tag{2.9}$$

$$\phi'' + Lef\phi' + \frac{Nt}{Nb}\theta'' = 0, \tag{2.10}$$

$$\chi'' + Scf\chi' - Pe\left(\phi'\chi' + \chi\phi''\right) - Pe_0\phi'' = 0, \tag{2.11}$$

subject to the boundary conditions

$$\begin{aligned} &f = f_w, f' = \lambda, \theta = 1, \phi = 1, \chi = 1 \text{ at } \eta \to 0, \\ &f' \to 1, \theta \to 0, \phi \to 0, \chi \to 0 \text{ as } \eta \to \infty, \end{aligned} \tag{2.12}$$

where the prime denotes differentiation with respect to η, $M = \frac{Gr_x}{Re_x^2}$ is the convection parameter, $Gr_x = \frac{g\beta_0(T_w - T_\infty)x}{\nu^2}$ is the Grassof number, $Re_x = \frac{U_\infty x}{\nu}$ is the Reynolds number, $\gamma = \frac{16T_\infty^3\sigma}{3k\nu}$ is the radiation parameter, $Pr = \frac{\nu}{\alpha_m}$ is the Prandtl number, $Nb = \frac{\tau D_B \Delta C}{\alpha_m}$ is the Brownian motion parameter, $Nt = \frac{\tau D_t \Delta T}{\alpha_m T_\infty}$ is the thermophoresis parameter, $Le = \frac{\nu}{D_B}$ is the Lewis number, $Sc = \frac{\nu}{D_n}$ is the Schimdt number, $Pe = \frac{bW_c}{D_n}$ is the bioconvection Peclet number, $Pe_0 = \sigma Pe$ with a dimensional constant $\sigma = \frac{n_\infty}{\Delta n}$, $f_w = -v_0/\sqrt{U_\infty \nu}$ is the suction ($f_w > 0$) or injection ($f_w < 0$) parameter and $\lambda = U_w/U_\infty$ is the stretching λ>0) and shrinking (λ<0) parameter.

The skin friction (C_f), local Nusselt number (Nu_x) the local Sherwood number (Sh_x) and the local density number of motile microorganisms (Nn_x) are defined as

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$$C_f = \frac{x\tau_w}{\rho u_e^2},\ Nu_x = \frac{xq_w}{k\Delta T},$$
$$Sh_x = \frac{xq_m}{D_B \Delta C},\ Nn_x = \frac{xq_n}{D_n \Delta n}, \tag{2.13}$$

where

$$\tau_w = \mu\left(\frac{\partial u}{\partial y}\right)_{y=0},\ q_w = -k\left(\frac{\partial T}{\partial y}\right)_{y=0},\ q_m = -D_B\left(\frac{\partial C}{\partial y}\right)_{y=0},$$
$$q_n = -D_n\left(\frac{\partial n}{\partial y}\right)_{y=0}.$$

Using the above non-dimensional and similarity transformation we get,

$$Ra_x^{1/2} C_f = f''(0),\ Re_x^{-1/2} Nu_x = -\theta'(0),$$
$$Re_x^{-1/2} Sh_x = -\phi'(0),\ Re_x^{-1/2} Nn_x = -\chi'(0), \tag{2.14}$$

where $Re_x = u_e(x)x/\nu$ is the local Reynolds number.

Results and Discussion

The system of governing equations (2.8)-(2.11) subject to the boundary conditions (2.12) was solved using the fourth order Range-Kutta and shooting method. In the present problem, we have calculated a unique solution only in the range $\lambda > \lambda_c$, where λ_c is a critical value of λ which depends on other parameters. There is no solution for $\lambda < \lambda_c$. In some case, we have expected a dual solution appears for both stretching and shrinking wedges which earlier observed by Merkin [16]. In general, the first solution is stable and physically realizable, while the second solution is unstable and it came due the mathematical establishment. Although the second solution seems to be deprived of physical significance, it is more interesting to study the stability in nonlinear differential equation theory since a familiar equation may reappear in some other situations where the corresponding solution could have a more realistic meaning. To reduce the maximum error, we have considered the value Nb and Nt each taking values in the range [0, 0.5] (as recommended by Nield and Kuznetsov [17]) and the range for Pe is [0, 5] (Kuznetsov [18]).

The numerical results have been compared with earlier works of those reported by Wang [19] and Bachok et al. [20] and displayed in Table 3.1. The comparison of results gave a very good agreement there by lending confidence as to the accuracy of the present method.

The velocity and temperature profiles due to different wedge angles are shown in Fig. 3.1. The wedge angle highly influences the velocity profiles rather that temperature profiles. The momentum boundary layer thickness enhances with increase in wedge angle. The thermal boundary layer thickness reduces with an increase in wedge angle. We get an opposite phenomena for the second solution.

The radiation parameter appears directly in the energy equation and so it is obvious that the temperature profiles would be susceptible to any changes in the radiation parameter. This clearly is the case as shown in Fig. 3.2. The thermal boundary layer thickness increases with introduction of thermal radiation in the system and continuously increases with increase of the thermal radiation parameter. The motile microorganisms are living creatures and cannot withstand heating above a certain temperature. So it becomes necessary to control radiation for the microorganisms to survive.

The skin friction, local Nusselt number, local Sherwood number and local density of the motile microorganism for different suction parameter values are shown in Fig. 3.3. Dual solutions are obtained for all value of λ. The first solution is shown in solid line and second solution in a dashed line. The solutions coincide at some critical point (λ_c), which depends on the suction parameter. For suction parameter values 3, 3.5 and 4, the values of λ_c are -4.4, -5.309 and -6.299. A loop appears in the first and second solutions for all the cases. The upper part of the loop defines the first solution while the lower part defines the second solution. A similar phenomena was observed (by Bhattacharyya [15]) in the absence of bio-convection.

Suction aids the function of bringing large quantities of ambient fluid near to the immediate neighbourhood of the wedge which enhanced the shear stress at the surface. Physical significance of the negative sign is that the surface employs a drag force on the fluid. It is evident that the skin friction coefficient is zero when λ=1 irrespective of the value of the other governing parameters. It is clear from the boundary condition that for λ=1, the fluid and surface of the wedge move with the same velocity and hence it not producing any shear stresses at the surface. As a consequence, the presence of fluid at near-ambient temperature close to the sheet and enhanced heat transfer rate. It is also obvious that the Sherwood number enhanced in the presence of the nanoparticles because of the contributions of the Brownian motion, thermophoresis and the buoyant motion prompted by the difference in the densities of the nanoparticles in the base fluid. But with an increase in the suction parameter, the nanoparticle Sherwood number decreases as shown in Fig. 3.4(a). This is because an increase in suction reduces the nanoparticle flux in the convective flow field. The density of the motile microorganism near the surface reduces when there is an increment in the suction parameter (see Fig. 3.4(b)).

The effect of convection and radiation parameters on the skin friction, heat transfer rate, mass transfer rate and local density of the motile microorganism are displayed through Table 3.2 by considering different values of the wedge angle. The second solutions are given in brackets. It is evident that the skin friction, local Nusselt number, local Sherwood number and local density of the motile microorganism increase as the wedge angle increases for both the first and the second solutions. A similar result is appearing for the convection parameter. This is because with an increase in the convection parameter, both the convective heat and mass transfer rates increase. This influences and enhances the skin friction, heat transfer rate and mass transfer rate at the surface. The skin friction increases since radiation increases the velocity of the fluid and the shear stress at the wall. Similarly, the local Sherwood number and density of the motile microorganisms

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increase with radiation. However the Nusselt number decreases as the temperature of the fluid increases due to the radiation parameter.

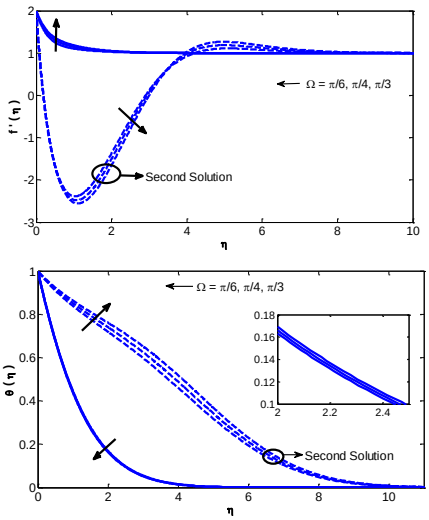

Fig 3.1: Effect of wedge angle on (a) velocity and (b) temperature with $M = 5$, $\gamma = 1$, $Pr = 6.2$, $Nb = Nt = 0.1$, $Le = 2$, $Sc = 1$, $Pe = 1$, $\sigma = 1$, $fw = 3$, $\lambda = 2$.

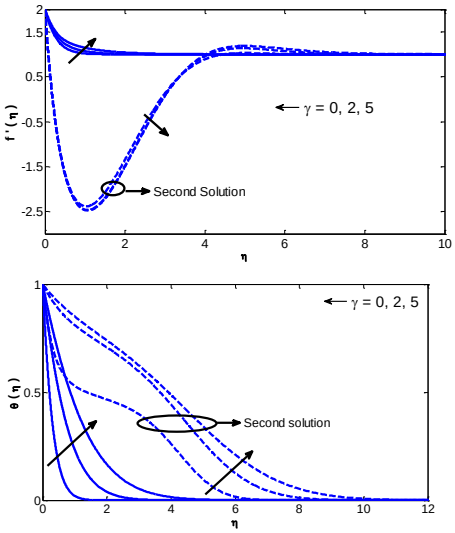

Fig 3.2: Effect of radiation parameter on (a) velocity and (b) temperature with $M = 5$, $\Omega = \pi/4$, $Pr = 6.2$, $Nb = Nt = 0.1$, $Le = 2$, $Sc = 1$, $Pe = 1$, $\sigma = 1$, $fw = 3$, $\lambda = 2$.

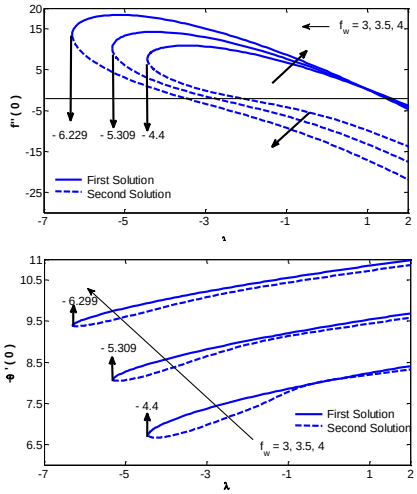

Fig 3.3: Effect of the suction / injection parameter on (a) skin friction coefficient (b) local Nusselt number .vs. λ with $M = 5$, $\Omega = \pi/4$, $\gamma = 1$, $Pr = 6.2$, $Nb = Nt = 0.1$, $Le = 2$, $Sc = 1$, $Pe = 1$, $\sigma = 1$.

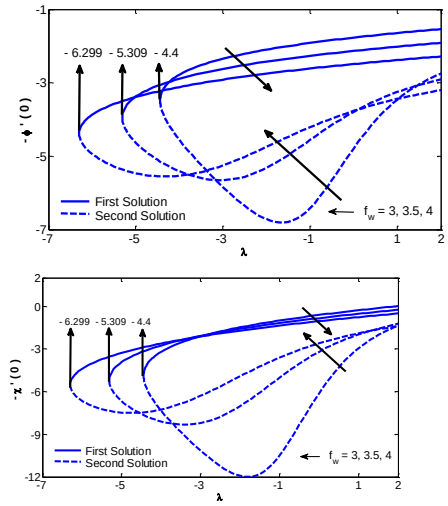

Fig 3.4: Effect of the suction / injection parameter on (a) local Sherwood number and (b) local density of the motile microorganism .vs. λ with $M = 5$, $\Omega = \pi/4$, $\gamma = 1$, $Pr = 6.2$, $Nb = Nt = 0.1$, $Le = 2$, $Sc = 1$, $Pe = 1$, $\sigma = 1$.

Conclusions

We have studied the stagnation point flow over a stretching / shrinking wedge with a nanofluid containing motile micro-organisms with water as base fluid. The influence of the suction parameter, wedge angle, convection parameter, radiation and stretching / shrinking parameter are discussed. The present results are compared with those of well-established

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publications available in the literature with some limiting values. Dual solutions were observed for both stretching and shrinking wedges. It was observed that the velocity and density of the motile microorganisms increase with suction for the case of a stretching wedge while they decrease in the case of a shrinking wedge. Temperature and nanoparticle concentration decrease with increase in suction parameter for both stretching and shrinking sheets. Radiation enhances both the velocity and temperature of the microorganism-rich nanofluid. A loop like graph observed due to the existence of the dual solution for all the physical parameters e.g., skin friction, local Nusselt number, local Sherwood number and local density of the motile microorganisms. An increase in the suction parameter, increases the skin friction and the local Nusselt number while the local Sherwood number and local density of the motile microorganism decreases

Table 3.1: Comparison of the present results with existing literature for different stretching parameter

λ	Wang [19]		Bachok et al. [20]		Present	
					$M = \lambda = f_w = Nb = Nt = 0$	
	$f_1''(0)$	$f_2''(0)$	$f_1''(0)$	$f_2''(0)$	$f_1''(0)$	$f_2''(0)$
-0.25	1.40224		1.4022408		1.40267084	
-0.5	1.49567		1.4956698		1.49566977	
-0.75	1.48930		1.4892983		1.48929824	
-1	1.32882		1.3288170	0	1.32881688	0
-1.1			1.1866805	0.0492290	1.18668029	0.04922896
-1.15	1.08223	0.116702	1.0822315	0.1167022	1.08223117	0.11670214
-1.2			0.9324739	0.2336497	0.93247336	0.23364973
-1.2465	0.55430		0.5842956	0.5542825	0.58428167	0.55429619
-1.24657			0.5639733		0.57452578	0.56401249

Table 3.2: Values of $f''(0)$, $-\theta'(0)$, $-\phi'(0)$, $\chi'(0)$ for different value of Ω, M, γ when $Pr = 6.2$, $Nb = Nt = 0.1$, $Le = 1$, $Sc = 1$(Second solution in next row)

Ω	M	γ	$f''(0)$	$-\theta'(0)$	$-\phi'(0)$	$-\chi'(0)$
$\pi/6$	5	1	-3.83595048	8.19597689	-2.69497850	-2.27970326
			-13.87873953	8.15350761	-3.82719202	-3.49750774
$\pi/4$			-3.77353449	8.19643314	-2.69433385	-2.27821274
			-13.80858003	8.15376909	-3.82316528	-3.48897130
$\pi/3$			-3.71444108	8.19686489	-2.69372413	-2.27680289
			-13.74218910	8.15401639	-3.81936286	-3.48091952
$\pi/4$	0	1	-3.96645204	8.19502220	-2.69632850	-2.28282433
			-14.02555090	8.15295989	-3.83564564	-3.51546132
	2		-3.88925621	8.19558705	-2.69552959	-2.28097736
			-13.93868770	8.15328406	-3.83063947	-3.50482406
	5		-3.77353449	8.19643314	-2.69433385	-2.27821274
			-13.80858003	8.15376909	-3.82316528	-3.48897130
$\pi/4$	5	0	-3.85443823	15.55408580	-11.70934200	-20.24130228
			-13.91087947	15.80353636	-12.75476392	-21.30312166
		2	-3.70574837	5.73764038	0.31873382	3.71702098
			-13.70229122	5.47104505	-1.02297038	2.02360466
		5	-3.54195831	3.17148490	3.35839299	9.75813296
			-13.91789195	1.55066582	1.65752706	6.55224864

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